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SOM

Strength of Material

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CHAPTER - 1

(STRESS AND STRAIN)

Strength of Material :- SOM is that branch of science which deals with the behaviour of deformable body which are subjected to stress and strain

→ Difference between SOM and Engineering mechanics is

SOM

- (1) SOM deals with deformable body
- (2) Energy will store in body i.e. strain energy

Mechanics

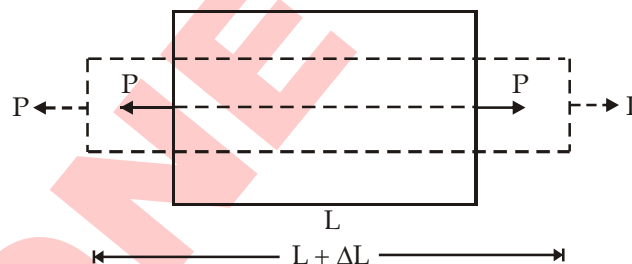
- (1) Mechanics deals with rigid body
- (2) Energy will not store in body

Strain :- Strain is a measure of deformation which represents relative displacement with respect to original length or natural length [separation between molecules of body]

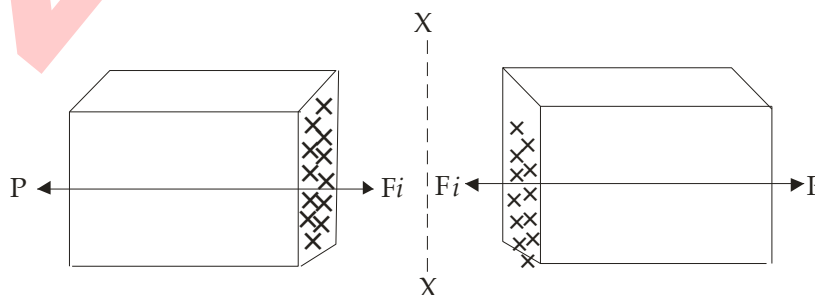
→ Strain is represented as the ratio of change in dimension to the original dimension if load is acting along the length of body then

$$\text{Strain} = \frac{\text{Final length} - \text{Original length}}{\text{Original length}}$$

$$\text{Strain} = \frac{\Delta L}{L}$$



Stress :- When a body is subjected to external force then deformation tendency will develop in a body. To resist this deformation molecules of body will provide an internal resistive force, this internal resistive force per unit area is known as stress.



Free Body Diagram

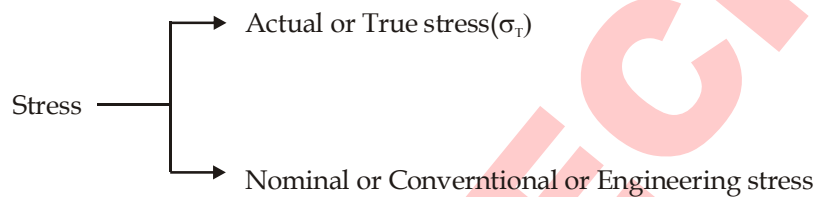
$$\text{Stress } (\sigma) = \frac{\text{Internal Resistance Force}}{\text{Original Cross Sectional Area}} = \frac{F_i}{A}$$

At equilibrium : $f_i = P$

Hence $\sigma = \frac{F_i}{A} = \frac{P}{A}$

→ Stress is a tensor quantity which means it is represented with the help of direction, magnitude and plane of a body.

Type of Stress :-



Actual stress:-

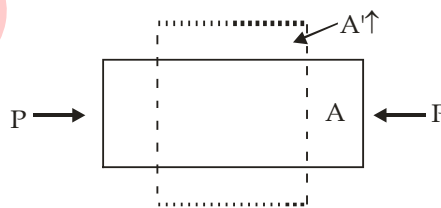
- It is defined as the load divided by actual area.
- It is represented with the help of σ_T
- For tension case $\sigma_{\text{Actual}} > \sigma_E$

For tension case $A_{\text{actual}} < A_{\text{initial}}$

Engineering stress:-

- It is defined as the load divided by original area.
- It is represented with the help of σ_E
- For compression case $\sigma_{\text{Actual}} < \sigma_E$

For compression case $A_{\text{actual}} > A_{\text{initial}}$



Relation between True stress and Engineering stress

$$\sigma_T = \sigma_{\text{Engg}} (1 \pm \epsilon) \quad \text{For tension (+) sign is used and vice versa.}$$

Relation between True strain and Engineering strain

$$\epsilon_{\text{True}} = \ln(1 + \epsilon_{\text{Engg}})$$

Engineering Stress - Strain curve for mild steel

OR

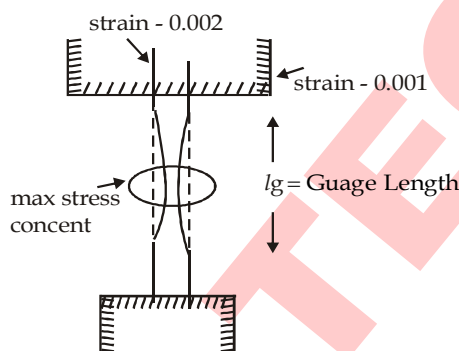
Tensile Test Diagram

- This test is always performed on mild steel because it exhibit all phenomenon like yielding, ultimate etc.
- For this test dog bone or dumble shape specimen is used.
- Specimen may be circular or non circular.

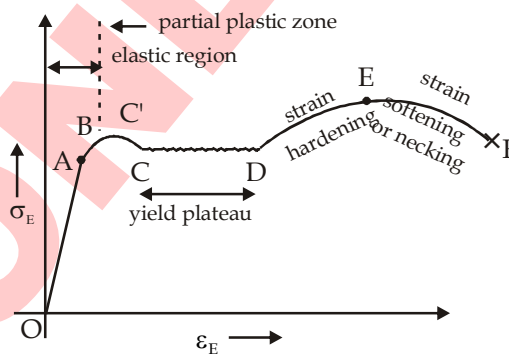
For circular section gauge length = $5.D_0$

For rectangular section gauge length = $5.65 \sqrt{A_0}$

- Deformation in gauge length is measured with the help of Extensometer.
- This test is performed on Universal Testing Machine (UTM).



Stress-Strain Curve



where

Point A (**Proportional Limit**) :- Hook's law is valid upto proportional limit where stress $\propto$ strain

Point B (**Elastic Limit**) - Material will regain its original shape after unloading upto this point

Point C' (**Upper Yield Point**) :- Upper Yield point depends on rate of loading

Point C (**Lower Yield Point**)

C-D Yield Plateau - In this zone strain will occur at constant stress or load

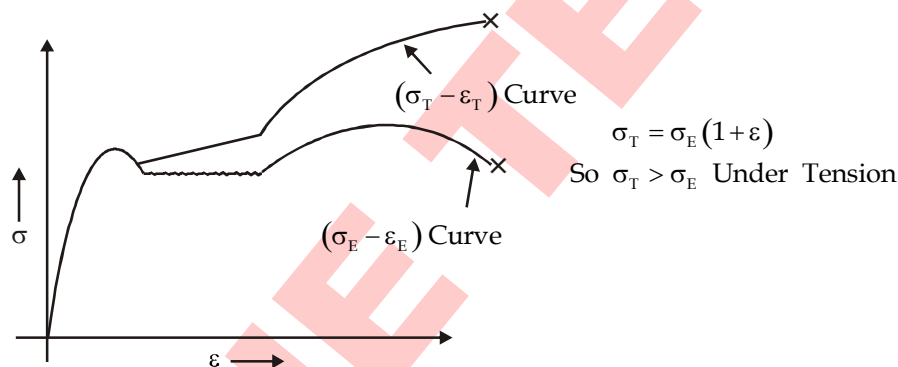
D-E strain hardening zone :- This zone occurs due to reorientation of molecules

Point E (Ultimate point) - Maximum stress is known as ultimate stress

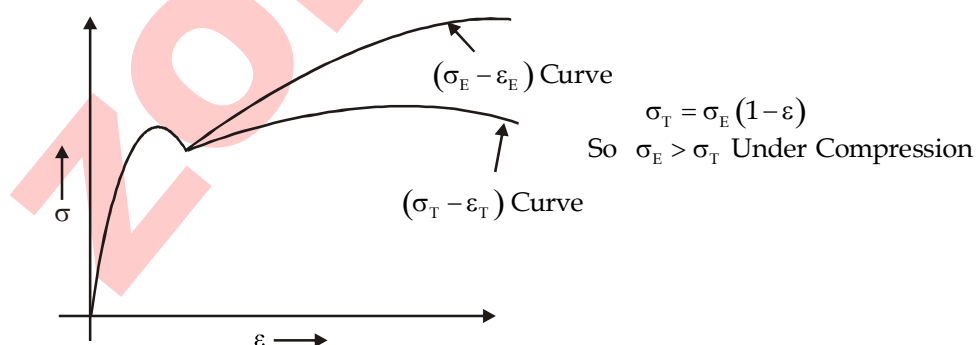
E-F strain softening zone :- It is also known as necking region that means reduction of area at cross section of the body.

Point F Fracture point or Break Point or Failure Point or Rupture Point

Engineering and True Stress - Strain curve for mild steel in tension



Engineering and True Stress - Strain curve for mild-steel in compression



CHAPTER - 2

(PROPERTIES OF MATERIAL)

Important Properties :-

(I) **Elasticity** :- Ability of material by virtue of which material will regain its original shape after unloading is known as elasticity and such type of materials are known as an elastic material.

(II) **Ductility** :- The property of material due to which material can be covered into a thin wire under uniaxial tensile load without fracture is known as ductility and such type of materials are known as ductile material.

Those material which under goes a large plastic deformation before fracture are known as ductile material

Eg - Mild Stel, Al, Cu, etc

$$\sigma_c \geq \sigma_t > \tau$$

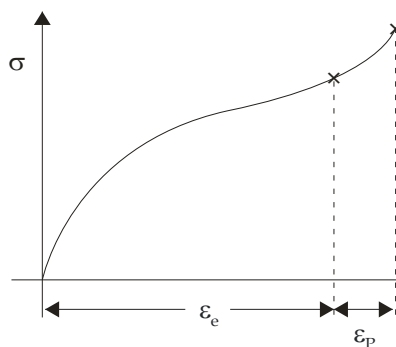
- Ductile materials are weak in shear
- Most Ductile material is platinum
- With increase in temperature ductility of material increases but strength decreases.
- With increase in carbon content ductility will decrease and strength of material will increase.

(III) **Brittle Material** :- Those material which have very less plastic deformation [almost negligible] before fracture are known as brittle material. Brittle materials are weak in tension.

Ex:- Concrete , Glass, Rubber

- For brittle material

$$\sigma_c > \tau > \sigma_t$$



(IV) Malleability :- Property of material due to which material can be converted into thin sheet under uniaxial compression is known as malleability.

Ex:- Most malleable material is Gold

'Hg' is malleable but not ductile,

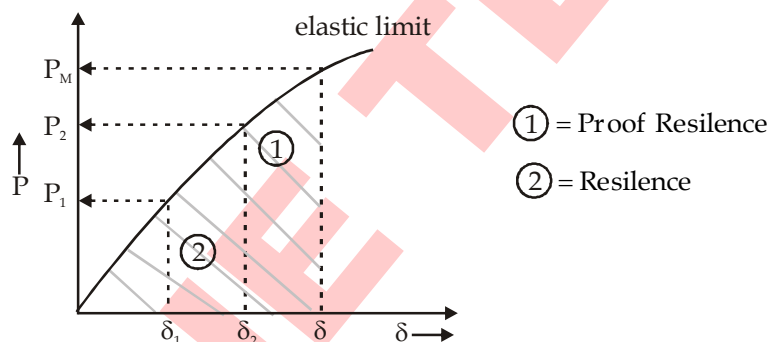
(V) Hardness :- Resistance of any material agency skretches or abraision is known as hardness
Hardness is measured with the help of following test:-

- (a) Rockwell Hardness Test
- (b) Brinell Hardness Test
- (c) Vicker's Hardness Test

(VI) Resilience :- Energy stored in a material upto elastic limit is known as resilience.

Area under the load v/s deformation curve represent resilience in a body. Resilience is applicable upto elastic limit.

(a) Proof Resilience :- Maximum energy which can be stored in a material upto elastic limit is known as proof resilience. Maximum value of resilience is known as proof resilience.



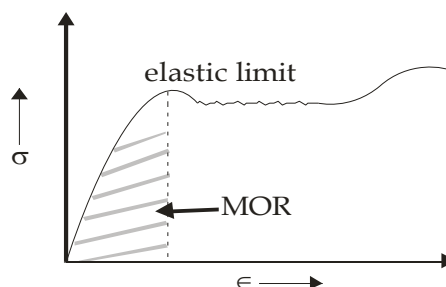
(b) Modulus of Resilience :- Max energy per unit volume which can be stored in a material upto elastic limit is known as modulus of resilience

OR

It is represented by proof resilience per unit volume

OR

Area under stress - Strain curve upto elastic limit represents modulus of resilience



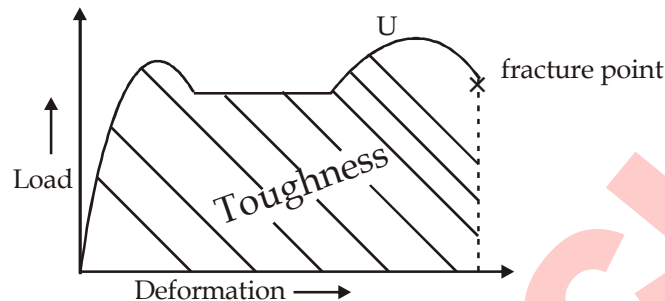
$$\text{Proof Resilience} = (\text{volume} \times \text{MOR})$$

(VII) Toughness :- Resistance of material against impact loading is known as toughness
OR

Total energy stored upto fracture point in material is known as toughness.

OR

Area under load v/s deformation curve upto fracture point is known as toughness



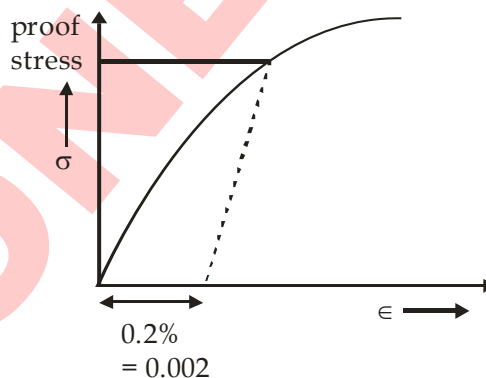
(a) Modulus of Toughness :- Max energy stored per unit volume upto fracture point is known as modulus of toughness

Area of stress - strain curve upto fracture point represents modulus of toughness.

$\text{Toughness} = \text{Volume} \times \text{MOT}$

(VIII) Proof Stress :- This term is used for those ductile materials which do not have clearly defined yield point

Proof stress is determined by offset method in which a offset line is constructed drawn at 0.2% strain



Note 1:-

Loading and unloading curve - When a material is loaded upto elastic limit then after unloading elastic material regain its original shape (during unloading material follow the same path as loading)

Refer fig.1

Note 1:-

If a material is loaded beyond elastic limit then after unloading material does not follow the same curve as loading but unloading curve is parallel to initial tangent of loading curve. This effect/phenomenon is given by Bauehinger hence known as Bauehinger effect. Refer fig. 2 and 3

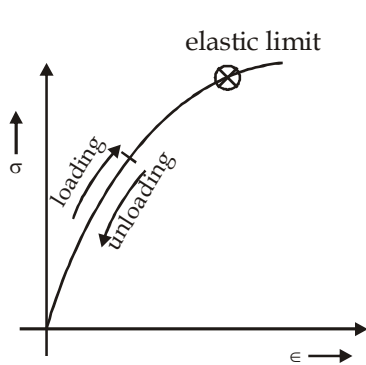


Fig.1

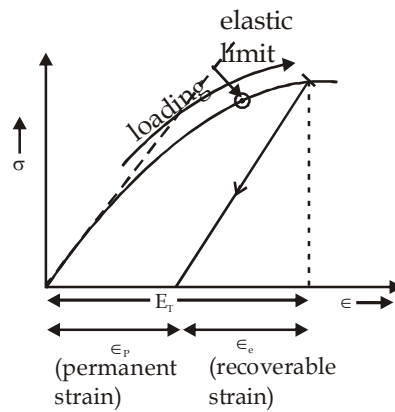


Fig.2

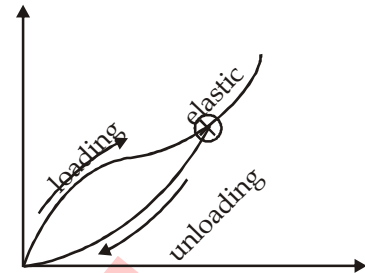
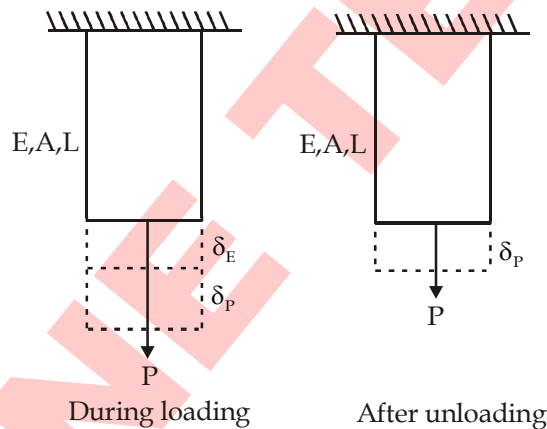


Fig.3

(IX) CREEP :- It is a time dependent permanent deformation of material when it is subjected to constant load for a long period of time. It is also called a "Time Dependent Strain".



δ_e - Elastic deformation

δ_p - Permanent deformation / creep deformation

Creep is mainly function of following factor:-

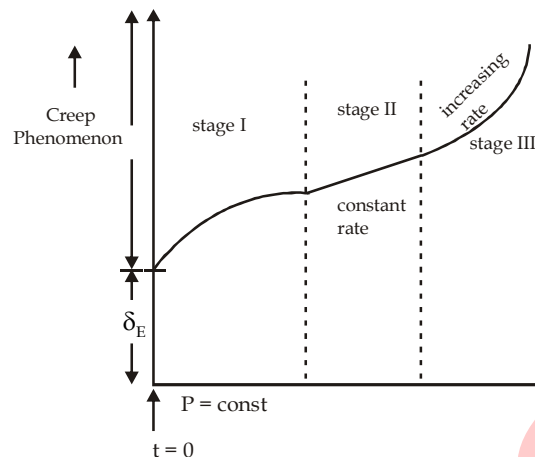
- Magnitude of load
- Temperature
- Time

Note :-

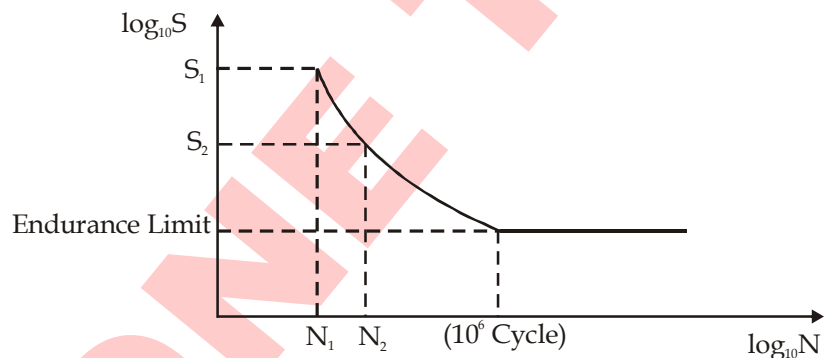
Temperature at which creep deformation become significant is known as homologous temp

$$\text{Homologous temp} \approx \left(\frac{\text{Melting point}}{2} \right)$$

Different stages of Creep:-

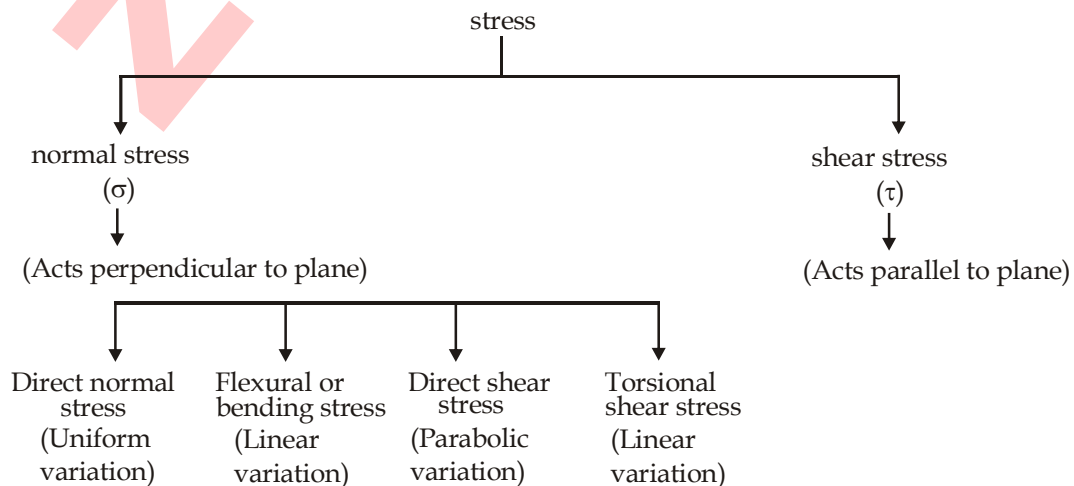


- (X) **Fatigue** :- When a material is subjected to a cyclic loading then material fails at lower stress and such type of failure is known as fatigue failure.
- (XI) **Endurance Limit** :- It is the value of stress at which a material has almost negligible probability of failure under infinite no of (10^7) loading cycles
Ex:- Mild steel - 181 - 186 MPa end.



Endurance limit is determined by S-N curve

Type of Stress :-



CHAPTER - 3

(ELASTIC CONSTANTS)

Elastic Constants :- Elastic constants are used to determine the relation between engineering stress and engineering strain. For homogenous and isotropic material the number of elastic constants are four i.e.

- (1) **Modulus of Elasticity or Young's Modulus (E):-** As per hooke's law upto proportional limit under uniaxial stress condition Normal stress is directly proportional to normal strain.

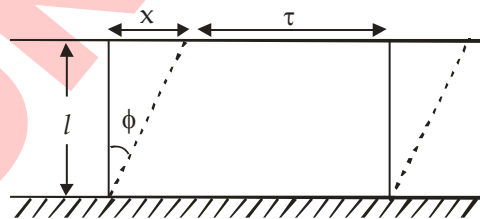
$$E = \frac{\text{Normal stress } (\sigma)}{\text{Normal strain } (\epsilon)}$$

$$E = \frac{\sigma}{\epsilon}$$

- (2) **Modulus of Rigidity or Shear Modulus (G) :-** As per hooke's law upto proportional limit under biaxial stress condition shear stress is directly proportional to shear strain.

$$G = \frac{\text{Shear stress } (\tau)}{\text{Shear strain } (\phi)}$$

$$G = \frac{\tau}{\phi}$$



$$\text{Shear strain } (\phi) = \frac{x}{l}$$

Bulk Modulus (K) - As per hooke's law under hydrostatic condition of stress normal stress is directly proportional to volumetric strain.

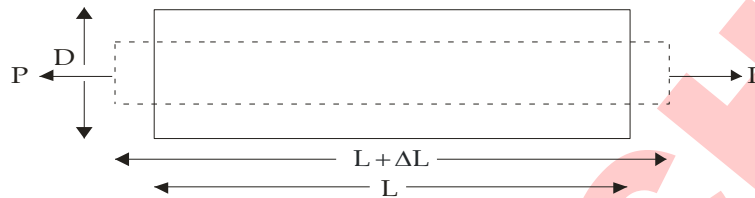
$$K = \frac{\sigma}{\epsilon_v} = \frac{\sigma}{\left(\frac{\delta_v}{V}\right)}$$

$$\text{Volumetric strain} = \epsilon_v = \frac{dv}{V}$$

Note:- If the value of $\mu = 0.5$, then volumetric strain will be zero and such type of material is known as incompressible.

Poisson's Ratio $\left(\mu \text{ or } \frac{1}{m}\right)$:- Upto proportional limit it is used to determine lateral strain and volumetric strain. It is unitless quantity.

Lateral strain $\propto$ Longitudinal strain



Longitudinal Strain = $\frac{\Delta L}{L}$ Strain in the direction of applied load

Lateral Strain = $\frac{\Delta D}{D}$ Strain in the perpendicular direction of applied load

$\frac{1}{m} \text{ or } \mu = \frac{-\text{lateral strain}}{\text{long. strain}} = \frac{\Delta D / D}{\Delta L / L}$ (-) sign represents the nature of lateral strain which is opposite to longitudinal

Range $-1 \leq \mu \leq 0.5$

Generally Range = $0 \leq \mu \leq 0.5$

Note:- If the value of Poisson ratio is zero, then lateral strain become zero and infinite change will occur in longitudinal strain.

Type of Material Poissons Ratio

Cork	0
Glass	0 - 0.02
Concrete	0.1 - 0.2
Cast Iron	0.2 - 0.3
Copper	0.355
Steel	0.27 - 0.3
Mild steel	0.3
Al	0.33
Au(Gold)	0.44
Rubber, wax, Paraffin	0.5

Generally with increase in ductility, poisson's ratio increases exception "Rubber"

Relationship Between Elastic Constants :-

$$(i) \quad E = 2G (1 + \mu) \dots (i)$$

$$(ii) \quad E = 3K (1 - 2\mu) \dots (ii)$$

$$\text{From (i)} \quad \mu = \frac{E}{2G} - 1$$

$$\text{From (ii)} \quad \mu = \left(1 - \frac{E}{3K}\right) \frac{1}{2}$$

$$\text{By equating both-} \quad \left[1 - \frac{E}{3K}\right] = \frac{E}{G} - 2$$

$$3 = \frac{E}{G} + \frac{E}{3K}$$

$$\text{So that } \frac{E(3K + G)}{3KG} = 3$$

$$(iii) \quad E = \frac{9KG}{3K + G}$$

Homogenous Material :- When a material exhibit same elastic properties at any point in a given direction then the material is known as homogeneous material.

Hetrogenous Material :- Those material in which composition is not uniform through out the body are known as heterogenous material.

Isotropic Material :- Those material which have same properties in all direction at a particular point are known as isotropic material

Anisotropic Material :- Those materials which have different properties in different direction at a particular point are known as anisotropic material.

Orthotropic Material :- Those material which have different properties in three mutually perpendicular direction are known as orthotropic material

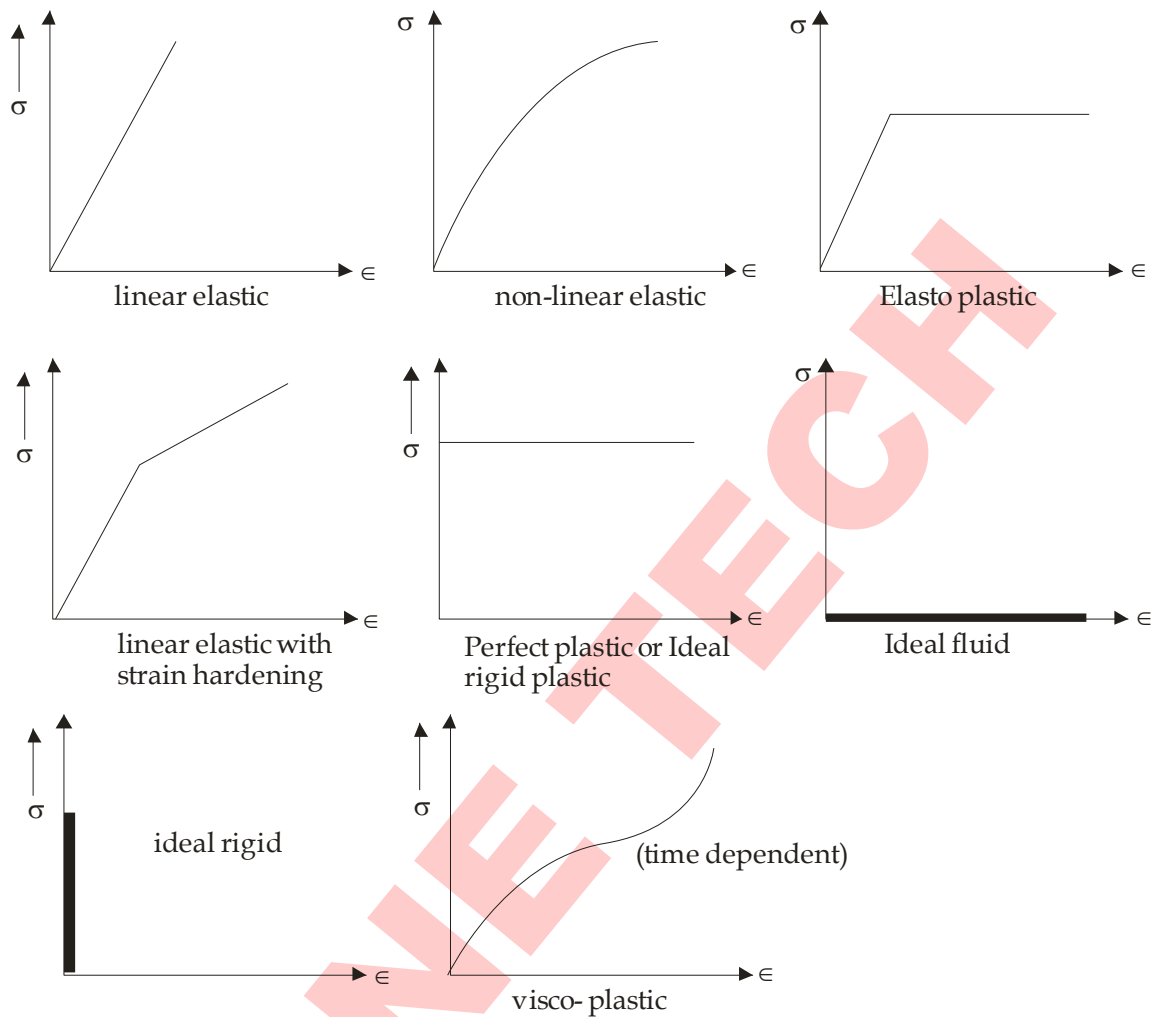
Note :-

Type of Material	No of Elastic Constants	No of Independent elastic constants
(i) Homogenous & Isotropic	4 (E, G, K, μ)	2 (with help of E and μ)
(ii) Homogenous & orthotropic	12	9
(iii) Homogenous & anisotropic	∞	21

Examples:-

- (i) **Isotropic:-** Ceramic, wood, slate (mineral)
- (ii) **Orthotropic:-** One layer material plywood, graphite and many crystals.
- (iii) **Anisotropic:-** Fibre, Reinforced composite, etc.

Different Type of Materials :-



Hook's Law :- According to this law for a homogenous and isotropic material, within elastic limit stress is directly proportional to strain

$$\sigma \propto \epsilon$$

$$\sigma = E \epsilon$$

Elongation of a prismatic bar subjected to tensile load 'P'



If elongation of bar = ΔL

then $\epsilon = \frac{\Delta L}{L}$

Stress in bar $= \sigma = \frac{P}{A}$

ACC. to Hook's Law :-

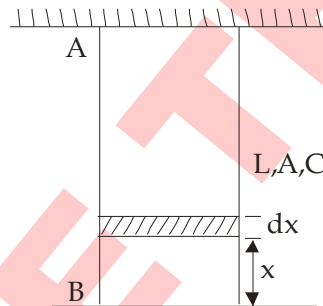
$$\sigma = E \epsilon$$

$$\frac{P}{A} = \frac{\Delta L}{L} \times E$$

$$\Delta L = \frac{PL}{AE}$$

Elongation of a prismatic bar due to its self weight

If unit weight of bar is γ



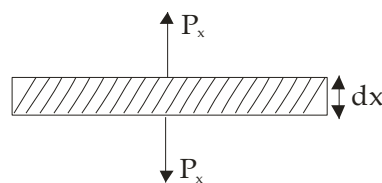
For strip at distance 'x' from

C/S area of strip $= A$

Length of strip $= dx$

Total Force acting on strip $= \gamma \cdot A \cdot L = \text{Weight of body}$

$$P_x = (\gamma \times A \times x)$$



then elongation of strip at x

$$d\delta = \frac{P_x \cdot dx}{A \cdot E}$$

$$d\delta = \frac{\gamma A x dx}{AE} = \frac{\gamma x}{E} dx$$

$$\text{Total elongation of bar} = \Delta l = \int dx = \int_0^L d\delta = \int_0^L \frac{\gamma x}{E} dx$$

$$\Delta L = \frac{\gamma}{E} \left[\frac{x^2}{2} \right]_0^L$$

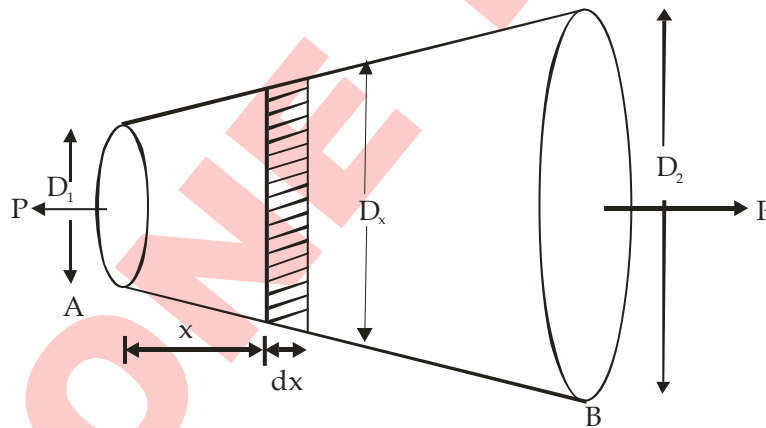
$$\Delta L = \frac{\gamma \cdot L \cdot L}{2E} = \frac{\gamma \cdot L^2}{2E} \quad \text{Or}$$

$$\Delta L = \frac{\gamma \cdot L \cdot L}{2E} \times \frac{A}{A} = \frac{(\gamma \cdot A \cdot L) \times L}{2AE} = \frac{WL}{2AE}$$

$$\Delta L = \frac{WL}{2AE}$$

Where W - Total weight of bar = ($\gamma \times A \times L$)

Elongation of a uniform tapered bar having circular C/S due to external load 'P'.



Consider a strip at distance x from 'A'

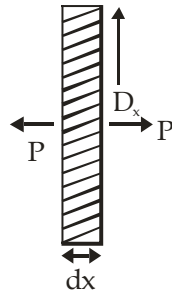
$$\text{Dia of strip} = D_x = D_1 + \left(\frac{D_2 - D_1}{L} \right) x$$

$$D_x = K_x = \left\{ \text{where } K = \frac{D_2 - D_1}{L} \right\}$$

Length of strip = dx

force acting on strip = Px = P

Then elongation of strip



$$d\delta = \frac{Pdx}{A_x E} = \frac{Pdx}{\frac{\pi}{4}(D_x)^2 \times E}$$

$$d\delta = \frac{4Pdx}{\pi E (D_1 + Kx)^2}$$

Then total elongation of $\Delta l = \int_0^L d\delta = \int_0^L \frac{4Pdx}{\pi E (D_1 + Kx)^2} dx$

$$\Delta l = \frac{4P}{\pi E} \left[\frac{1}{K(D_1 + Kx)} \right]_0^L \left[\frac{(D_1 + Kx)^{-2+1}}{-1} \times \frac{1}{K} \right]$$

$$= \frac{4P}{\pi E K} \left[-\left(\frac{1}{D_1 + KL} \right) + \frac{1}{D_1} \right]$$

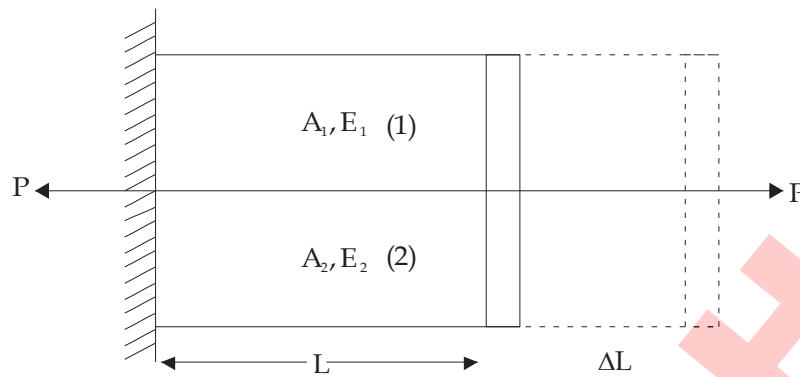
$$= \frac{4P}{\pi E \left(\frac{D_2 - D_1}{L} \right)} \left[\frac{1}{D_1 + \left(\frac{D_2 - D_1}{L} \right)} + \frac{1}{D_1} \right]$$

$$= \frac{4PL}{\pi E (D_2 - D_1)} \left[-\frac{1}{D_2} + \frac{1}{D_1} \right]$$

$$= \frac{4PL}{\pi E (D_2 - D_1)} \left[\frac{D_2 - D_1}{D_1 D_2} \right]$$

$$\Delta l = \frac{4PL}{\pi E D_1 D_2}$$

Elongation of a composite bar :-



Since both bars are attached with the help of rigid plate. Hence final elongation of both bars will be equal

$$\Delta_1 = \Delta_2 = \Delta$$

If P_1 = force carried by bar (1)

P_2 = force carried by bar (2)

then
$$\Delta_1 = \frac{P_1 L}{A_1 E_1} \quad \Delta_2 = \frac{P_2 L}{A_2 E_2}$$

$$P_1 + P_2 = P \dots (1)$$

from $\Delta_1 = \Delta_2$

$$\frac{P_1 L}{A_1 E_1} = \frac{P_2 L}{A_2 E_2}$$

$$\frac{P_1}{P_2} = \frac{A_1 E_1}{A_2 E_2} \dots (2)$$

from equation (1) and (2)

$$P_1 = \left[\frac{A_1 E_1}{A_1 E_1 + A_2 E_2} \right] \times P$$

$$P_2 = \left[\frac{A_2 E_2}{A_1 E_1 + A_2 E_2} \right] \times P$$

Total elongation = $\Delta = \Delta_1 = \Delta_2$

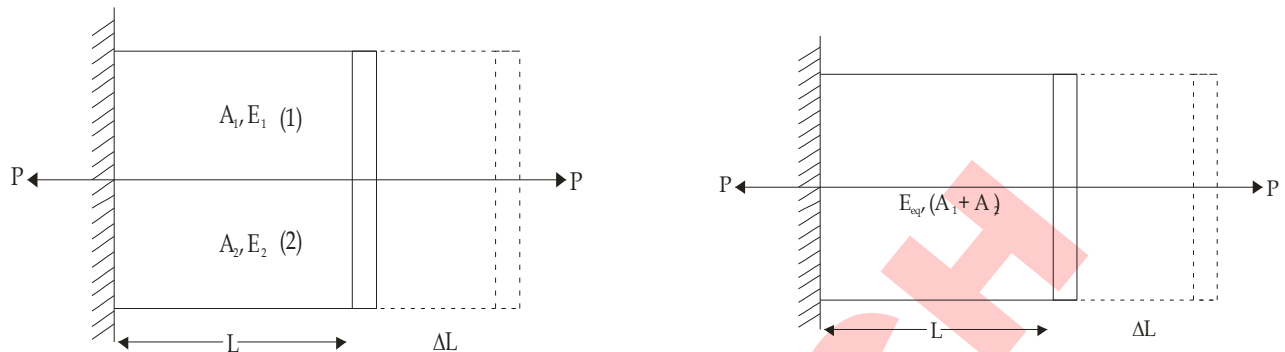
$$\Delta_1 = \frac{P_1 L}{A_1 E_1} = \left(\frac{A_1 E_1 \times P}{A_1 E_1 + A_2 E_2} \right) \times \frac{L}{A_1 E_1}$$

$$\Delta_1 = \frac{P L}{A_1 E_1 + A_2 E_2}$$

$$\Delta_1 = \Delta_2 = \Delta = \frac{P L}{A_1 E_1 + A_2 E_2}$$

Note :-

- If given composite bar is replaced by a single bar having modulus of elasticity equation, cross section area $A_1 + A_2$, length 'L' which have same elongation under same tensile load



Elaongation of equivalent bar

$$\Delta^1 = \frac{PL}{(A_1 + A_2)E_{eq}}$$

For $\Delta_1 = \Delta_2$

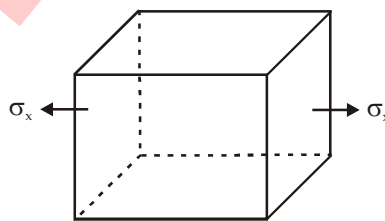
$$\frac{PL}{(A_1 + A_2)E_{eq}} = \frac{PL}{(A_1E_1 + A_2E_2)}$$

$$(A_1 + A_2)E_{eq} = A_1E_1 + A_2E_2$$

$$E_{eq} = \frac{A_1E_1 + A_2E_2}{A_1 + A_2}$$

Expression for volumetric strain under Tri-axial loading:-

When load is applied in x direction only



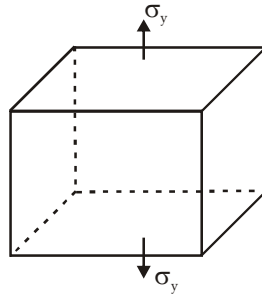
Strain in x direction will be $\epsilon_x = \frac{\sigma_x}{E}$

$$\mu = \frac{-\epsilon_y}{\epsilon_x} = \frac{-\epsilon_z}{\epsilon_x}$$

Strain in y direction will be $\epsilon_y = -\mu\epsilon_x = -\frac{\mu\sigma_x}{E}$

Strain in z direction will be $\epsilon_z = \frac{-\mu\sigma_x}{E}$

When load is applied in y direction

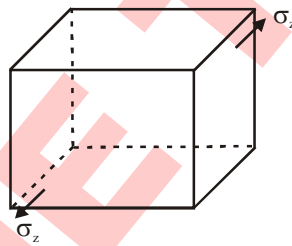


Strain in y direction will be $\epsilon_y = \frac{\sigma_y}{E}$

Strain in x direction will be $\epsilon_x = -\frac{\mu\sigma_y}{E}$

Strain in z direction will be $\epsilon_z = -\frac{\mu\sigma_y}{E}$

When load is applied in z direction only

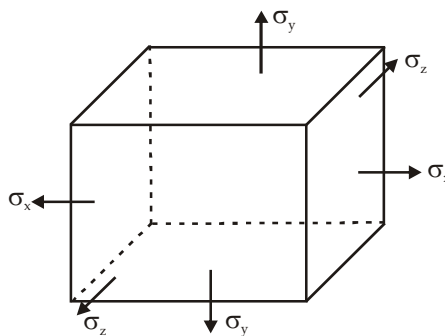


Strain in z direction will be $\epsilon_z = \frac{\sigma_z}{E}$

Strain in x direction will be $\epsilon_x = -\frac{\mu\sigma_z}{E}$

Strain in y direction will be $\epsilon_y = -\frac{\mu\sigma_z}{E}$

When load is applied in all three directions i.e. for 3-D loading condition



Total strain in x-direction will be $\varepsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E}$

Total strain in y-direction will be $\varepsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_z}{E}$

Total strain in z-direction will be $\varepsilon_z = \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E}$

Then Volumetric Strain will be

$$\varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$E_V = \left(\frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E} \right) + \left(\frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_z}{E} \right) + \left(\frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E} \right)$$

$$E_V = \frac{(\sigma_x + \sigma_y + \sigma_z)(1 - 2\mu)}{E}$$

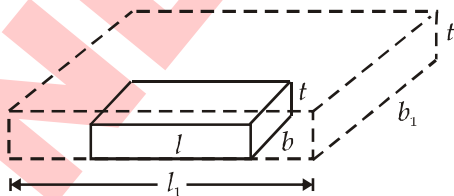
$$E_V = \frac{\Delta V}{V} = (E_x + E_y + E_z) = \frac{\sigma_x + \sigma_y + \sigma_z}{E} (1 - 2\mu)$$

For incompressible body, vol. strain = 0

$$\mu = 0.5$$

Thermal Stresses:- Total stress developed on the cross-section at a component is equal to mechanical normal stress + thermal stress.

Case I:- (Free expansion) - No Reaction



"Free Expansion of rectangular Body"

Thermal deformation in bar:-

$$\delta L = l_1 - l = (\delta_{th})_{XX} = \alpha \Delta T \cdot l$$

$$\delta b = b_1 - b = (\delta_{th})_{ZZ} = \alpha \Delta T \cdot b$$

$$\delta t = t_1 - t = (\delta_{th})_{YY} = \alpha \Delta T \cdot t$$

where α = linear coefficient of thermal expansion

Thermal strain in bar:-

$$(\varepsilon_{th})_{XX} = \alpha \Delta T$$

$$(\varepsilon_{th})_{YY} = \alpha \Delta T$$

$$(\varepsilon_{th})_{ZZ} = \alpha \Delta T$$

$$\text{Hence } (\varepsilon_{th})_{XX} = (\varepsilon_{th})_{YY} = (\varepsilon_{th})_{ZZ} = \pm \alpha \Delta T$$

Volumetric Strain in bar:-

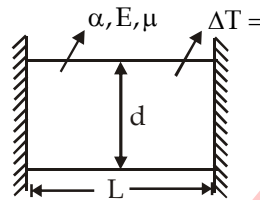
$$\delta_V = \frac{\delta V}{V} = \epsilon_x + \epsilon_y + \epsilon_z = 3\epsilon_{th} = \pm 3\alpha \cdot \Delta T$$

Volume Increase become

$$\delta_V = (3\alpha \cdot \Delta T)(l.b.t) = \text{_____} m^3$$

To maintain dimensional stability in presence of temperature variation Invar (Ni and steel) is selected as the material because α of Invar is equal to $1.2 \times 10^{-6} \text{ } ^\circ/\text{C}$

Case II:- Completely restricted expansion in one direction.



$$\epsilon_{Total_x} = 0$$

$$\epsilon_{Total_{y/z}} = \alpha \cdot \Delta T + \mu \cdot \alpha \cdot \Delta T$$

$$\epsilon_{Total} = \alpha \cdot \Delta T (1 + \mu)$$

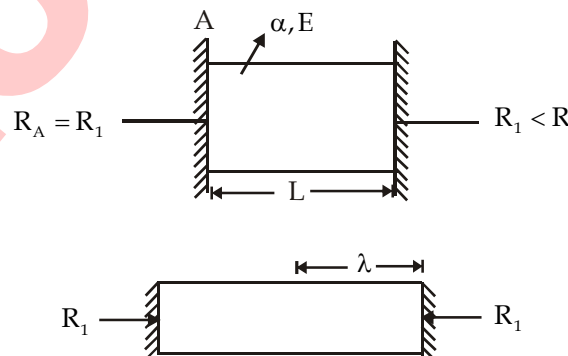
Reaction force:-

$$(\delta_{Total}) = \delta_{th} + \delta_{ad} = \text{zero}$$

$$\Rightarrow (\alpha \cdot \Delta T L) + \left(-\frac{RL}{AE} \right) = 0$$

$$R = \alpha \cdot \Delta T \cdot E \cdot A$$

$$\sigma = \frac{R}{A} = \alpha \cdot \Delta T \cdot E$$

Partially Restricted expansion of a bar:-

λ = Change in length of bar

$$\text{Change in length} = (\delta_{Total})_X = \delta_{th_X} + (\delta_{axial def})_X = \lambda$$

$$= (\alpha \cdot \Delta T \cdot L) + \left(-\frac{R_1 L}{AE} \right) = \lambda$$

$$\alpha \cdot \Delta T \cdot L - \frac{\sigma_{th} \cdot L}{E} = \lambda$$

$$\sigma_{th} = \left(\frac{\alpha \cdot \Delta T \cdot L - \lambda}{L} \right) \times E$$

$$\sigma_{th} = \pm \left(\frac{\delta_{th} - \lambda}{L} \right) \times E$$

Thermal stress

$$(\sigma_{th})_x = \frac{P}{A} = \frac{R}{A} = \alpha \cdot \Delta T \cdot E$$

$$\sigma_{thx} = \alpha \cdot \Delta T \cdot E$$

Hence

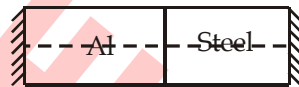
When temperature increases $\rightarrow \sigma_{th}$ is compression nature

ϵ_{th} is tensile nature

Temperature decreases $\rightarrow \sigma_{th}$ is tensile nature

ϵ_{th} is compression nature

Thermal stress in compound bar:-

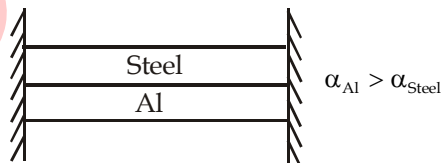


When temperature will rise $\Rightarrow$ Compress in both shaft

When temperature will drop $\Rightarrow$ Tensile in both bar

Thermal stress in composite bar:-

(1) When temperature will increase



$T \uparrow$ Al (α) $\uparrow \Rightarrow$ Compression thermal stress

Steel (α) $\downarrow \Rightarrow$ Tensile thermal stress

$T \uparrow$ Al (α) $\uparrow \Rightarrow$ Tension thermal stress

Steel (α) $\downarrow \Rightarrow$ Compression thermal stress

CHAPTER - 4

(CENTROID & MOMENT OF INERTIA)

CENTROID AND MOMENT OF INERTIA

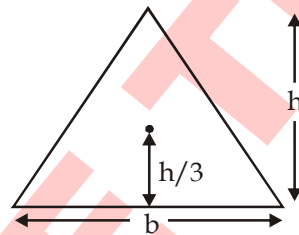
Centre of gravity and Centroid:-

The centre of gravity of a body is that point through which the resultant of system of parallel force formed by the weight of all practice passes

Centroid:- Centre of area for the plane x-sectional area like circle, rectangle, triangle etc. It is represented by C or G.

Centroid for same standard shape/Areas:-

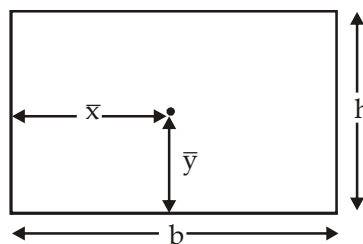
(I) Triangle



$$\bar{y} = \frac{h}{3}$$

$$\text{Area} = \frac{1}{2}bh$$

(II) Rectangle

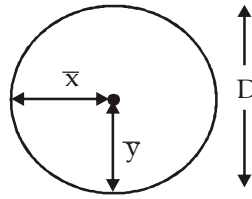


$$\bar{x} = \frac{b}{2}$$

$$\bar{y} = \frac{h}{2}$$

$$\text{Area} = b \times h$$

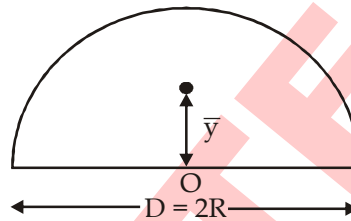
(III) Circle



$$\text{Area} = \frac{\pi}{4} D^2$$

$$\bar{x} = \bar{y} = \frac{D}{2}$$

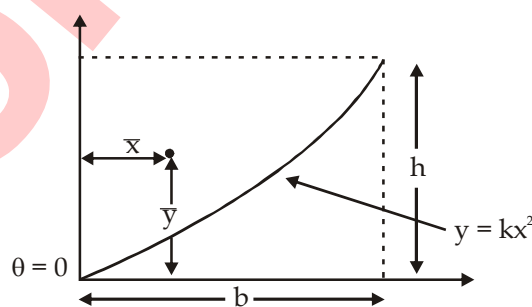
(IV) Semi - Circular Disc



$$\text{Area} = \frac{1}{2} \left(\frac{\pi D^2}{4} \right)$$

$$\bar{y} = \frac{4R}{3\pi}$$

(V)

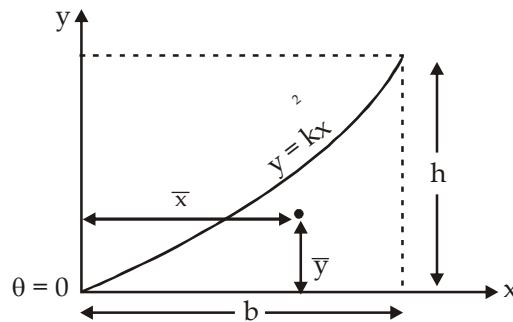


$$\text{Area} = \left[\frac{2}{3} \times b \times h \right]$$

$$\bar{x} = \frac{3}{8} b$$

$$\bar{y} = \frac{3}{5} h$$

(VI)



$$\text{Area} = \left[\frac{1}{3} \times b \times h \right]$$

$$\bar{x} = \frac{3}{4}b$$

$$\bar{y} = \frac{3}{10}h$$

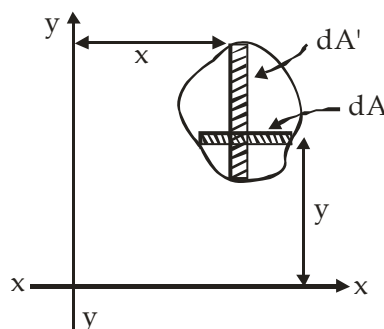
Note :-

Centroid for any non-symmetric body:-

$$\bar{x} = \frac{A_1 X_1 + A_2 X_2 + \dots + A_n X_n}{A_1 + A_2 + \dots + A_n}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + \dots + A_n y_n}{A_1 + A_2 + \dots + A_n}$$

Area - Moment of Inertia :- Second order moment of area about avg axis is known as area - moment of inertia



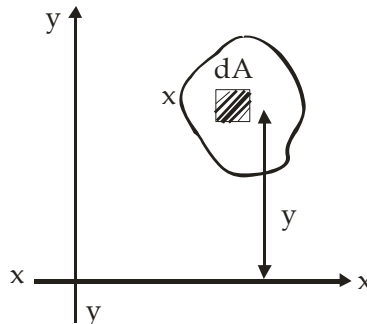
$$I_{xx} = \int y^2 dA \quad I_{xx} - \text{MOI about } xx - \text{axis}$$

$$I_{yy} = \int x^2 dA \quad I_{yy} - \text{MOI about } yy - \text{axis}$$

Note :-

Product of Moment of Inertia :-

Product of MOI about x - x and y - y axis -



$$I_{xy} = \int yx \, dA \quad I_{xy} = \iint xy \, dx \, dy$$

→ Those axis about which product of inertia is zero are known as principal axis.

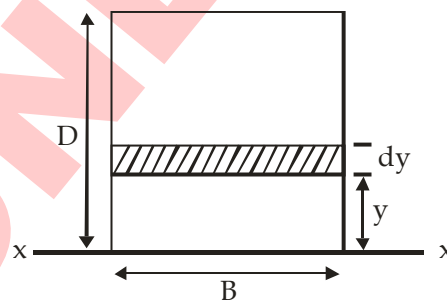
Note :-

Unit of area MOI = mm⁴, m⁴, cm⁴

$$\text{Unit of } \frac{\text{mass MOI}}{(\text{mr}^2)} = \text{kgm}^2$$

Rectangle :-

→ Consider a strip at distance 'y' from x - x axis



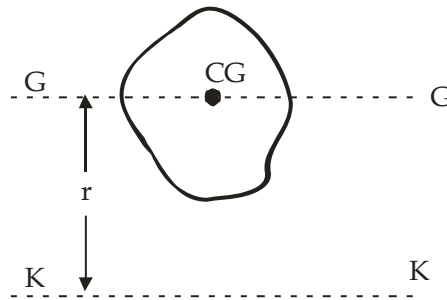
- Area of strip = dA = Bdy
- MOI of strip about x-axis = dI = y²dA = y²(Bdy)
- Total MOI of rectangle about x-axis

$$I_{xx} = \int dI = \int_0^D y^2 B dy$$

$$I_{xx} = B \left[\frac{y^3}{3} \right]_0^D$$

$$I_{xx} = \frac{BD^3}{3}$$

Parallel Axis Theorem :- Moment of inertia with respect to any axis parallel to the centroidal axis is equal to the moment of inertia with respect to centroidal axis plus the product of area of figure and square of the distance between axis.



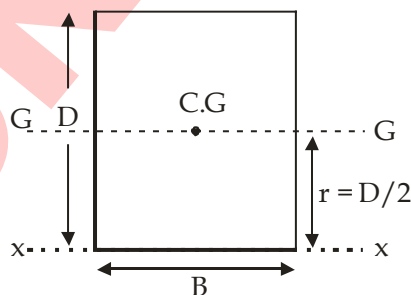
→ $I_{KK} = I_{GG} + Ar^2$
 I_{GG} = MOI about C.G axis
 axis passing through C.G
 I_{KK} = MOI about K.K axis
 A = Total area
 r = distance between G - G & K - K axis

Ex.

$$I_{xx} = \frac{BD^3}{3}$$

$$I_{GG} = ?$$

Acc. to 11 axis theorem :-



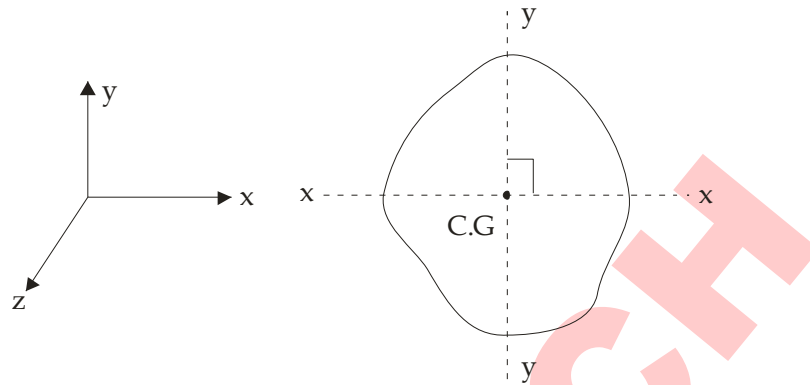
$$I_{xx} = I_{GG} + Ar^2$$

$$\frac{BD^3}{3} = I_{GG} + BD \times \left(\frac{D}{2}\right)^2$$

$$I_{GG} = \frac{BD^3}{3} - \frac{BD^3}{4}$$

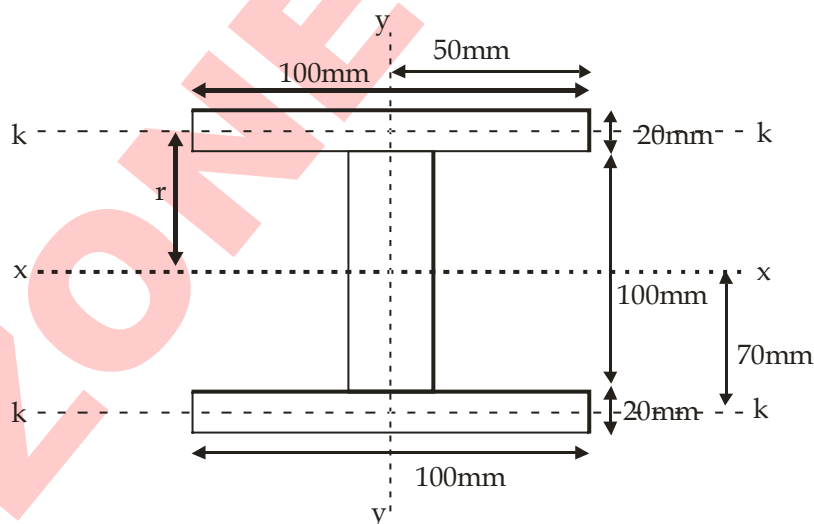
$$I_{GG} = \frac{BD^3}{12}$$

Perpendicular Axis Theorem :- Moment of inertia about an axis perpendicular to the plane and passing through the intersection of the other two axis X-X and Y-Y contained by plane is equal to the sum of Moment of Inertia about XX and YY.



$$I_{zz} = I_{xx} + I_{yy}$$

where ;

$$I_{xx} = \text{MOI about x-axis}$$
$$I_{yy} = \text{MOI about y-axis}$$
$$I_{zz} = \text{MOI about z-axis}$$


For given I-section calculate MOI about x - axis and y - axis

(1) **MOI about X-axis :-**

$$I_{xx} = (I_{xx})_{web} + (I_{xx})_{flange \dots (1)} + (I_{xx})_{flange}$$

$$(I_{xx})_{web} = \frac{(20)(100)^3}{12} = 1666666.67$$

$$(I_{xx})_{\text{flange} \dots (1)} = [I_{xx} + Ar^2]_{\text{flange} \dots (1)}$$

$$= \left[\frac{100(20)^3}{12} + 100 \times 20 \times 60^2 \right]$$

$$= 7266666.67$$

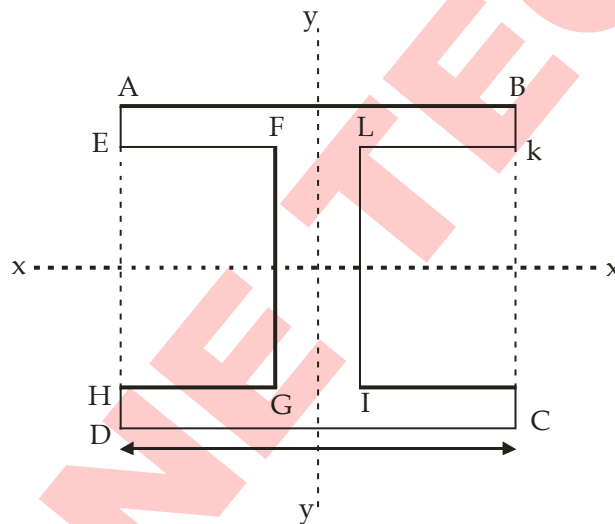
$$(I_{xx})_{\text{flange} \dots (1)} = (I_{xx})_{\text{flange} \dots (2)}$$

$$\rightarrow I_{xx} = (I_{xx})_{\text{web}} + (I_{xx})_{\text{flange} \dots (1)} + (I_{xx})_{\text{flange} \dots (2)}$$

$$= [1666666.67 + 7266666.67 \times 2]$$

$$= 16.2 \times 10^6 \text{ mm}^4$$

Method-2 :-



$$I_{xx} = (I_{xx})_{ABCD} = (I_{xx})_{EFGH} = (I_{xx})_{IJKL}$$

$$= \frac{100(140)^3}{12} = \frac{40(100)^3}{12} = \frac{40(100)^3}{12}$$

$$I_{xx} = 16.2 \times 10^6$$

$$I_{yy} = (I_{yy})_{ABCD} - (I_{yy})_{IJKL} - (I_{yy})_{EFGH}$$

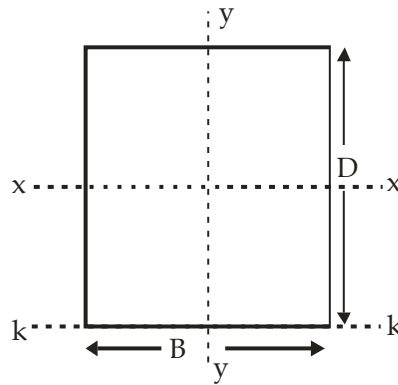
$$= \frac{(140)(100)^3}{12}$$

$$I_{yy} = (I_{yy})_{\text{web}} + (I_{yy})_{\text{flange} \dots (1)} + (I_{yy})_{\text{flange} \dots (2)}$$

$$= \frac{100(20)^3}{12} + \frac{20(100)^3}{12} + \frac{20(100)^3}{12}$$

$$I_{yy} = 03.4 \times 10^6 \text{ mm}^4$$

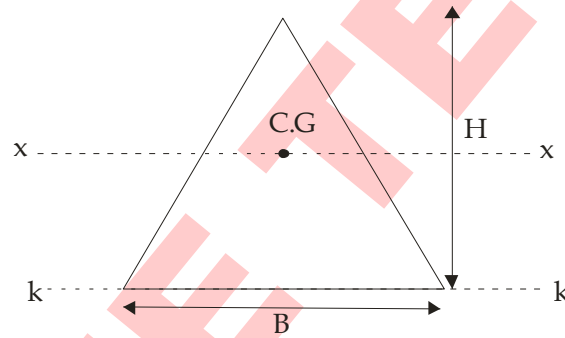
Standard Results :-



$$I_{xx} = \frac{BD^3}{12}$$

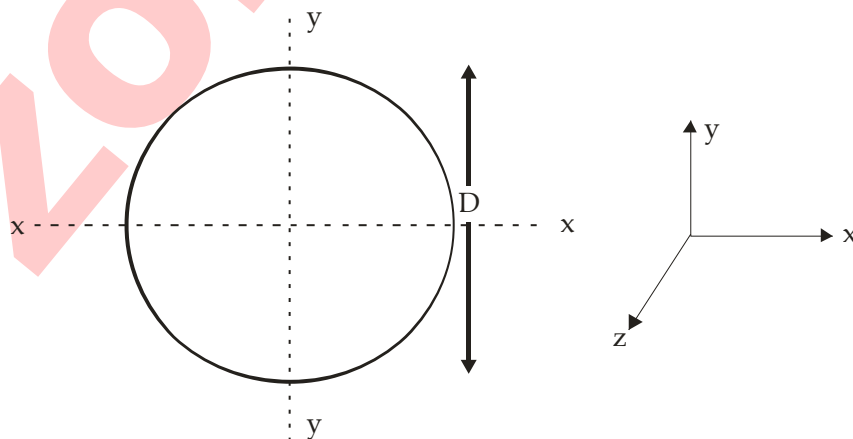
$$I_{yy} = \frac{DB^3}{12}$$

$$I_{kk} = \frac{BD^3}{3}$$



$$I_{xx} = \frac{BH^3}{36}$$

$$I_{kk} = \frac{BH^3}{12}$$



$$I_{xx} = I_{yy} = \frac{\pi D^4}{64}$$

$$I_{zz} = I_{xx} + I_{yy}$$

$$I_{zz} = \frac{\pi D^4}{32}$$

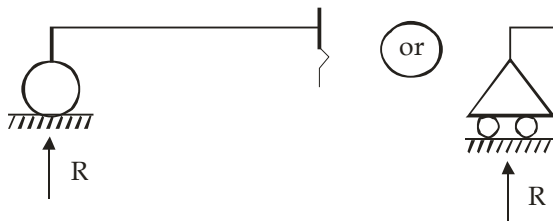
CHAPTER - 5

(BENDING MOMENT AND SHEAR FORCE DIAGRAM)

S.F.D And B.M.D

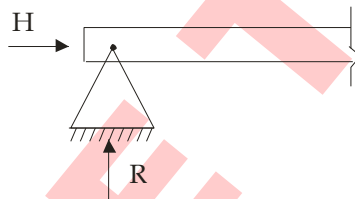
Different type of supports :-

(I) Roller Support :-



only vertical reaction will be there

(II) Hinge Support :-



Moment reaction will be zero

Only horizontal & vertical reaction will be there

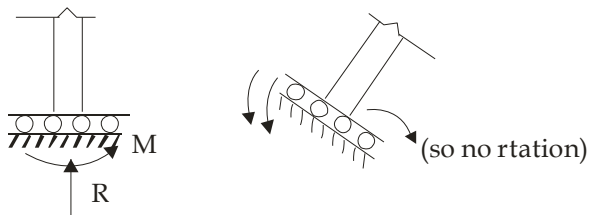
(III) Fixed Support



Horizontal & vertical reaction will be there

Moment reaction will also there

(IV) Guided Roller



Types of Beam :-**(1) Simply Supported Beam :-**

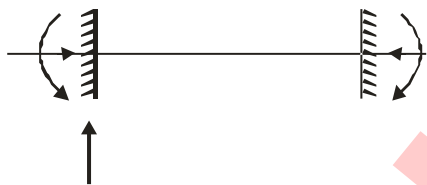
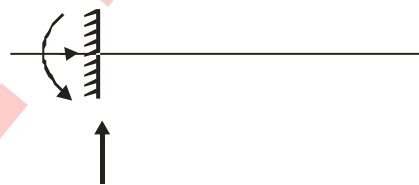
Will not permit thermal expansion so we will not use this



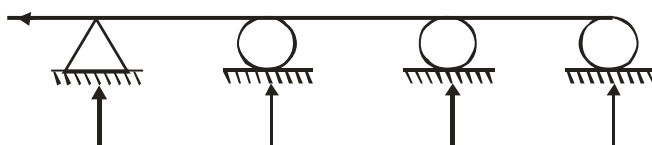
Horizontal displacement can't be resist

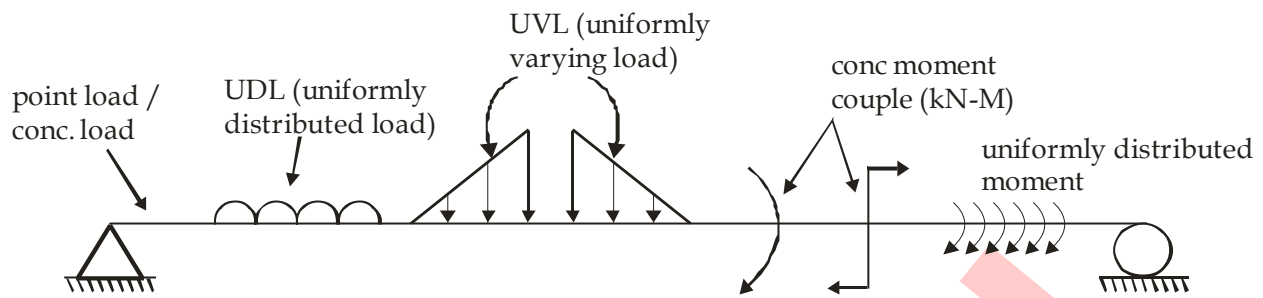
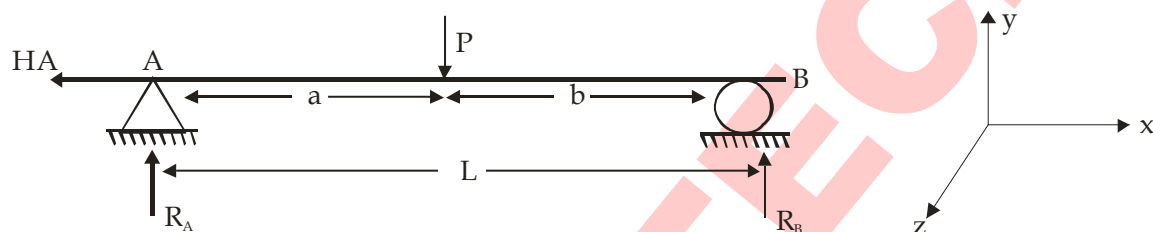


Best combination for SSB so one end is hinged & another is roller support

(2) Fixed Beam**(3) Cantilever Beam****(4) Propped Cantilever****(5) Over hanging beam**

(a) single side

**(b) Double side overhang****(6) Continuous beam**

Type of Loading :-**Calculation of Support Reaction :-****By using Static Equilibrium :-**

$$\sum f_x = 0 \Rightarrow -H_A = 0 \Rightarrow H_A = 0$$

$$\sum f_y = 0 \Rightarrow R_A + R_B - P = 0 \Rightarrow R_A + R_B = P \dots (1)$$

$$\sum M_z = 0, \sum M_A = 0$$

$$P(a) - R_B L = 0$$

$$Pa = R_B L$$

$$R_B = \frac{Pa}{L}$$

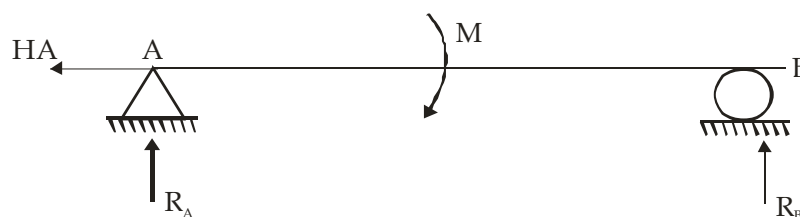
From equation (1)

$$R_A + \frac{Pa}{L} = P$$

$$R_A = \frac{P - Pa}{L}$$

$$R_A = \frac{Pb}{L}$$

Ex. Calculate the reaction in given arrangement



$$\Sigma f_x = 0, H_A = 0$$

$$\Sigma f_y = 0, R_A + R_B = 0$$

$$\Sigma M_A = 0$$

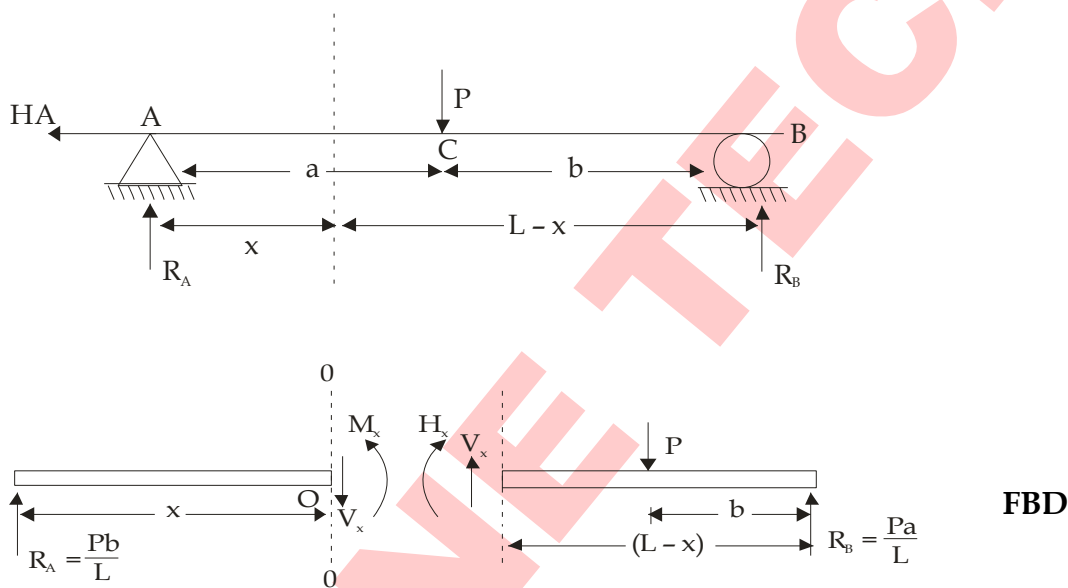
$$M - R_B L = 0$$

$$R_B = M/L$$

$$R_A = R_B$$

$$R_A = -M/L$$

where, (-ve) sign represents that direction of ' R_A ' will be opposite



Shear Force :- Shear force at any section is the algebraic sum of total transverse forces which are acting to left side or to right side of the given section

Bending Moment :- Bending moment at any section is the algebraic sum of total moment due to transverse forces or couples to the left side or to right side of given section.

For above figure in left side portion -

$$\Sigma f_y = 0$$

$$V_x = R_A \Rightarrow V_x = \frac{Pb}{L}$$

$$\Sigma M_0 = 0$$

$$M_x = R_A X$$

$$M_x = \frac{Pb}{L} x$$

For right side portion :-

$$\sum f_y = 0 \Rightarrow R_B + V_x = P$$

$$V_x = P - R_B$$

$$\Rightarrow V_x = \left(P - \frac{Pa}{L} \right)$$

$$\sum M_0 = 0$$

$$\Rightarrow M_x + P [(L - x) - b] = R_B (L - x)$$

$$M_x = R_B (L - x) - P [(L - x) - b]$$

S.F.D (Shear Force Diagram) :-

A curve which represents the shear force on different section along the length of member is known as shear force diagram.

B.M.D (Bending Moment Diagram) :-

A curve which represents the bending moment of diff sections along the length of member is known as bending moment diagram.

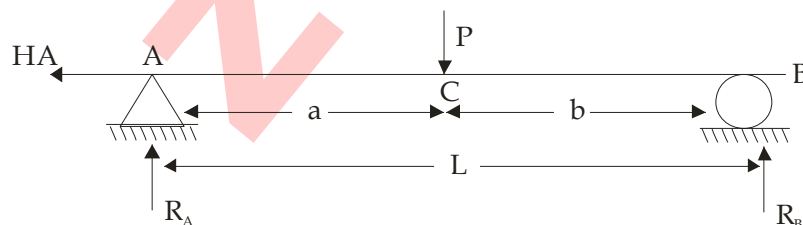
Axial Thrust Diagram :-

A curve which represents axial thrust on diff sections along the length of member is known as axial thrust diagram.

Steps to Draw SFD and BMD :-

Step - I

Calculate, Support Reactions.

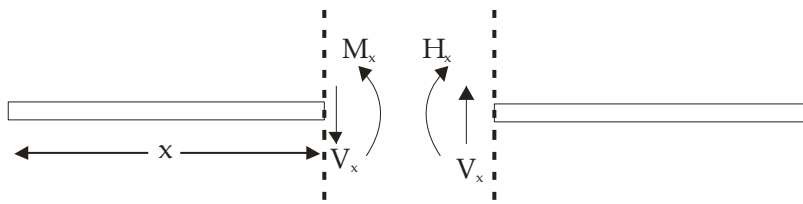


$$H_A = 0$$

$$R_A = \frac{Pb}{L}$$

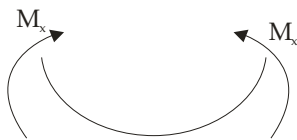
$$R_B = \frac{Pa}{L}$$

Step - II: Sign Convention :-



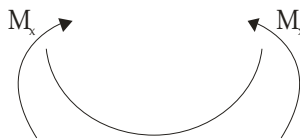
$$V_x = (+ve)$$

$$M_x (\text{sagging}) = +ve$$



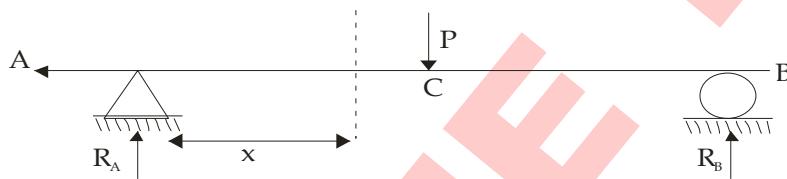
$$V_x = +ve$$

$$M_x (\text{sagging}) = +ve$$



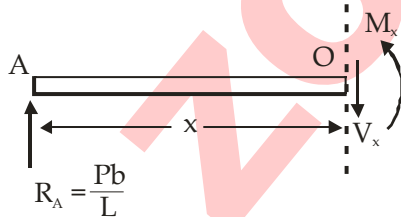
Step - III :-

Write the equation for shear force and bending moment for different segment



For segment AC ($0 < x < a$)

where 'x' from 'A'



For equilibrium :-

$$\sum f_y = 0 \Rightarrow V_x = R_A = \frac{Pb}{L}$$

$$\sum M_0 = 0 \Rightarrow M_x = R_A \times x$$

$$M_x = \frac{Pbx}{L}$$

At Pt A, $x = 0$

$$V_A = \frac{Pb}{L}$$

$$M_A = 0$$

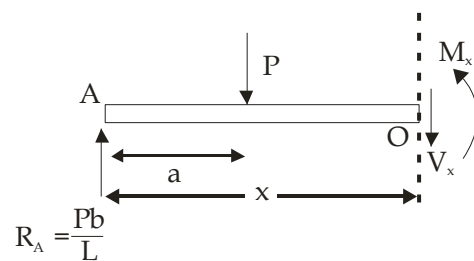
at Pt c, $x = a$

$$(V_c)_{\text{left}} = \frac{Pb}{L}$$

$$(M_c)_{\text{left}} = \frac{Pb}{L} (a) = \frac{Pab}{L}$$

For segment CB ($a < x < L$)

where x from A



for equilibrium :-

$$\sum f_y = 0 \Rightarrow V_x + P = R_A$$

$$V_x = \frac{Pb}{L} - P = \frac{Pa}{L}$$

$$\sum M_O = 0$$

$$M_x + P(x - a) = R_A x$$

$$M_x = \frac{Pb}{L} x - P(x - a)$$

at Pt C, $x = a$

$$(V_c)_{\text{right}} = \frac{-Pa}{L}$$

$$(M_c)_{\text{right}} = \frac{Pab}{L}$$

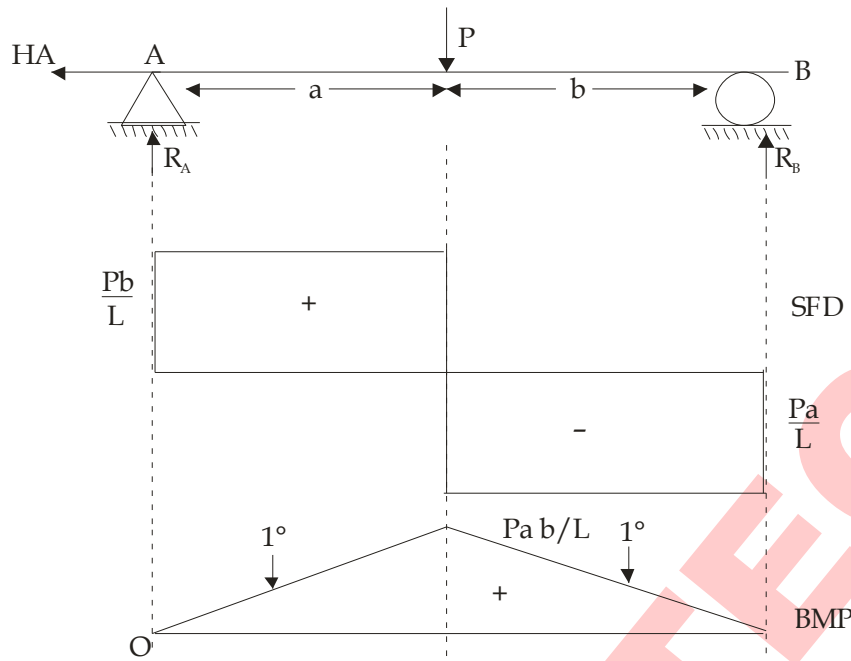
at Pt B, $x = L$

$$(V_B) = \frac{-Pa}{L}$$

$$(M_B) = \frac{Pab}{L} (L) - P(L - a)$$

$$= Pb - Pb = 0$$

Step-IV :- Draw SFD and BMD :-



Properties of SFD and BMD :-

(I) $\frac{dM}{dx} = V$ (SF) 'A' to 'C'

$$\frac{dV}{dx} = -W$$

$$\frac{dV}{dx} = 0$$

$$dV = 0$$

where : $M = \text{B.M}$ $V = C$ (const)

$$V = \text{S.F} \quad \frac{dM}{dx} = C$$

$$W = \text{loading} \quad M = C(x') + c_2$$

Loading

SFD

BMD

n^0

$(n+1)^0$

$(n+2)^0$

Point load

Const (0°)

linear (1°)

(const) UDL

1°

2°

(linear) UVL

2°

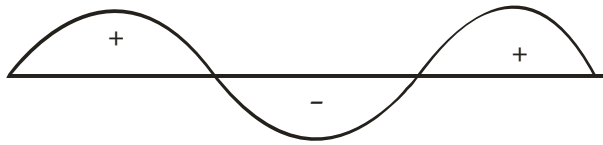
3°

(II) Point at which a concentrated force is acting, there will be a sudden change in SFD and slope of BMD.

(III) A point at which a couple is acting there will be a sudden change in BMD

Note :-

- (1) A point at which SFD changes its sign, bending moment will be max (max sagging) or min (max hogging) at that point



- (2) A point at which BMD changes its sign is known as point of contraflexure sometimes it is also known as point of inflexion

Step for SFD and BMD without using equation :-

Step-I

Calculate support reactions

Step-II

Calculate shear force and bending moment on critical section (point)

→ Critical points are

- (a) Point load, starting and end point of UDL and UVL point at which moment is acting support reactions, point at which SFD change its sign.

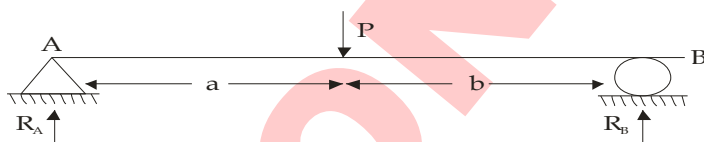
Step-III :- Determination of Shear Force.

Case-I :

If left portion of given section is considered

→ Shear force at given section = total upward force to left of section.

Ex.



$$R_A = \frac{Pb}{L}, R_B = \frac{Pa}{L}$$

Case-II

If right portion of given section is considered

→ Shear force at given section = total downward force to the right of section

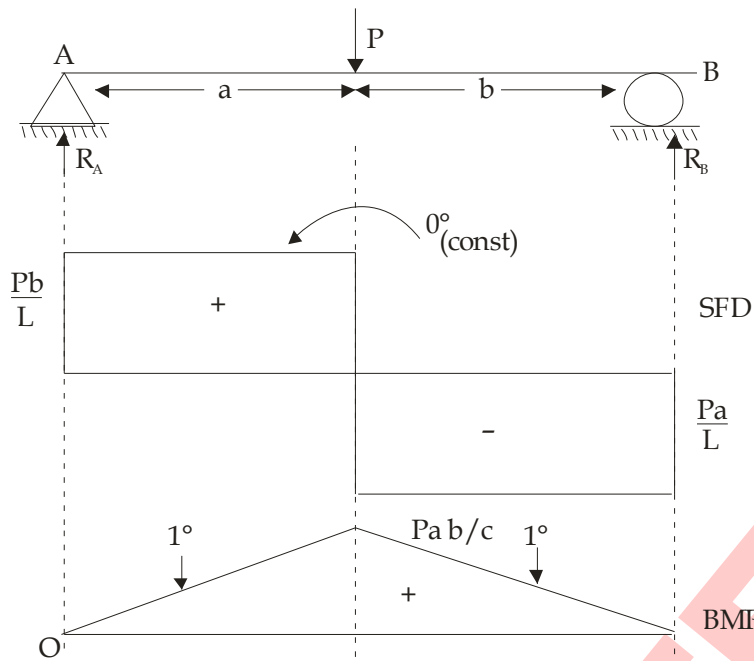
Step-4 : Determination of bending section is considered

→ Bending moment at given section = total clockwise moment due to forces & couples to the left side of section.

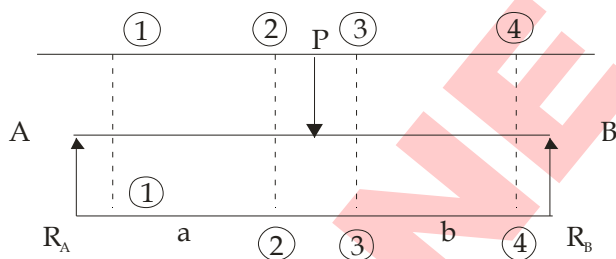
Case-II : If right portion of given section is considered

→ Bending moment at given section = total anticlockwise moment due to forces & couples to the right side of section.

Step-5 : By using properties of SFD and BMD draw the SFD & BMD



Mathematical equation for above problem



V_{A_R} = total upward force on left of (1) (1)

$$V_A = R_A = \frac{Pb}{L}$$

V_{C_l} = total upward force on left of (2) (2)

$$R_A = \frac{Pb}{L}$$

V_{C_R} = total upward force on left portion of (3) (3)

$$R_A = -P$$

$$= \frac{Pb}{L} - P = \frac{-Pa}{L}$$

V_{B_l} = total downward force on right portion (4) (4)

$$-R_B = \frac{-Pa}{L}$$

Bending Moment :-

$$BM)_A = BM \text{ at section (1) (1)}$$

→ total clockwise moment due to forces & couples to the left side of section

$$MA = 0$$

$$B)_l = BM \text{ at section (2) (2)}$$

→ Total clockwise moment due to forces acting to left side of section

$$M_{C_l} = R_A \times a = \frac{Pb}{L} \times a = \frac{Pab}{L}$$

$$M_{C_R} = B.M \text{ at section (3) (3)}$$

Total clockwise moment due to force acting to left side of (3).... (3)

$$= R_A \times a - P(0) = R_A \times a$$

$$M_{C_R} = \frac{Pab}{L}$$

$$M_B = B.M \text{ at section (4) (4)}$$

Total anticlockwise moment due to forces acting on right side of (4) (4)

$$M_B = M_B(0) = 0$$

Note :-

Case for double side over hanging beam with UDL load for most economical section (+ve) B.M at mid span should be equal to (-ve) B.M at supports

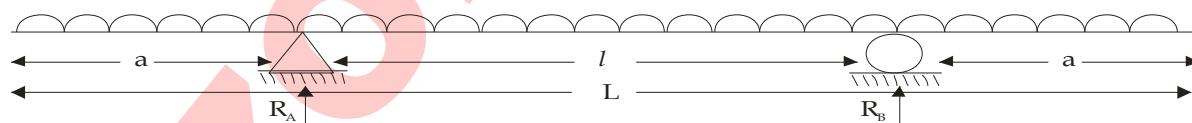
$$\frac{w}{8} [\ell^2 - 4a^2] = \frac{w}{2} a^2$$

$$\frac{1}{4} [\ell^2 - 4a^2] = a^2$$

$$\ell^2 = 8a^2$$

$$\ell = 2\sqrt{2}a$$

[less strength requires at this length]



$$L = l + 2a$$

for most economical section

$$\ell = 2\sqrt{2}a$$

$$L - 2a = 2\sqrt{2}a$$

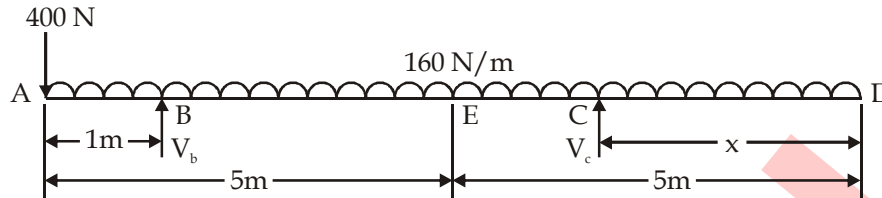
$$L = a(2 + 2\sqrt{2})$$

$$a = \frac{L}{2 + 2\sqrt{2}} = 0.207L$$

$$\ell = 2\sqrt{2}a$$

$$a = \frac{L}{2 + 2\sqrt{2}} = 0.207L$$

Ex. A horizontal beam AD, 10 metres long carries a uniformly distributed load of 160 N/m together with a concentrated load of 400 N at the left end A. The beam is supported at a point B which is 1 m from A and at C which is one the right-hand, half of the beam and x metres from the end, D.



Determine the value of x, if the mid-point is a point of contraflexure and for this arrangement draw S.F. and B.M. diagrams.

Sol. Fig. shows the beam carrying the load system mentioned in the problem. Let V_b and V_c be the reaction at the supports B and C.

Resolving the forces on the beam vertically

$$V_b + V_c = 400 + 160 \times 10 = 2000 \text{ N}$$

$$\therefore V_b + V_c = 2000 \text{ N}$$

It is given the B.M. at the mid-point E = 0. Taking moments about E of the forces on L.H.S. of E,

$$V_b \times 4 - 400 \times 5 - 160 \times \frac{5^2}{2} = 0$$

$$\therefore V_b = 1000 \text{ N}$$

Since, $V_b + V_c = 2000 \text{ N}$

We have $V_c = 1000 \text{ N}$

Taking moments about E of the forces on the RHS of E,

$$1000(5-x) - 160 \times \frac{5^2}{2} = 0$$

$$\therefore x = 3 \text{ m}$$

Fig. shows the loaded beam showing the actual position of the support C.

S.F. analysis

S.F. just on RHS of A = -400 N

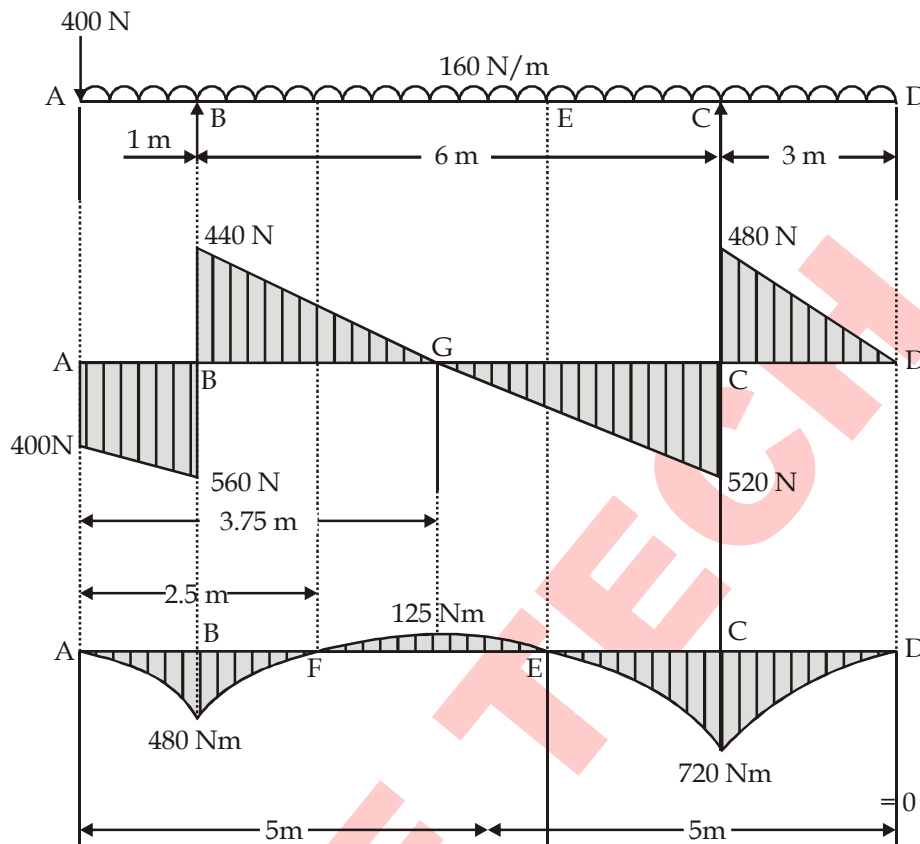
S.F. just on LHS of B = -400 - 160 × 1 = -560 N

S.F. just on RHS of B = -560 + 1000 = +440 N

S.F. just on LHS of C = 1000 - 400 - 160 × 7 = -520 N

S.F. just on RHS of C = -520 + 1000 = +480 N

S.F. at D = 0



Section of zero shear. Let G be the section of zero shear. If this section is x metres from A, equating the S.F. to zero,

$$1000 - 400 - 160x = 0$$

$$\therefore x = 3.75 \text{ m}$$

B.M. of analysis B.M. at A = 0

$$\begin{aligned} \text{B.M. at B} &= -400 \times 1 - \frac{160 \times 1^2}{2} \\ &= -480 \text{ Nm} \end{aligned}$$

$$\text{B.M. at G} = 1000 \times 2.75 - 400 \times 3.75 - \frac{160 \times 3.75^2}{2} = 125 \text{ Nm}$$

$$\text{B.M. at C} = -160 \times \frac{3^2}{2} = -720 \text{ Nm}$$

$$\text{B.M. at D} = 0$$

Points of contraflexure between B and C

Let a point of contraflexure be x metres from A

Equating the B.M. to zero,

$$1000(x-1) - 400x - 160 \times \frac{x^2}{2} = 0$$

$$\therefore 2x^2 - 15x + 25 = 0$$

$$\therefore (2x - 5)(x - 5) = 0$$

$$\therefore x = 2.5 \text{ m and } 5 \text{ m.}$$

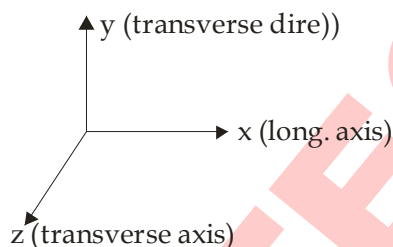
ZONE TECH

CHAPTER - 6

(BENDING STRESS & SHEAR STRESS IN BEAM)

BENDING STRESSES

- bending = along transverse axis load
- twisting = along longitudinal axis load



- When a moment is applied about a transverse axis of beam then bending will generate in beam due to this bending compressive and tensile stress (normal stresses σ) will generate which are known as bending stress or flexural stress.

Bending Equation Assumptions :-

- Material is homogenous, isotropic and obey hooks law
- Beam is initially straight and prismatic
- Plane section before bending remains plane after bending also i.e strain in any fibre is proportional to the distance of that fibre from neutral axis.
- Resultant of applied loading is in plane of symmetry
- Modulus of elasticity is same in tension and compression
- Beam is subjected to bending only

Note :-

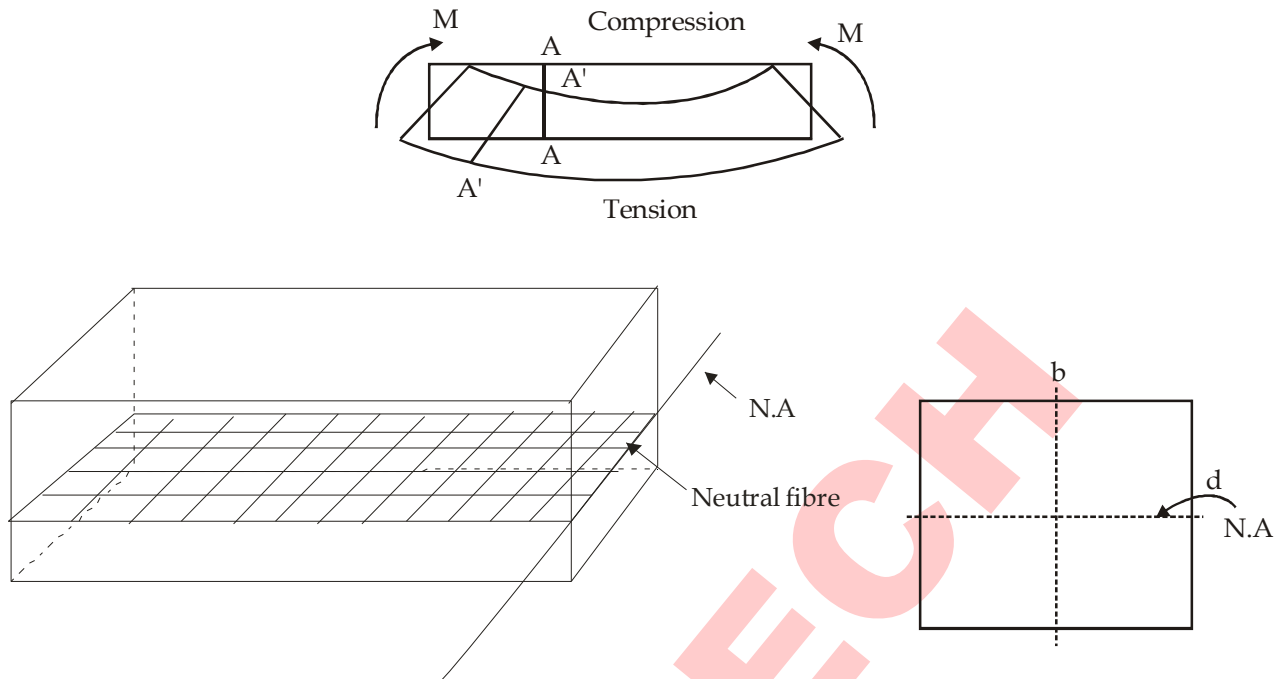
Neutral axis?

While bending, one fibre exists such that the length of which remains unchanged. This layer is neutral layer. The line of intersection of the neutral layer with plane of cross-section of the beam is called neutral axis.

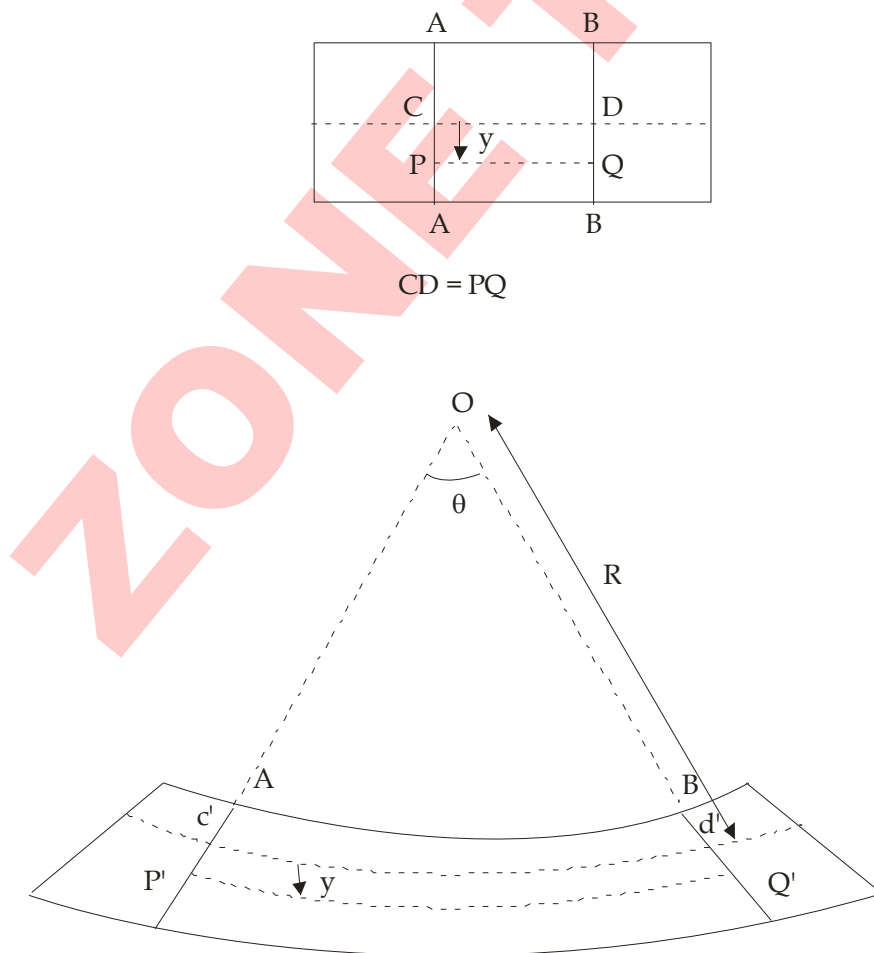
It represents the location of zero flexural stress in the member subjected to bending.

Neutral fibre - A fibre in which total strain is zero (i.e. no change in length of that fibre) is known as neutral fibre

- A line which passes through the intersection of neutral fibre and any c/s is known as neutral axis of that cross-section



Bending equation Derivation



$C'D' = CD$ (at NA - No strain, No stress)

$$C'D' = R\theta$$

$$P'\theta' = (R + y)\theta$$

$$PQ = CD' = C'D' = R\theta$$

Strain in fibre at distance 'y' from NA

$$\epsilon = \frac{P'Q' - PQ}{PQ}$$

$$\epsilon = \frac{(R + y)\theta - R\theta}{R\theta}$$

$$\epsilon = \frac{y}{R}$$

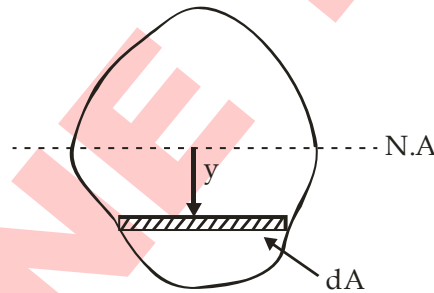
$$\epsilon \propto y$$

where $R \rightarrow$ Radius of curvature

$$\frac{1}{R} \rightarrow \text{Curvature}$$

$$\epsilon = \frac{\sigma}{E} = \frac{y}{R} \dots (1)$$

Stress in fibre at distance 'y' from N.A



$$\sigma = \frac{y}{R} E$$

Force resisted by strip $dm = y \times df = y \times \sigma \times dA$

$$dM = y \times \frac{yE}{R} dA$$

Total moment carrying capacity of $M = \int dM$ section

$$= \int \frac{E}{R} y^2 dA$$

$$M = \frac{E}{R} \int y^2 dA$$

$$M = \frac{E}{R} I$$

$$\frac{M}{I} = \frac{E}{R} \dots (ii)$$

From equation (1)

$$\frac{\sigma}{E} = \frac{y}{R}$$

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$$

Bending equation

Where

I = area MOI about N.A

R = Radius of Curvature

$$\frac{1}{R} = \text{Curvature}$$

Case - I

$$\frac{\sigma}{y} = \frac{M}{I}$$

$$\sigma = \frac{M}{I} \times y$$

$$\sigma_{\max} = \frac{M}{I} \times y_{\max} = \frac{M}{Z}$$

where, Z = section modulus

$$= I$$

$$= \frac{I}{y_{\max}}$$

Case - II

$$\frac{M}{I} = \frac{E}{R}$$

so

$$R = \frac{EI}{M}$$

Case - III

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$R = \frac{E \cdot y}{\sigma}$$

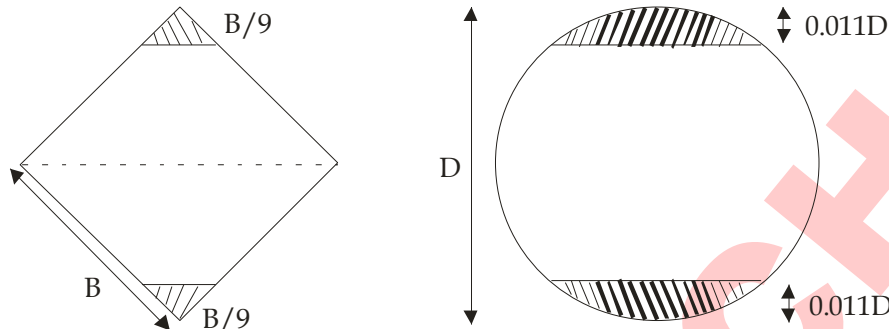
→ **Beam of Uniform Strength :-**

A beam is which value of maximum bending stress at all cross section is equal is known as beam of uniform strength.

Flitched Beam :- Flitched beam are used to increase the strength of section

Note :-

- (1) Top and bottom flined beam has more strength in comparison of side flined beam [by analysis]
- (2) When shaded portion of diamond or circular section is removed then value of section modulus increases (due to which moment carrying capacity increases).



$\uparrow \text{Moment} = \uparrow Z = \frac{I \downarrow (\text{initially dominant})}{y \downarrow \text{after that more affect}}$ (In these sections if not chamfered cotness then bearing stresses

Ex. A beam of square section of the side 'a' is placed such that

- (i) Two sides are horizontal
- (ii) One diagonal is horizontal

Find the ratio of moment of resistance for case (i) & (ii).

Case-I



$$(\text{MOR})_I = \sigma \cdot Z_1$$

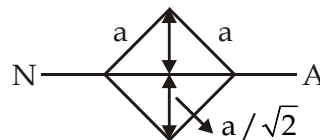
$$= \sigma \cdot \left(\frac{a^4}{12} \right) \cdot \frac{a}{2}$$

$$(\text{MOR})_I = \frac{a^3}{6} \cdot \sigma$$

$$(\text{MOR})_{II} = \frac{a^3}{6\sqrt{2}} \cdot \sigma$$

$$\frac{(\text{MOR})_I}{(\text{MOR})_{II}} = \sqrt{2}$$

Case-II



$$(\text{MOR})_{II} = \sigma \cdot Z_2$$

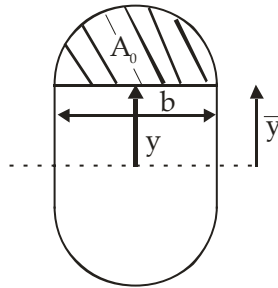
$$= \sigma \cdot \left(\frac{I}{y_{\max}} \right)$$

$$= \sigma \cdot \frac{\frac{a^4}{12}}{\frac{a}{\sqrt{2}}}$$

Shear Stress

Let shear force at any cross section = P

then shear stress in fibre at distance ' y ' from N.A. will be



$$\tau = P \cdot \left(\frac{A \cdot \bar{y}}{I_{NA} \cdot b} \right)$$

where :

b = width of fibre

I = M.O.I about N.A

$Q = A \bar{y}$

A = Area of shaded portion

$\bar{y}$ = distance of C.G of shaded portion from N.A

Shear stress distribution for a rectangular section

Let shear force = V

$$\tau = \frac{V \cdot A \cdot \bar{y}}{I \cdot b}$$

$$\tau = \frac{V \cdot \left[\left(\frac{d}{2} - y \right) \cdot b \left(y + \frac{\frac{d}{2} - y}{2} \right) \right]}{I \times b}$$

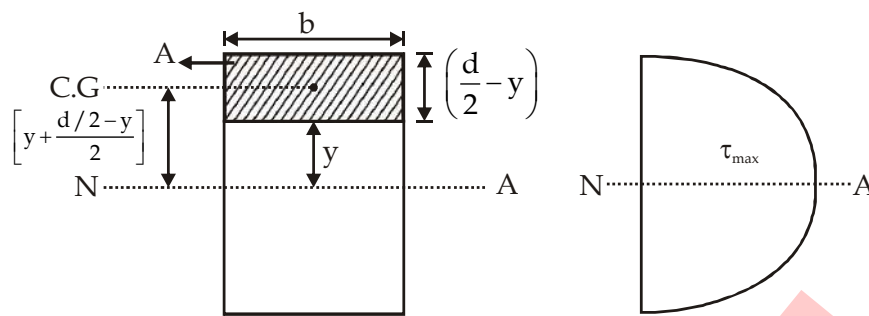
$$\tau_{\text{parabolic}} = \frac{V}{2I} \left(\frac{d^2}{4} - y^2 \right)$$

at outer fiber, $y = \frac{d}{2}$

$$\tau = 0$$

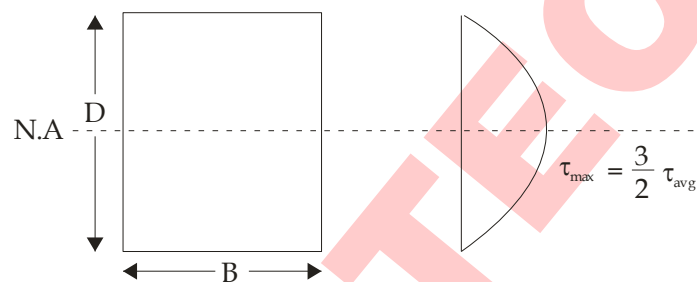
at N.A., $y = 0$,

$$\therefore \tau = \frac{3}{2} \frac{V}{bd}$$



Standard Case :-

(1) Rectangular



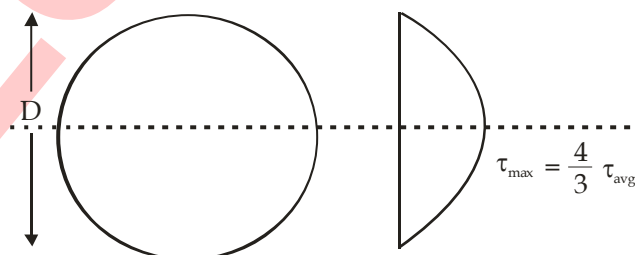
where

$$C_{avg} = \frac{P}{BD}$$

Note :- For square, rectangular and triangle section

$$\tau_{max} = \frac{3}{2} \tau_{avg}$$

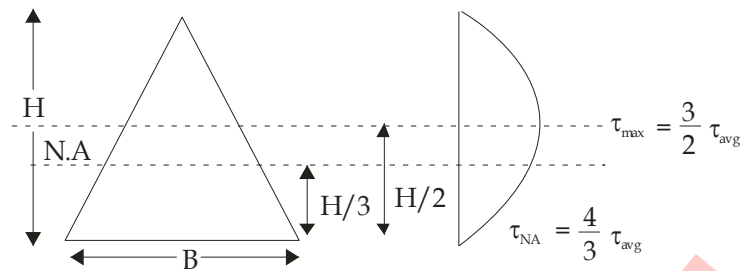
(2) Circular section



where

$$C_{avg} = \frac{P}{\frac{\pi}{4} D^2}$$

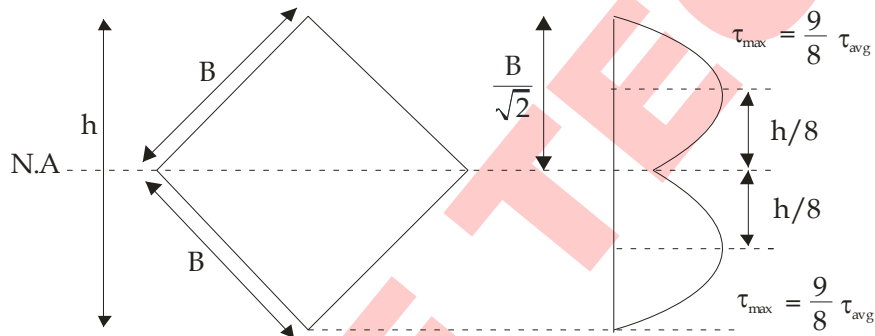
(3) Triangle section



where

$$\tau_{max} = \frac{3}{2} \tau_{avg}$$

(4) Diagonal square

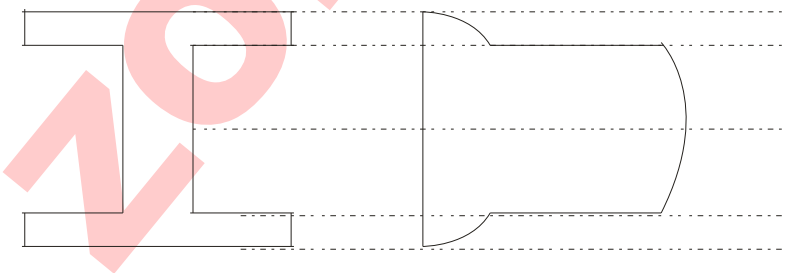


where

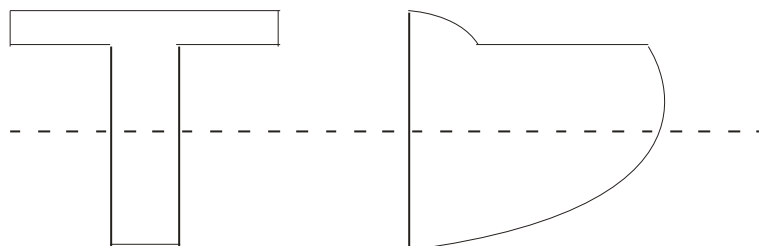
$$\tau_{avg} = \frac{V}{B^2}$$

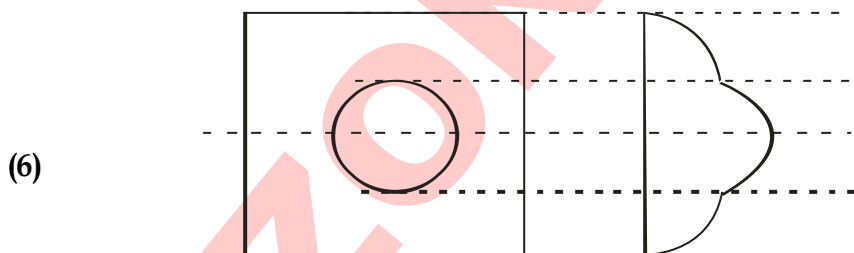
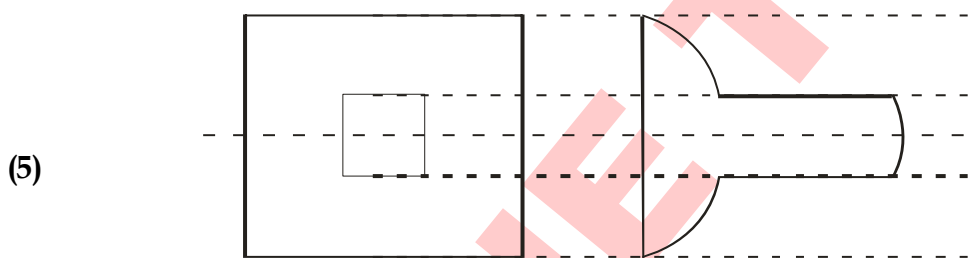
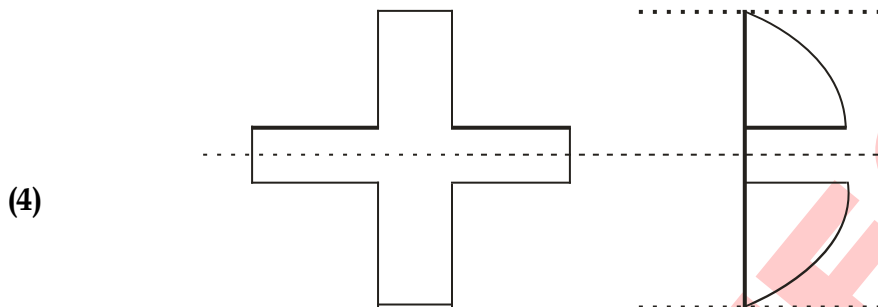
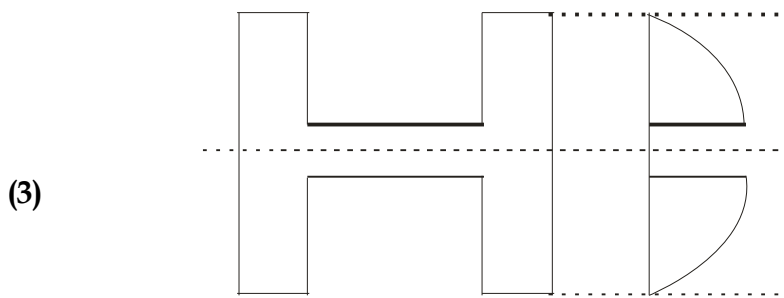
→ Variation of Shear Stress For Different Section :-

(1)



(2)





Ex A circular section of steel has diameter 'D'. Determine the dimension of the strongest rectangular section in bending, that can be cut from this circular section?

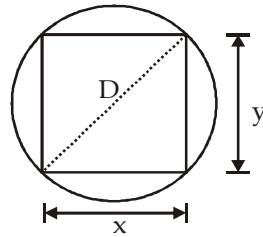
Sol. $\therefore \text{MOR} = Z \times \sigma_{\text{per.}}$

Since for given material σ_{per} will be constant.

Hence $\text{MOR} \propto Z$

Section with maximum Z will be stronger

$$x^2 + y^2 = D^2$$



$$\text{Section modulus (Z)} = \frac{I}{y}$$

$$Z = \frac{xy^3 / 12}{y / 2} = \frac{xy^2}{6}$$

$$Z = \frac{x(D^2 - x^2)}{6} \quad \left\{ \because x^2 + y^2 = D^2 \right\}$$

$$\text{For Z to be maximum } \frac{dz}{dx} = 0 \rightarrow$$

$$(D^2 - x^2) + (-2x)(x) = 0$$

$$\Rightarrow x = \frac{D}{\sqrt{3}} \quad \& \quad y = \sqrt{\frac{2}{3}}D$$

Ex. Determine the concentrated load which when placed at the free end of a cantilever of length 1 m would produce a shear stress of 1.5 N/mm^2 at the level of the neutral axis of the section carrying the maximum shear. Assume that the beam has uniform rectangular cross-section $200 \text{ mm} \times 400 \text{ mm}$.

Hence compute the maximum compressive and tensile stresses due to being over the cross-section at the fixed end of the cantilever.

Sol. Let W be the concentrated load applied at the free end of the cantilever.

$$\text{Max. S.F.} = S = W$$

$$\text{Max. shear stress} = \frac{3}{2} \times \text{average shear stress}$$

$$= \frac{3}{2} \times \frac{W}{200 \times 400} = 1.5 \text{ N/mm}^2$$

$$\therefore W = 8000 \text{ N}$$

$$\therefore \text{Max. B.M.} = M = 8000 \times 1 \times 1000 = 8 \times 10^7 \text{ Nmm.}$$

Equating the moment of resistance of the maximum bending moment.

$$\frac{1}{6} f b d^2 = M$$

$$\frac{1}{6} f \times 200 \times 400^2 = 8 \times 10^7$$

$$\therefore f = 15 \text{ N/mm}^2$$

Hence maximum compressive or tensile stress at fixed end will be 15 N/mm^2 .

CHAPTER - 7

(TORSION IN SHAFT)

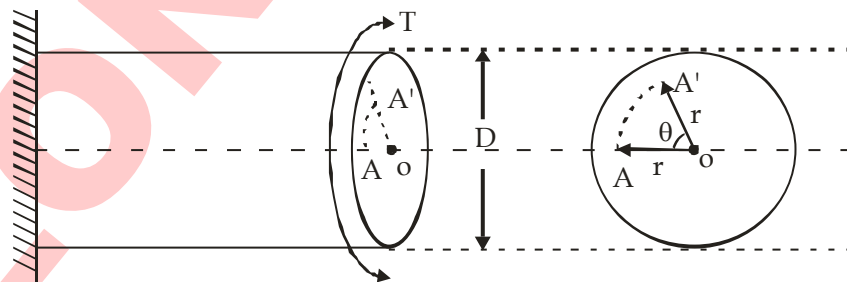
When a moment is applied about the longitudinal axis of any shaft then twisting will generate due to which shear stresses generates in shaft

Difference Between Bending and Twisting :-

Bending	Twisting
When moment is applied about transverse axis then bending generates	When moment is applied about longitudinal axis then twisting occurs
In bending section rotates about transverse axis	In twisting c/s rotates about longitudinal
In bending normal stresses generates	In twisting shear stress generates

Assumptions :-

- (1) Material is homogeneous, isotropic and obey hook's law
- (2) C/S of shaft is circular. (which may be hollow or solid)
- (3) Plane and vertical section before twisting remains plane and vertical after twisting also.



Consider a fiber at distance 'r' from centre

$$\text{shear strain in fibre} = \phi = \frac{AA'}{L}$$

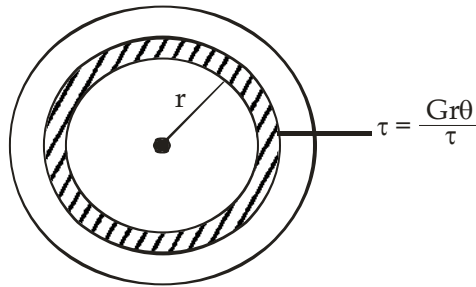
$$AA' = r\theta$$

where, θ = angle of rotation

$$\phi = \frac{r\theta}{L}$$

$$\phi = \frac{\tau}{G} \Rightarrow \tau = \phi G$$

Hence, shear stress in fibre at distance 'r' from NA

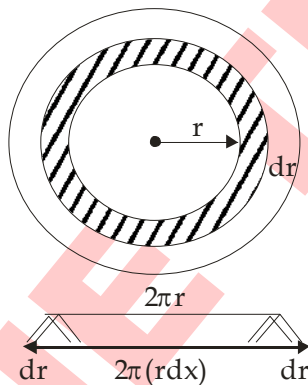


$$\tau = \phi G$$

$$\tau = \frac{Gr\theta}{L}$$

Shear stress in fibre at distance 'r' from centre

$$\tau = \frac{Gr\theta}{L} \quad \text{or} \quad \frac{\tau}{R} = \frac{G\theta}{L} \quad \dots (i)$$



Shear force resisted by strip

$$dF = \tau \times dA$$

$$= \tau(2\pi r dr)$$

Torque carried by strip = $dT = dF \times r$

$$dT = \frac{Gr\theta}{L} \times (2\pi r dr) r$$

$$= \frac{2G\pi\theta}{L} r^3 dr$$

$\therefore \theta$ is not depend on r

→ Torque carrying capacity of section

$$T = \int dT$$

$$T = \frac{2G\pi\theta}{L} \int_0^R r^3 dr$$

$$T = \frac{2G\pi\theta}{L} \left[\frac{r^4}{4} \right]_0^R = \frac{2G\pi\theta}{L} \times \frac{R^4}{4}$$

$$T = \frac{G\pi\theta}{2L} \left(\frac{D}{2}\right)^4$$

$$T = \frac{G\pi\theta D^4}{32L} = \frac{G\theta}{L} J$$

$$T = \frac{GJ\theta}{L} \text{ or } \frac{T}{J} = \frac{G\theta}{L} \dots (ii) \text{ by equation (i) \& (ii) } \frac{\tau}{R} = \frac{T}{J} = \frac{G\theta}{L}$$

where,

J = polar MOI about longitu axis

θ = angle of twist

G = shear modulus

Angle of twist will be

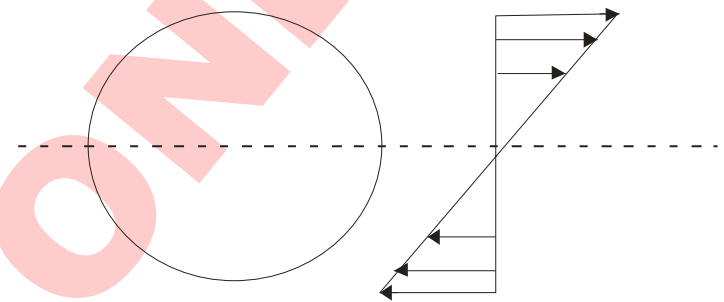
$$\theta = \frac{TL}{GJ}$$

Shear stress in fibre at distance 'r' from centre

$$\tau = \frac{Gr\theta}{L} = \frac{Gr}{L} \times \frac{TL}{GJ}$$

$$\tau = \frac{Tr}{J}$$

$$\tau_{\max} = \frac{T_{r_{\max}}}{J} = \frac{TR}{J}$$



variation of shear stress

Power Transmission By Shaft :-

Torque in shaft = T

angular velocity of shaft = ω

Power = $P = T \times \omega$

ω = angular velocity

1 round $\rightarrow 2\pi$ angle subtended at centre by N - rotation

$$\omega = \frac{2\pi N}{60}$$

where

N = rpm

Note :

(1) Angle of twist $\theta = \frac{TL}{GJ}$

(2) Shear stress $\tau = \frac{T_r}{J}$

(3) Power transfer $P = T \times \omega$

where ;

θ = angle of twist

T = Torque in shaft

J = polar MOI

G = Shear modulus

P = Power

ω = angular velocity

$$J = \frac{\pi d^4}{64} + \frac{\pi d^4}{64} \text{ (axis theorem)} = \frac{2\pi N}{60} \text{ (N in rpm)}$$

$$\frac{\pi d^4}{32} \text{ (along longitudinal direction) if N in rps - then}$$

$$\omega = 2\pi N$$

Connection of Shaft :- there are two type of connection which may be possible

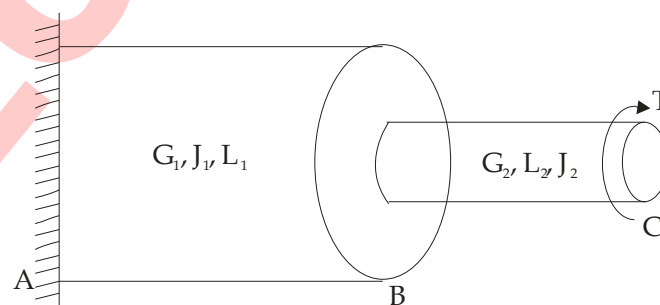
(1) **Series Connection :-**

In series connection

(a) torque will be same in each section

(b) angle of twist will be added from free end to fixed end

Series connection is described following

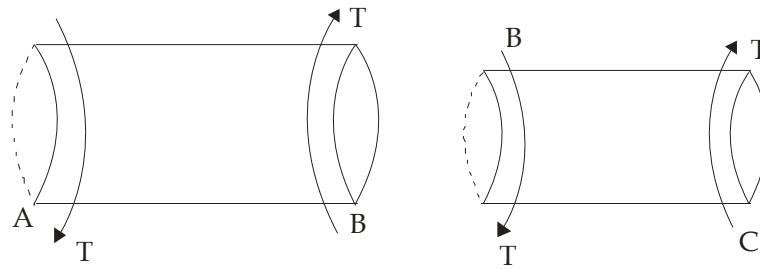


Q_{BA} = Rotation at 'B' wrt 'A'

Q_{CB} = Rotation at 'C' wrt 'B'

Q_{CA} = Rotation at 'C' wrt 'A'

$$\theta_{CA} = \theta_{CB} + \theta_{BA}$$



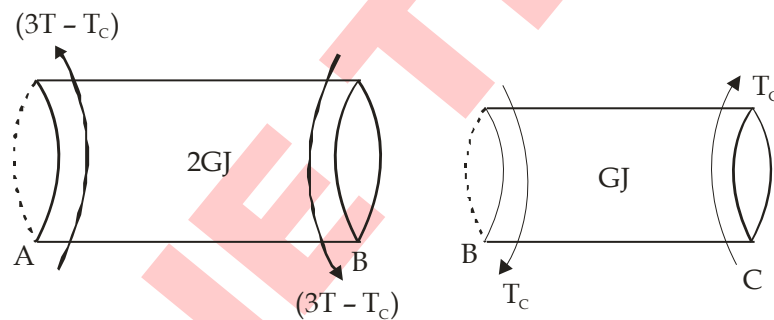
$$\theta_{BA} = \frac{(T_{AB})(L_{AB})}{(G_{AB})(J_{AB})} = \frac{TL_1}{G_1J_1} \text{ (clockwise)}$$

$$\theta_{CB} = \frac{(T_{BC})(L_{BC})}{(G_{BC})(J_{BC})} = \frac{TL_2}{G_2J_2} \text{ (clockwise)}$$

$$\theta_{CA} = \theta_{CB} + \theta_{BA} \left(\frac{TL_2}{G_2J_2} + \frac{TL_1}{G_1J_1} \right) \text{ (clockwise)}$$

For Equilibrium :-

$$T_A + 3T = T_C \dots (i)$$



$$\theta_{BA} = \frac{(3T - T_C)L}{2GJ} \text{ (anticlockwise)}$$

$$\theta_{CB} = \frac{T_{BC} \times L_{BC}}{G_{BC} \times J_{BC}} = \frac{T_C L}{GJ} \text{ (clockwise)}$$

Since shaft is fixed at A & C hence angle of rotation at C wrt A should be zero

$$\theta_{CA} = 0$$

$$\theta_{CA} = \theta_{CB} + \theta_{BA} = \frac{T_{CL}}{GJ} - \frac{(3T - T_C)}{2GJ} = 0$$

$$\frac{T_C L}{GJ} = \frac{(3T - T_C)L}{2GJ}$$

$$2T_C = 3T - T_C$$

$$3T_C = 3T$$

$$T_C = T$$

from equation (1)

$$T_A + 3T = T$$

$$T_A = -2T$$

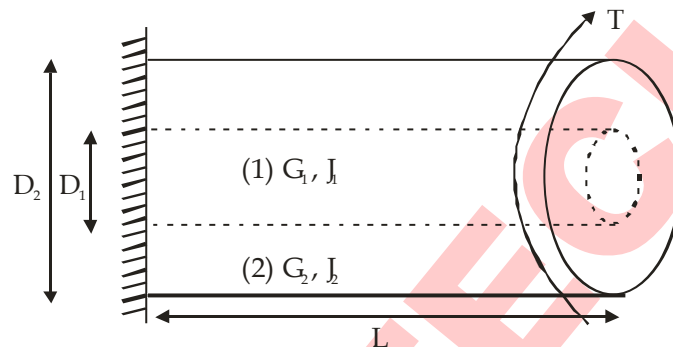
Since (-ve) sign represents that torque provided by support is opposite.

Parallel Connection :-**(1) Parallel Connection :-**

In parallel connection

- (a) angle of twist will be same in each section
- (b) torque will be added from free end to fixed end

Parallel connection is described following



T_1 = Torque resistance by shaft (1)

T_2 = Torque resistance by shaft (2)

then

$$T_1 + T_2 = T \text{ (1)}$$

Since both shafts are held together hence rotation of both shafts should be equal

$$\theta_1 = \theta_2$$

$$\frac{T_1 L}{G_1 J_1} = \frac{T_2 L}{G_2 J_2}$$

$$\frac{T_1}{T_2} = \frac{G_1 J_1}{G_2 J_2} \text{ (ii)}$$

from equation (i) and (ii)

$$T_1 = \left(\frac{G_1 J_1}{G_1 J_1 + G_2 J_2} \right) T$$

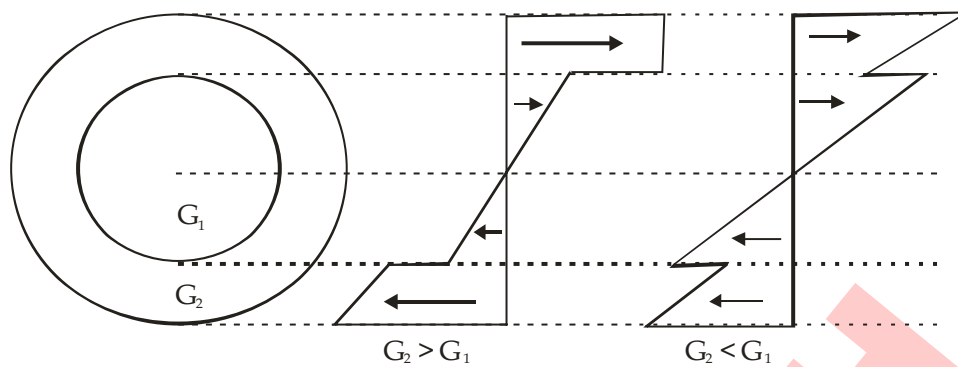
$$T_2 = \left(\frac{G_2 J_2}{G_1 J_1 + G_2 J_2} \right) T$$

→ Angle of rotation at free end

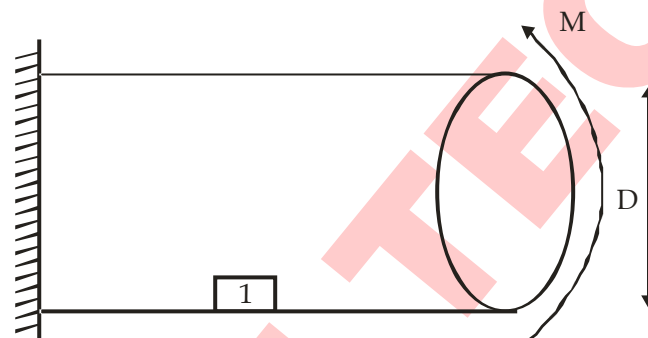
$$\theta_1 = \theta_2 = \theta = \frac{TL}{G_1 J_1 + G_2 J_2}$$

Note :-

Variation of shear stress in composite shaft



Equalent moment and torque in case of Combined Bending And Twisting :-
Case - I : Pure Bending :-



$$MOI = I = \frac{\pi D^4}{64}$$

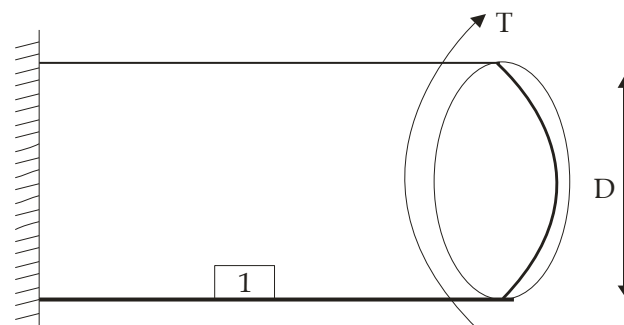
$$y_{\max} = \frac{D}{2}$$

$$\text{Max. Normal stress} = \sigma = \frac{M}{I} y_{\max}$$

$$\sigma = \frac{M}{\left(\frac{\pi D^4}{64}\right)} \times \frac{D}{2}$$

$$\sigma = \frac{32M}{\pi D^3}$$

Case-II Pure torsion :-



$$\text{Polar MOI} = J = \frac{\pi D^4}{32}$$

$$r_{\max} = \frac{D}{2}$$

$$\text{max shear stress at outer fibre } \tau = \frac{T r_{\max}}{J}$$

$$\tau = \frac{T \times (D/2)}{\frac{\pi D^4}{32}}$$

$$\tau = \frac{16T}{\pi D^3} \Rightarrow$$

Note

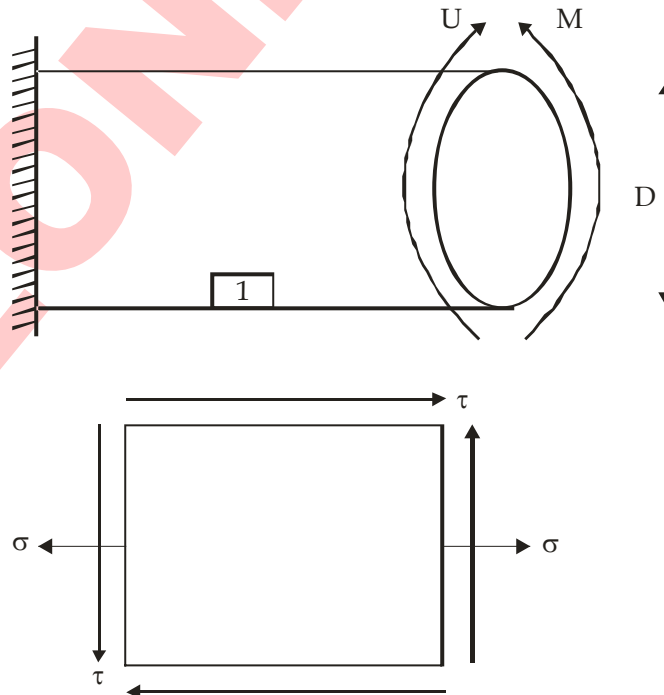
If a shaft of dia 'D' is subjected to a bending moment 'M' then max normal stress equal to

$$\sigma = \frac{32M}{\pi D^3}$$

If a shaft of dia 'D' is subjected to a torque 'T' then max shear stress in shaft equal to $\tau = \frac{16T}{\pi D^3}$

Ratio of max normal stress to max shear stress $\frac{\sigma}{\tau} = \frac{2M}{T}$

Case-III Combined bending and twisting :-



Here ' σ ' is not a P.stress

$$\sigma = \frac{32M}{\pi D^3}, \quad \tau = \frac{16T}{\pi D^3}$$

For element (1)

$$\sigma_x = \sigma = \frac{32M}{\pi D^3}$$

$$\sigma_y = 0$$

$$\tau_y = \tau = \frac{16T}{\pi D^3}$$

Major principal stress

$$\begin{aligned} \sigma_1 &= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \left(\frac{\frac{32M}{\pi D^3} + 0}{2}\right) + \sqrt{\left(\frac{\frac{32M}{\pi D^3} - 0}{2}\right)^2 + \left(\frac{16T}{\pi D^3}\right)^2} \\ &= \frac{16M}{\pi D^3} + \sqrt{\left(\frac{16M}{\pi D^3}\right)^2 + \left(\frac{16T}{\pi D^3}\right)^2} \\ \sigma_1 &= \frac{16}{\pi D^3} \left[M + \sqrt{M^2 + T^2} \right] \end{aligned}$$

Minor principal stress :-

$$\sigma_2 = \frac{16}{\pi D^3} \left[M - \sqrt{M^2 + T^2} \right]$$

Max shear stress

$$\tau_{\max} = \frac{16}{\pi D^3} \left(\sqrt{M^2 + T^2} \right)$$

Equivalent Bending Moment :- Bending moment which generates max normal stress equal to max normal stress in case of combined bending and twisting



max normal stress due to M_{eq}

$$s_1' = \frac{32M_{eq}}{\pi D^3}$$

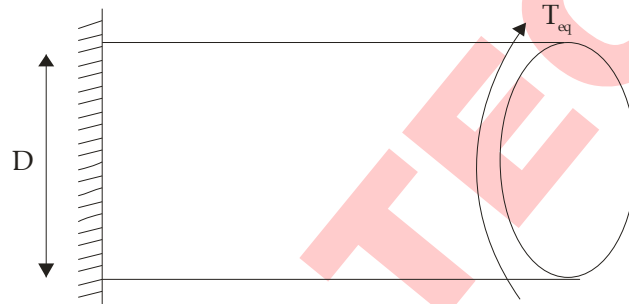
For

$$\sigma_1' = \sigma_1$$

$$\frac{32M_{eq}}{\pi D^3} = \frac{16}{\pi D^3} \left[M + \sqrt{M^2 + T^2} \right]$$

$$M_{eq} = \frac{1}{2} \left[M + \sqrt{M^2 + T^2} \right]$$

Equivalent Torque :- The torque which generates max shear stress equal to max shear stress in case of combined bending and twisting is known as T_{eq} .



Max shear stress due to T_{eq} -

$$\tau_{max} = \frac{16T_{eq}}{\pi D^3}$$

For

$$\tau_{max}' = \tau_{max}$$

$$\frac{16T_{eq}}{\pi D^3} = \frac{16}{\pi D^3} \left[\sqrt{M^2 + T^2} \right]$$

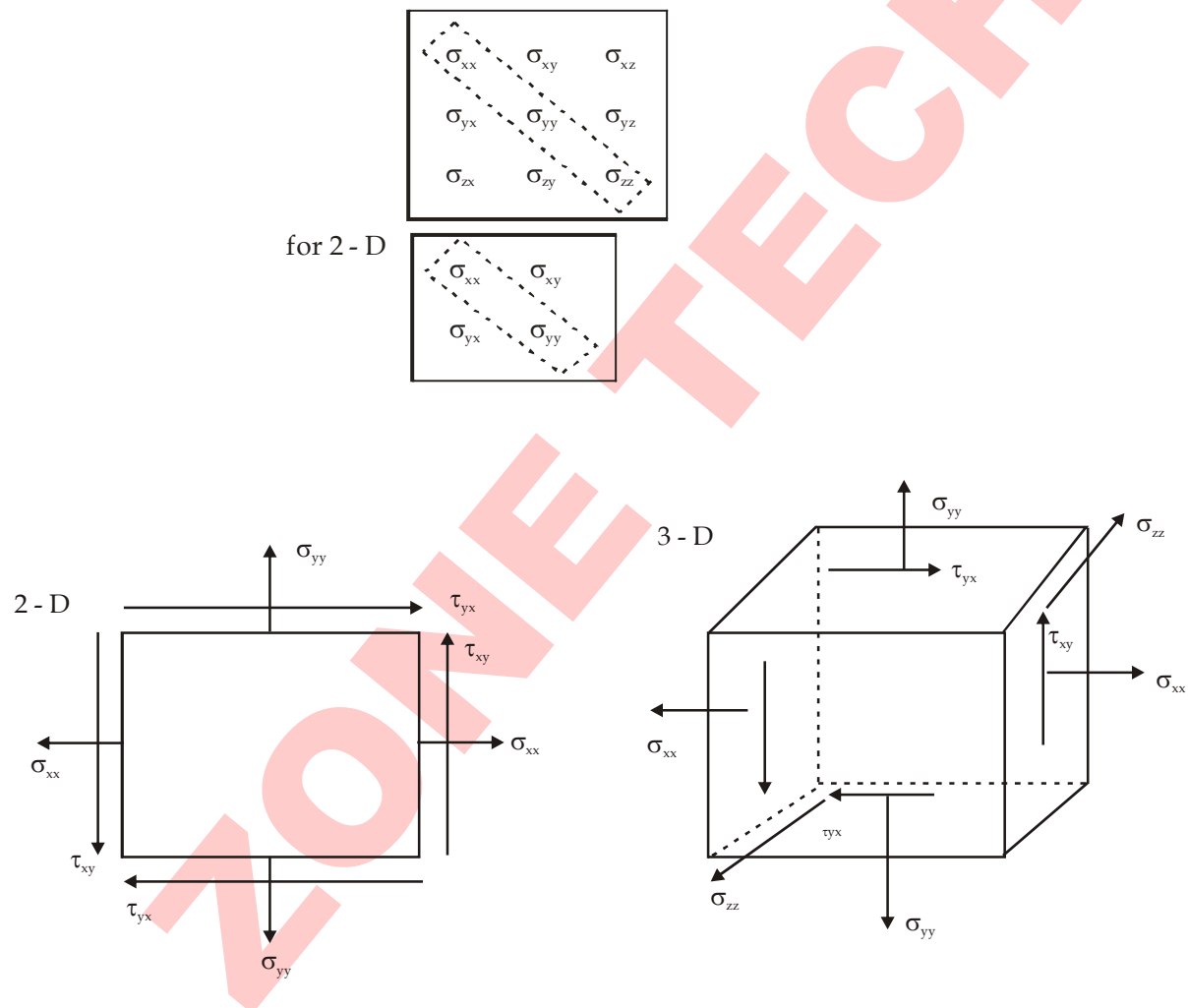
$$T_{eq} = \sqrt{M^2 + T^2}$$

CHAPTER - 8

(PRINCIPAL STRESS AND STRAIN)

Stress Tensor :- A matrix which is used to represent the stresses on different planes at a particular point is known as stress tensor.

Stress Tensor For 3 -D :-



Note :- Elements in leading diagonal of stress tensor represents normal stress.

Major principal stress & minor principal stress.

For a given stress condition at a point, out of various planes, there are two planes on which shear stress is zero, these planes are called principal planes. Out of these two planes the plane on which normal stress is maximum (magnitude wise) is called major principal plane & another is called minor principal plane. Normal stress, on major principal plane is known as major principal stress and stress at minor P.P. is known as minor principal stress.

Principal Plane :-

A plane which have zero shear - stress (i.e plane have only normal stress) is known as principal plane.

Normal stress acting on principal plane is known as principal stress.

Max shear plane :-

A plane which has max value of shear stress is known as max shear plane.

On max shear plane, normal stress may or may not be zero

A plane on which normal stress is zero (i.e plane has only shear stress) is known as pure shear plane.

Stress Analysis For 1-D :-

The analysis of 1-D stress can be understood for the help of following equations

$$p_n = p \cos^2 \theta$$

$$p_t = -\frac{p}{2} \sin 2\theta$$

Ex. A tie bar is subjected to a uniform tensile stress of 100 N/mm^2 . Find the intensity of normal stress, shear stress and resultant stress on a plane the normal to which is inclined 30° to the axis of the bar. Also estimate the maximum shear stress in the bar.

Sol.

$$p_n = p \cos^2 \theta = 100 \cos^2 30^\circ = 75 \text{ N/mm}^2$$

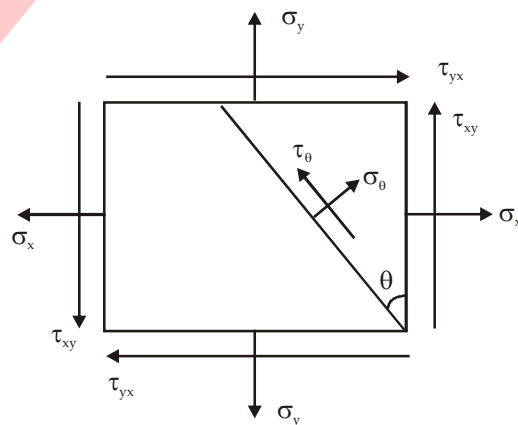
$$p_t = -\frac{p}{2} \sin 2\theta = -\frac{100}{2} \sin 60^\circ = -43.3 \text{ N/mm}^2$$

(i.e. in the clockwise direction)

$$p_r = \sqrt{p_n^2 + p_t^2} = \sqrt{(75)^2 + (43.3)^2} = 86.6 \text{ N/mm}^2$$

$$(p_t)_{\max} = -\frac{p}{2} = -\frac{100}{2} = -50 \text{ N/mm}^2 \text{ at } \theta = +45^\circ$$

Stress Analysis For 2-D :- Expression for normal stress and shear stress develop on in oblique plane passing through a point under bi-axial stress.



→ Normal stress acting on plane (at angle θ with vertical)

$$\sigma_v = \frac{1}{2}[\sigma_x + \sigma_y] + \frac{1}{2}[\sigma_x - \sigma_y] \cos 2\theta + \tau_{xy} \sin 2\theta$$

→ Shear stress on plane at angle θ

$$\tau_\theta = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos 2\theta$$

Direction of Principal Plane :-

If angle of principal plane with vertical is θ_p

$$\tau_\theta / \theta = \theta_p = 0$$

$$\tau_{\theta=\theta_p} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\tan 2\theta_p = \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

$$\theta_p = \frac{1}{2} \tan^{-1} \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y} \right)$$

Note :-

If angle of one principal plane (with vertical)

$$= \theta_{p_1} = \theta_p$$

then angle of another principal plane / with vertical

$$= \theta_{p_2} = \theta_{p_1} + 90^\circ$$

Note :

Summation of normal stresses acting on two mutually perpendicular planes is const

$$\sigma_\theta = \sigma_x \cos^2 \theta + \sigma_y \sin^2 \theta + \tau_{xy} \sin 2\theta \dots (1)$$

$$\sigma_{(\theta+90)} = \sigma_x \cos^2 (90 + \theta) + \sigma_y \sin^2 (90 + \theta) + \tau_{xy} (\sin 2(90 + \theta))$$

$$\sigma_{(\theta+90)} = \sigma_x \sin^2 \theta + \sigma_y \cos^2 \theta - \tau_{xy} \sin 2\theta \dots (ii)$$

eq (i) + (ii)

$$\sigma_\theta + \sigma_{(\theta+90)} = \sigma_x + \sigma_y$$

Direction of Max Shear Plane :-

Shear stress on a plane at angle θ

$$\tau_\theta = \left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos 2\theta$$

→ Max value of shear stress

$$\frac{d\tau_\theta}{d\theta} = 0$$

$$\frac{d\tau_\theta}{d\theta} = \left(\frac{\sigma_x - \sigma_y}{2}\right) \cos 2\theta \times 2 - \tau_{xy} (\sin 2\theta) \times 2 = 0$$

$$\frac{-\sigma_x + \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tan 2\theta = \left(\frac{-\sigma_x + \sigma_y}{2\tau_{xy}}\right)$$

If angle of max shear plane is θ_s then

$$\tan 2\theta_s = -\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right)$$

$$\theta_s = \frac{1}{2} \tan^{-1} \left[-\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) \right]$$

Note :- Max shear plane exist at bisector of principal planes.

→ If angle of principal plane is θ_p (with vertical) then

$$\tan 2\theta_p = \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) \dots (1)$$

If angle of max shear plane is θ_s (with vertical)

$$\tan 2\theta_s = \left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) \dots (ii)$$

from (i) $\times$ (ii)

$$(\tan 2\theta_p) \times (\tan 2\theta_s) = \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) \times \left[-\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right)\right]$$

$$2\theta_p \times 2\theta_s = -1$$

$$2\theta_s = 2\theta_p + 90^\circ$$

$$\theta_s = \theta_p \pm 45^\circ$$

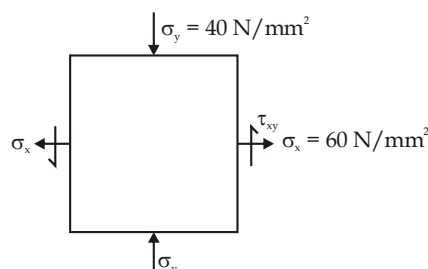
Principal stresses represent the min and max value of normal stress at a particular point hence the value of maximum and minimum principal stress will be

$$\sigma_{1/2} = \frac{1}{2}[\sigma_x + \sigma_y] \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

Mohr Circle :- It is the graphical representation of normal stress and shear stress at different planes at a particular point.

Mohr's circle of stress is a graphical representation which is extremely useful in visualizing the relationship between normal and shear stresses acting on various inclined planes at a point in a stressed body. It is also commonly known as circular diagram for stress. Some important features of **Mohr's circle of stress** are :

1. Mohr circle is a locus of normal and shear stresses at a point in a strained body with the changing value of θ at that point.
2. Radius of Mohr's circle represents the value of maximum shear stress.



3. If Y-axis represents shear stress (τ) and normal stress (σ) is represented on X-axis, then Mohr's circle cuts σ axis at two points. One of them represents major principal stress and the other represents minor principal stress. The centre of Mohr's circle always lies on σ axis.

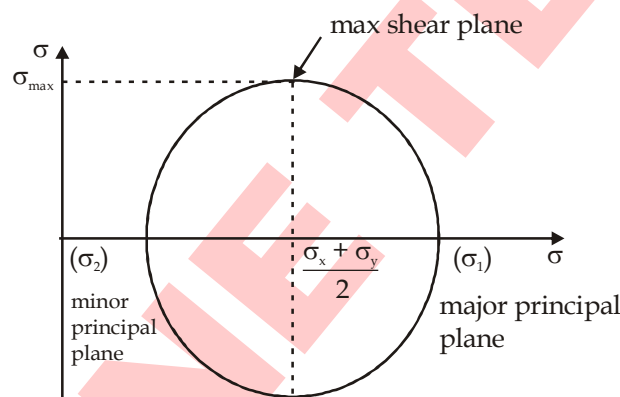
We know that $\sigma_1 / \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

where, σ_1 = major principal stress
 σ_2 = minor principal stress

Centre of Mohr's Circle = $\left(\frac{\sigma_x + \sigma_y}{2}, 0\right)$

Radius of mohr's circle

$$= R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$



(1) Major Principal Stress :-

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + R$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

(2) Minor Principal Stress :-

$$\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

(3) Max Shear Stress :-

$$\tau_{\max} = R$$

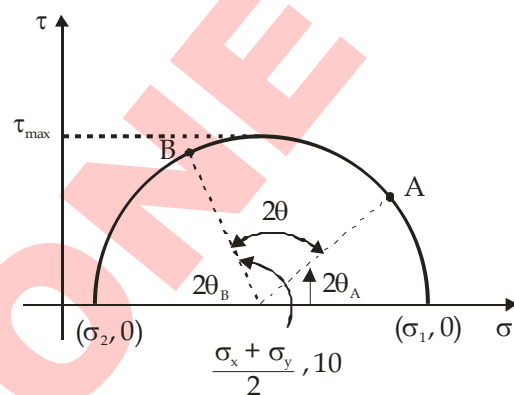
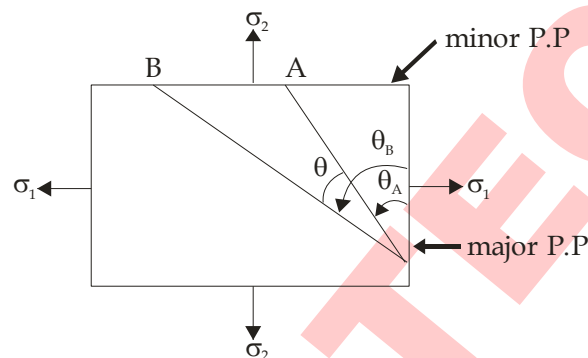
$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} = \left(\frac{\sigma_1 - \sigma_2}{2}\right)$$

(4) Normal stress on max shear plane :-

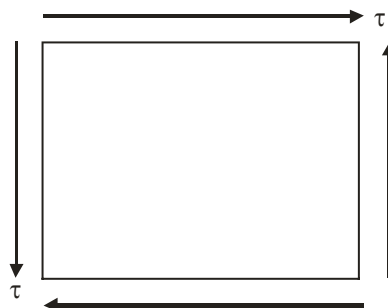
$$= \left(\frac{\sigma_x + \sigma_y}{2} \right)$$

Properties of Mohr Circle :-

- (I) Centre of mohr circle lies on " σ -axis"
- (II) Radius of mohr circle represents max shear stress ($\tau_{\max}$)
- (III) Normal stress on max shear planes is equal
- (IV) If the angle between two planer in elemental diagram (stress element) is ' θ ' then in mohr circle, angle between those planes will be ' 2θ '

**Case - I**

- (I) Mohr circle for pure shear condition



$$\sigma_x = \sigma_y = 0$$

$$\tau_{xy} = \tau$$

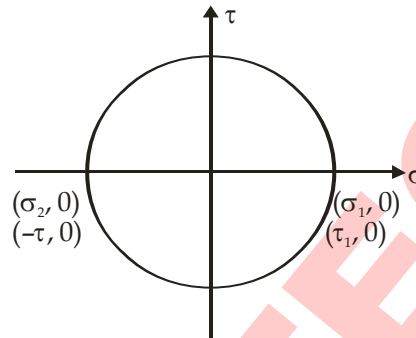
centre of mohr's

circle = (0, 0)

Radius of mohr

circle = $R = \tau$

Major principal



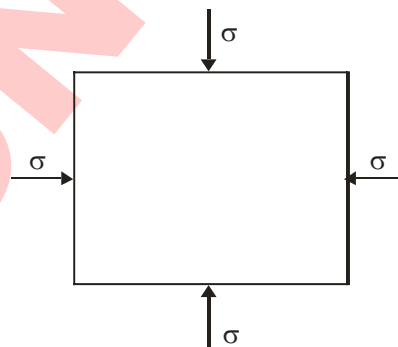
stress = $\sigma_1 = \tau$

minor p.s = $\sigma_2 = -\tau$

max shear stress = $\tau_{\max} = \tau$

Case - II

Mohr circle for hydrostatic loading



$$\sigma_x = \sigma_y = -\sigma$$

$$\tau_{xy} = 0$$

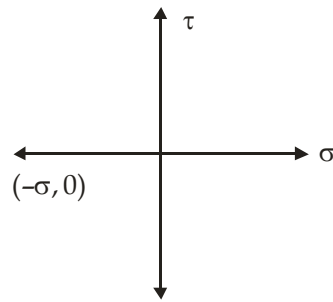
→ Centre of mohr's

circle = $(-\sigma, 0)$

→ Radius of mohr's circle

$R = 0$

→ major principal



$$\text{stress} = \sigma_1 = -\sigma$$

$$\text{minor principal stress} = \sigma_2 = -\sigma$$

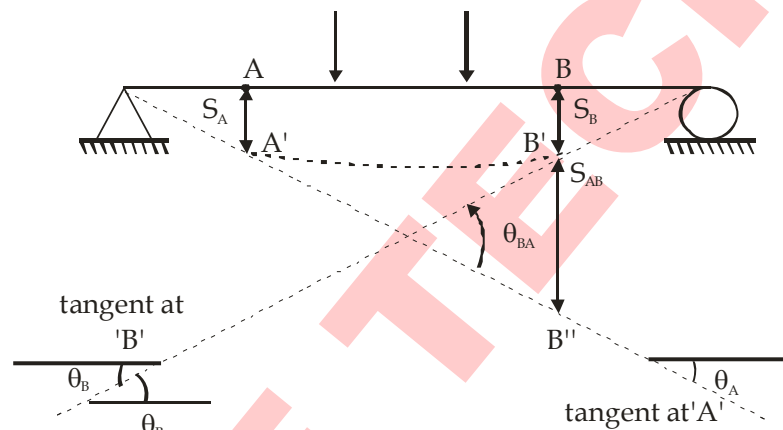
$$\text{max shear stress} = \tau_{\max} = 0$$

ZONE TECH

CHAPTER - 9

(DEFLECTION OF BEAM)

The aim of this chapter is to derive the expression for maximum slope and maximum deflection for various beam under different loading condition. There are many methods to find out slope and deflection in a beam. Some methods are described below for cantilever and simply supported beam.



θ_A = slope at 'A'

θ_B = slope at 'B'

θ_{BA} = slope at 'B' wrt tangent at 'A'

δ_A = deflection at 'A' = AA'

δ_B = deflection at B = BB'

δ_{BA} = deflection at 'B' wrt to tangent at 'A' = B'B''

(i) Double Integration Method :-

From bending equation

$$\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}$$

$$\frac{1}{R} = \frac{M}{EI} \quad \dots (i)$$

$$\frac{1}{R} = \left(\frac{d^2 y}{dx^2} \right) \frac{1}{R} = \frac{\left(\frac{d^2 y}{dx^2} \right)}{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}}$$

for small deflection

$$\left(\frac{dy}{dx}\right)^2 \ll 1$$

Hence,
$$\frac{1}{R} = \frac{d^2y}{dx^2} \dots (ii)$$

from (i) and (ii).....

$$\frac{1}{R} = \frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$M = (E.I_{NA})_{x-x} \left[\frac{d^2y}{dx^2} \right]$$

$$\frac{1}{R} = \text{curvature}$$

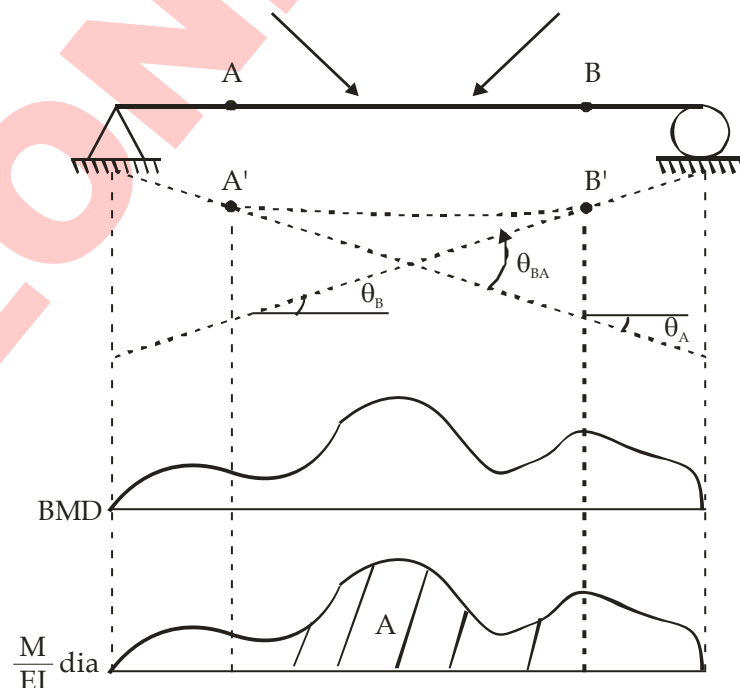
R → Radius of curvature

2nd derivative of deflection ∝ Bending moment

(ii) Area Moment Method :-

(I) Area - Moment Theorem - I :-

→ According to this theorem slope at any point 'B' wrt the tangent at other point 'A' is equal to area of M/EI diagram between those points



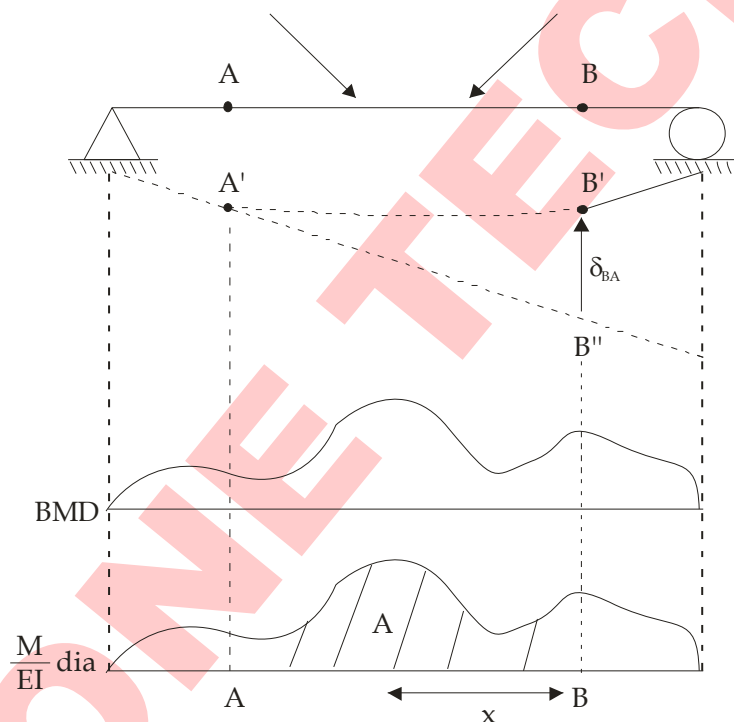
Acc to area moment theorem - I

$$\theta_{BA} = \text{area of } \frac{M}{EI} \text{ diagram between A + B}$$

where : θ_{BA} = slope at 'B' wrt tangent at 'A'

(II) Area Moment theorem - II :

According to this theorem the deflection of any point 'B' wrt tangent at any other point 'A' is equal to the moment of area of M/EI diagram between 'A' and 'B' about point 'B'.



$$\delta_{BA} = \text{deflection at B wrt tangent at 'A'}$$

Acc to area moment theorem - II

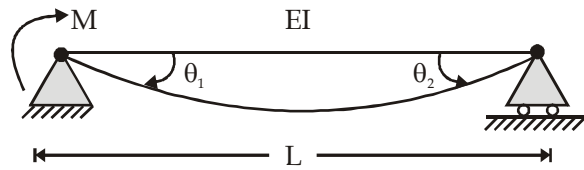
$$\delta_{BA} = \text{moment of area of } M/EI \text{ dia between A \& B about point B'}$$

$$[(\text{Area}) \times \bar{x}]$$

Ex. Calculate moment to be applied at the left support of a simply supported beam of span length L to cause unit rotation (1 radian) at that support.



Sol.



We know that

$$\theta_1 = \frac{ML}{3EI}$$

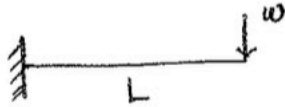
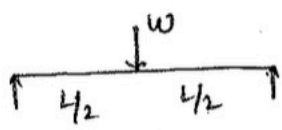
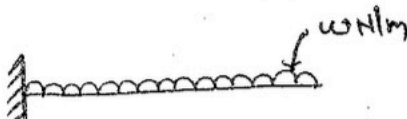
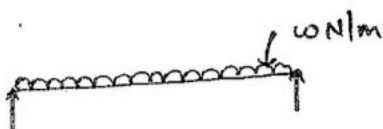
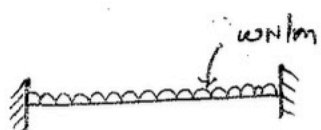
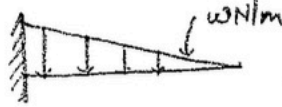
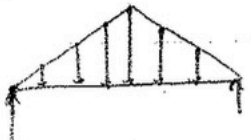
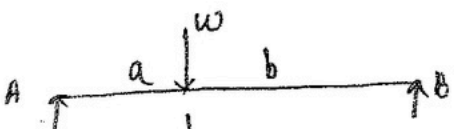
and

$$\theta_2 = \frac{ML}{6EI}$$

Given that $\theta_1 = 1$

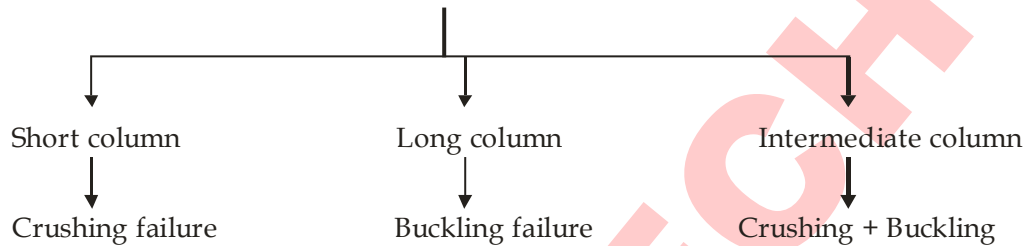
$$\Rightarrow \frac{ML}{3EI} = 1$$

$$\Rightarrow M = \frac{3EI}{L}$$

Beam	Slope	Deflection
 	$\theta_{\max} = \frac{ML}{EI}$ $\theta_{\max} = \frac{ML}{2EI}$	$y_{\max} = \frac{ML^2}{2EI}$ $y_{\max} = \frac{ML^2}{8EI}$
  	$\theta_{\max} = \frac{wL^2}{2EI}$ $\theta_{\max} = \frac{wL^2}{16EI}$ $\theta_{\max} = 0$	$y_{\max} = \frac{wL^3}{3EI}$ $y_{\max} = \frac{wL^3}{48EI}$ $y_{\max} = \frac{wL^3}{192EI}$
  	$\theta_{\max} = \frac{wL^3}{6EI}$ $\theta_{\max} = \frac{wL^3}{24EI}$ $\theta_{\max} = 0$	$y_{\max} = \frac{wL^4}{8EI}$ $y_{\max} = \frac{5wL^4}{384EI}$ $y_{\max} = \frac{wL^4}{384EI}$
  	$\theta_{\max} = \frac{wL^3}{24EI}$ $\theta_{\max} = \frac{5wL^3}{192EI}$ $\theta_{\max} = \frac{wL^3}{8EI}$	$y_{\max} = \frac{wL^4}{30EI}$ $y_{\max} = \frac{wL^4}{120EI}$ $y_{\max} = \frac{wL^4}{120EI}$
	$\theta_c = \frac{wb(a^2 - ab)}{3EIL}$	$y_c = \frac{wa^2b^2}{3EIL}$

CHAPTER - 10 (COLUMN)

column is a vertical member on which axial compressive load is applied. Column may be classified in following ways



Euler's Theory :- This theory is used for long column

Assumptions :-

- (1) Material is isotropic, homogenous and elastic
- (2) Column is long column
- (3) Column is initially straight and C/S area is constant throughout the length.
- (4) Load applied on column is concentric
- (5) Failure of column is due to buckling only
- (6) Self weight of column is neglected.
- (7) Hinge are frictionless

ACC to Euler's Theory :-

$$\text{Critical load for column} = P_{cr} = \frac{\pi^2 EI_{min}}{(L_{eff})^2}$$

where

I_{min} = minimum MOI

L_{eff} = effective length = kL

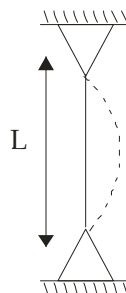
E = young's modulus

Effective Length :-

Support condition

eff. length ($L_{eff} = kL$)

k

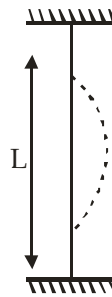


- (1) Both ends are winged

L

1

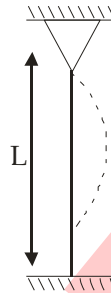
(2) Both ends are fixed



$$\frac{L}{2}$$

$$\frac{1}{2}$$

(3) One end hinged & other end is fixed



$$\frac{L}{\sqrt{2}}$$

$$\frac{1}{\sqrt{2}}$$

(4) One end is fixed and other end is free



$$2L$$

$$L$$

The limitation of Euler's theory.

- Euler's formula is not valid for short column.
- It determines only the critical load and not the working load.
- It doesn't consider the strength of material.

Definition of slenderness ratio.

Slenderness ratio is a quantity that measures tendency of a column to buckle. It is the ratio of effective length of the column to the appropriate radius of gyration i.e.

Slenderness ratio (λ) = Effective length of the column / Radius of gyration

Higher the slenderness ratio, greater is the tendency of column to buckle.

Factors governing slenderness ratio.

The various factors that affect slenderness ratio of a column are :

- Length of the column
- End conditions of the column
- Cross sectional dimensions of the column.

Rankine's Theory :- In this theory effect of crushing failure is also considered

If load carrying capacity of column is 'P' then

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_{cr}}$$

where :-

$$P_c = \text{crushing load} = \sigma_c A$$

$$\sigma_c = \text{crushing strength}$$

$$P_{cr} = \text{critical buckling load}$$

$$= \pi^2 EI_{\min} (\text{left})^2$$

$$P = \frac{P_c P_{cr}}{P_c + P_{cr}}$$

$$P = \frac{P_c}{1 + \left(\frac{P_c}{P_{cr}} \right)} = \frac{\sigma_c A}{1 + \left(\frac{\sigma_c A}{\frac{\pi^2 EI_{\min}}{(\text{left})^2}} \right)}$$

$$P = \frac{\sigma_c A}{1 + \frac{\sigma_c A}{\pi^2 EI_{\min}} (\text{left})^2}$$

$$I_{\min} = A r_{\min}^2$$

$$r_{\min} = \text{min radius of gyration.}$$

$$P = \frac{\sigma_c A}{1 + \frac{\sigma_c A}{\pi^2 EA (r_{\min})^2} \times (\text{left})^2}$$

$$P = \frac{\sigma_c A}{1 + \frac{\sigma_c}{\pi^2 E} \left(\frac{\text{left}}{r_{\min}} \right)^2} = \frac{\sigma_c A}{1 + \alpha (\lambda_c)^2}$$

where :- $\lambda_c = \text{critical slenderness ratio} = \frac{\text{left}}{r_{\min}}$

$$\alpha = \text{Rankine's coeff} = \frac{\sigma_c}{\tau^2} \in$$

End Condition
Value of 'n' and ' α '

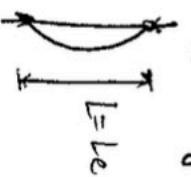
n

$\alpha = \frac{1}{\sqrt{n}}$

$L_e = \alpha \cdot L$

$P_c = \frac{n \pi^2 E I_{min}}{L^2}$

Both end Hinged



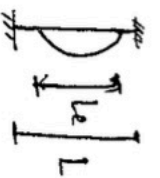
1

1

$L_e = L$

$P_c = \frac{\pi^2 E I_{min}}{L^2}$

Both end fixed



4

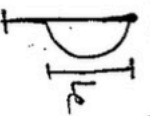
$1/2$

$L_e = L/2$

$P_c = \frac{4 \pi^2 E I_{min}}{L^2}$

maximum

one end fixed and one hinged



2

$\frac{1}{\sqrt{2}}$

$L_e = \frac{L}{\sqrt{2}}$

$P_c = \frac{2 \pi^2 E I_{min}}{L^2}$

one end fixed and one free



0.25 $\frac{2\pi}{4}$

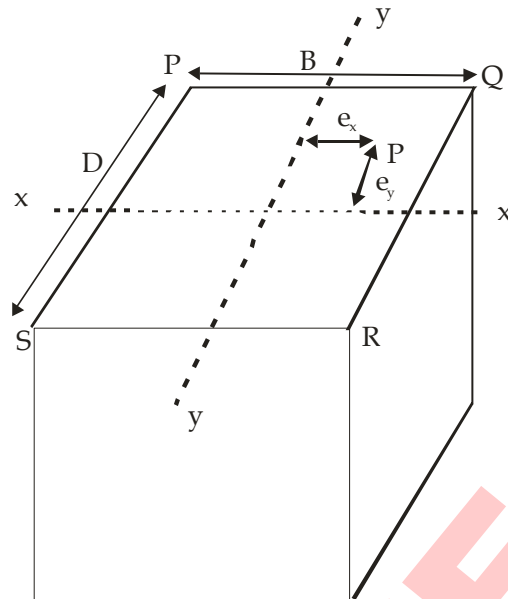
2

$L_e = 2L$

$P_c = \frac{\pi^2 E I_{min}}{4L^2}$

minimum

→ Eccentric Load On Column :-

→ e_x = eccentricity about y-axis M_y = moment about y-axis

$$M_y = P(e_x)$$

→ e_y = eccentricity about x-axis M_x = moment about x-axis

$$M_x = P(e_y)$$

→ I_x = MOI about x-axis I_y = MOI about y-axis

→ Max stress due to axial load P

$$= \sigma_1 = \frac{P}{\text{area}} = \frac{P}{B \times D}$$

→ Max stress due to moment M_x

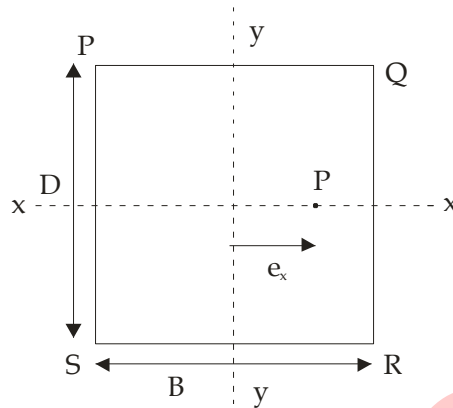
$$= \sigma_2 = \frac{M_x}{I_x} y_{\max} = \frac{P(e_y)}{\left(\frac{BD^3}{12}\right)} \left(\frac{D}{2}\right) = \frac{6P_{ey}}{BD^2}$$

→ Max stress due to moment M_y

$$= \sigma_3 = \frac{M_y}{I_y} y_{\max} = \frac{P(e_x)}{\left(\frac{BD^3}{12}\right)} \frac{B}{2} = \frac{6Pe_x}{DB^2}$$

Assume, Tensile stress as (+ve) then total stress at point 'P' = $\sigma_p = -\sigma_1 - \sigma_2 + \sigma_3$ Total stress at pt 'Q' = $\sigma_Q = -\sigma_1 - \sigma_2 + \sigma_3$ Total stress at pt 'R' = $-\sigma_1 + \sigma_2 - \sigma_3$ Total stress at pt 'S' = $-\sigma_1 + \sigma_2 + \sigma_3$

Kern / Core Area :-



If tensile stress is +ve then stress at 'P'

$$\begin{aligned}\sigma_P &= \frac{-P}{\text{Area}} + \frac{M_y}{I_y} (y_{\max}) \\ &= \frac{-P}{BD} + \frac{P(e_x)}{\left(\frac{DB^3}{12}\right)} \times \frac{B}{2} \\ &= \frac{-P}{BD} + \frac{6Pe_x}{DB^2}\end{aligned}$$

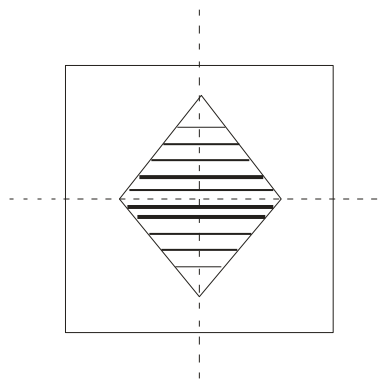
If max eccentricity for no tension at 'P' is $(e_x)_{\max}$ then for $e_x = (e_x)_{\max}$ $\sigma_P = 0$

$$\begin{aligned}\frac{-P}{BD} + \frac{6P(e_x)_{\max}}{DB^2} &= 0 \\ (e_x)_{\max} &= \frac{B}{6}\end{aligned}$$

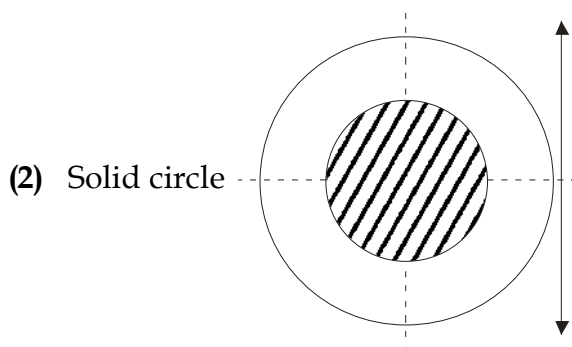
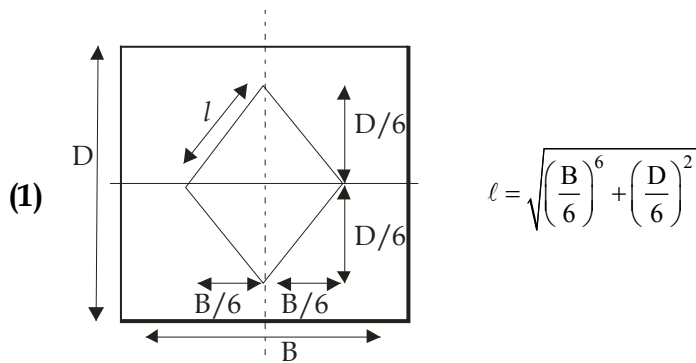
similarly

$$(e_y)_{\max} = \frac{D}{6}$$

→ Area in which when compressive load is applied then entire section will be under compression is known as core area/kern area

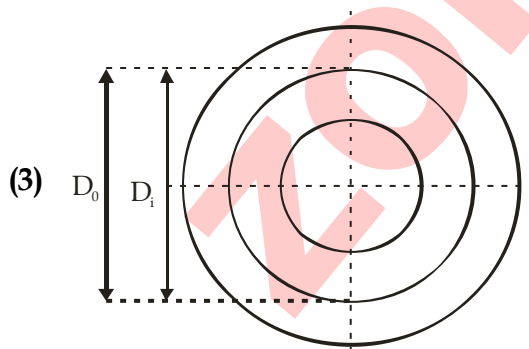


Kern Area For Diff Section :-



Dia of core area = $\frac{D}{4}$

Radius of core area = $\frac{D}{8}$



Dia of core area = $\frac{D_o^2 + D_i^2}{4D_o}$

Radius of core area = $\frac{D_o^2 + D_i^2}{8D_o}$

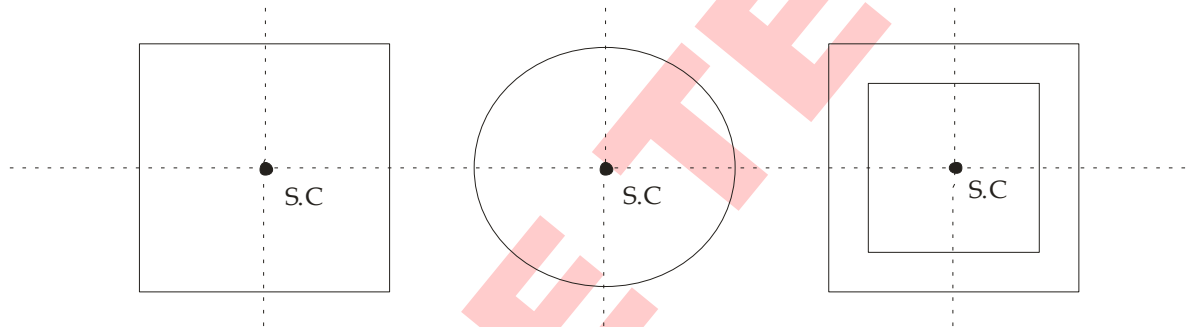
Note :-

Section	Shape of core area
(i) Rectangle	Rhombus
(ii) Square	Square / Rhombus
(iii) I-section	Rhombus
(iv) Solid circular section	Circular
(v) Hollow circular section	Circular

Note :- SHEAR CENTER

A point through which when load passes then no twisting (only bending) occurs in section, is known as shear centre

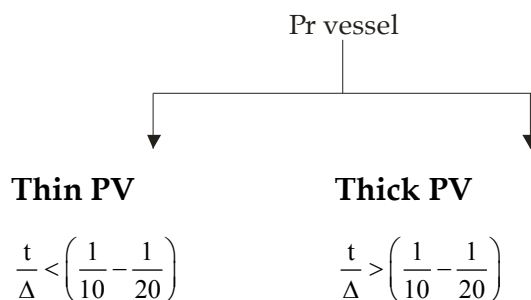
- Shear centre always lie on the axis of symmetric
- If any section has more than one axis of symmetry then shear centre will lie at the intersection of those axis of symmetry
- If a section has combination of plates then shear centre will lie at the intersection of axis of those plates



CHAPTER - 11

(THIN CYLINDER AND SPHERICAL SHELLS)

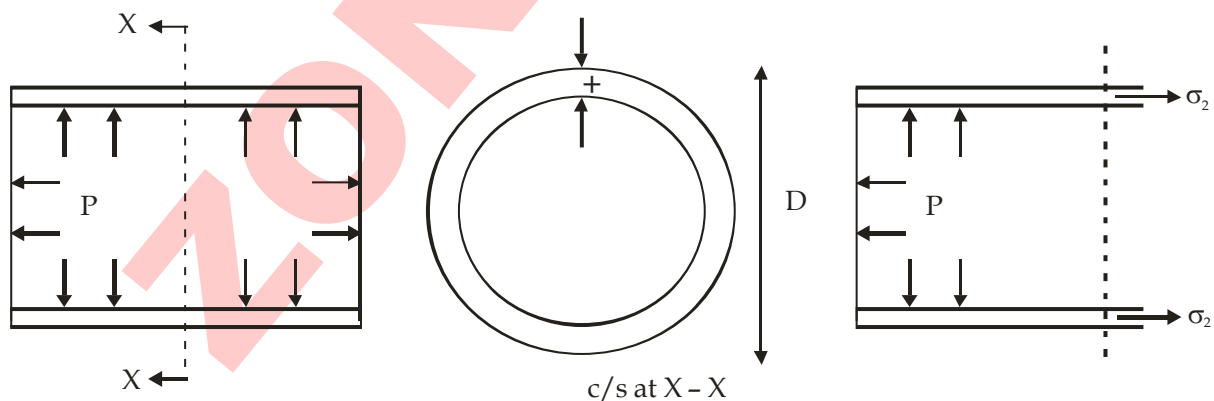
Thin Cylindrical and Spherical Pressure Vessel



Thin Pressure Vessel :-

- Stresses along the circumference, are known as circumferential stress or hoop stress.
- Stresses which generates in longitudinal direction are known as longitudinal stress or merdonial stress.
- Stresses which generates in radial direction are known as radial stresses.

Thin Cylinder :-



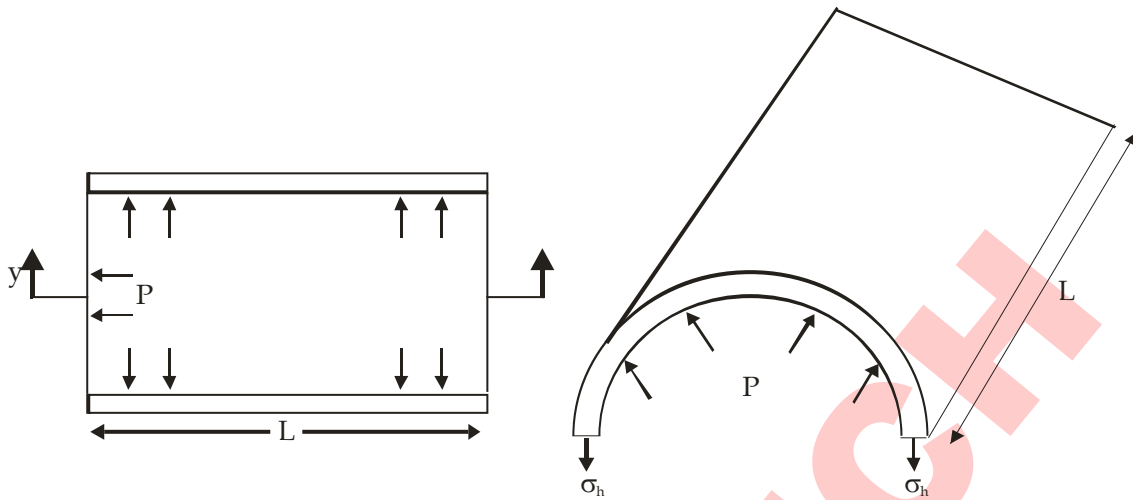
(a) Longitudinal Stress :-

For equilibrium

$$P \times \frac{\pi D^2}{4} = \sigma_L \times \pi D t$$

$$\sigma_L = \frac{pD}{4t}$$

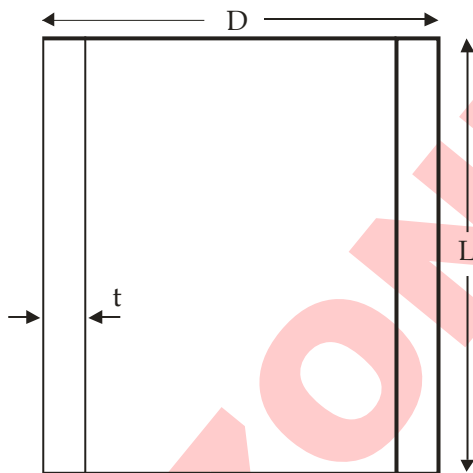
(b) Hoop Stress :-



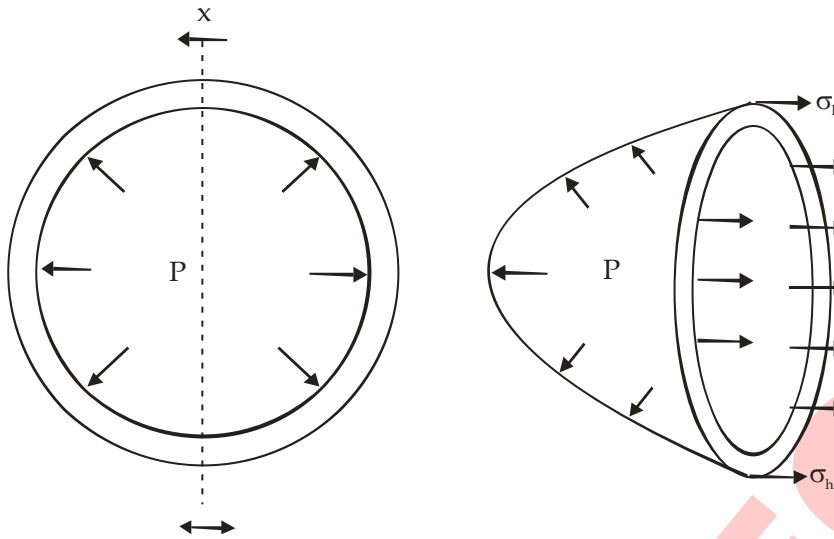
For equilibrium :-

$$P(\Delta \times L) = \sigma_h [(L \times t) + (L \times t)]$$

$$\sigma_h = \frac{p\Delta}{2t}$$



Thin Sphere :-



For equilibrium :-

$$\left(p \times \frac{\pi D^2}{4} \right) = \sigma_h \times \pi D \times t$$

$$\sigma_h = \frac{pD}{4t}$$

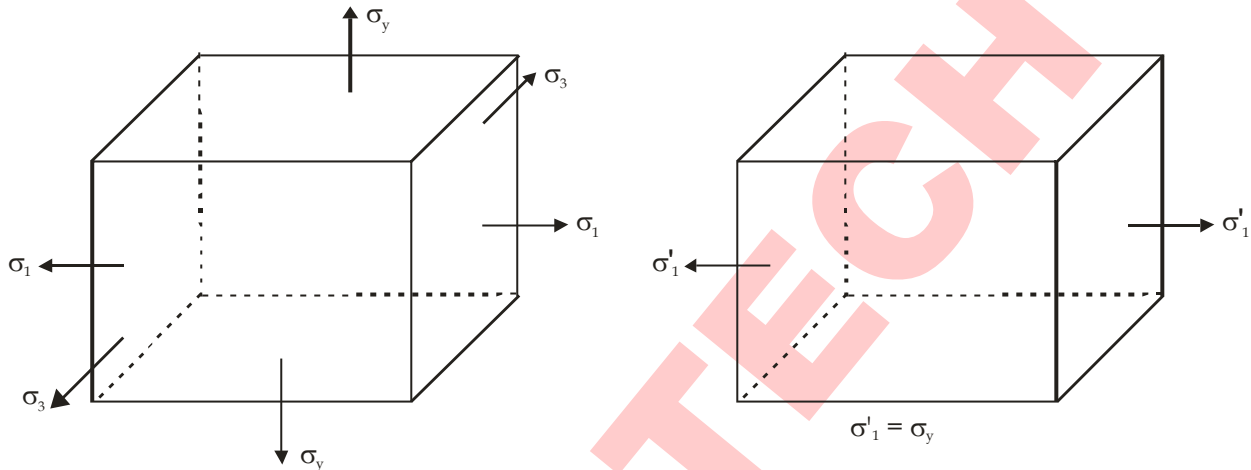
$$\sigma_L = \frac{pD}{4t}$$

CHAPTER - 12

(THEORIES OF FAILURE)

(I) Maximum Normal Stress Theory (Rankine's Theory) or Principal Stress Theory

3-D



According to this theory a material will fail when max normal stress (σ_1) reaches to max normal stress in uni-axial loading at failure (σ'_1)

Acc to this theory

At failure

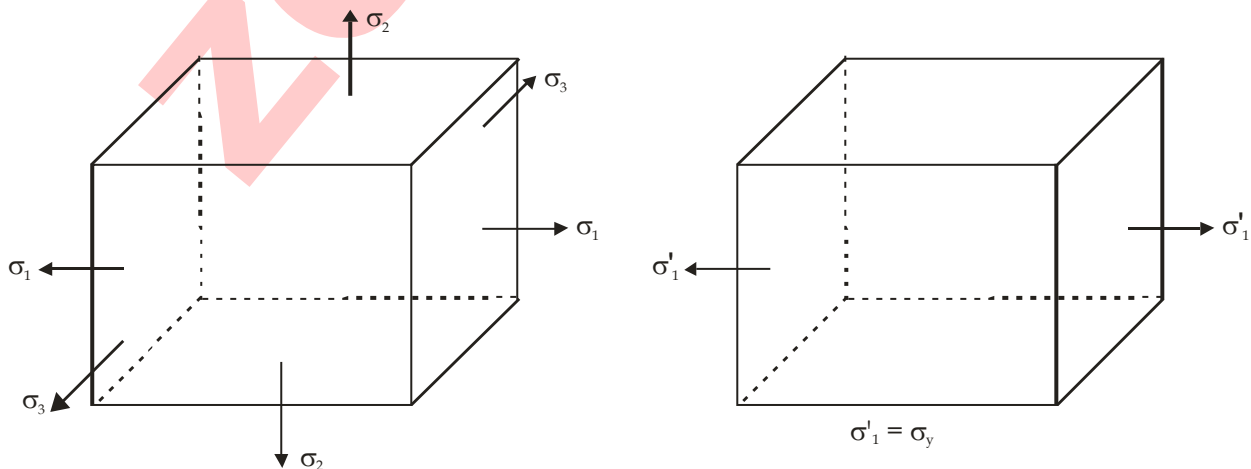
$$\sigma_1 = \sigma'_1 = \sigma_y$$

Hence,

$$\sigma_1 \leq \left(\frac{\sigma_y}{Fos} \right)$$

(II) Max normal strain theory (Strenant's theory) :- Acc to this theory a material will fail when max normal strain (ϵ_1) reaches to max normal strain at failure in uniaxial loading

3 - D



$$\sigma_1 > \sigma_2 > \sigma_3$$

$$\epsilon_1 = \frac{\sigma'_1}{E} = \frac{\sigma_y}{E}$$

$$\epsilon_1 = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E} - \frac{\mu\sigma_3}{E}$$

Acc. to this theory -

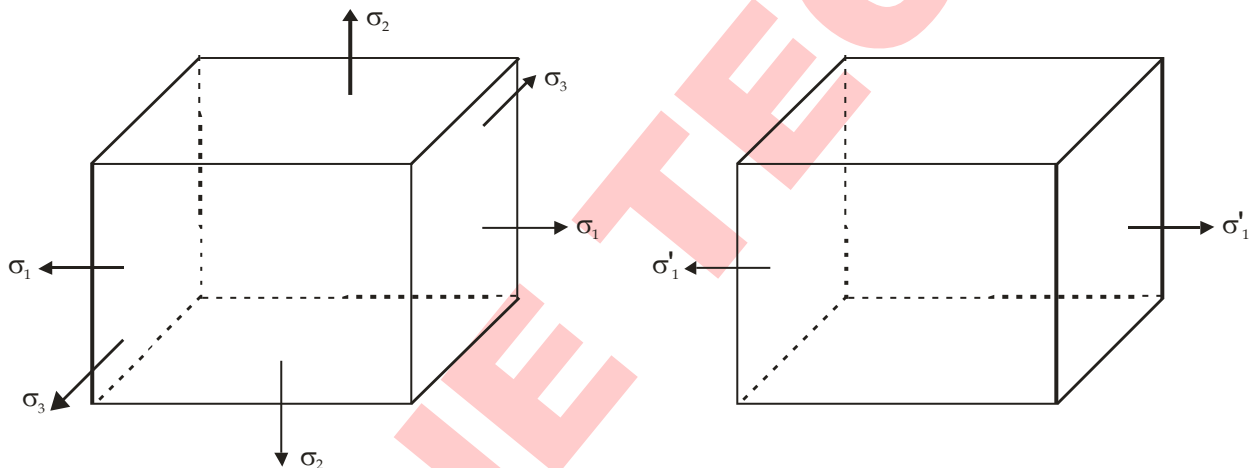
At failure

$$\epsilon_1 = \epsilon'_1 = \frac{\sigma_y}{E}$$

$$\text{Hence, } (\sigma_1 - \mu\sigma_2 - \mu\sigma_3) \leq \left(\frac{\sigma_y}{\text{FOS}} \right)$$

(III) Max Shear Stress Theory (Tresca's theory) :-

Acc. to this theory a material will fail when max shear stress ($\tau_{\max}$) reaches to max shear stress at failure in uniaxial loading.



$$\sigma_1 > \sigma_2 > \sigma_3$$

$$\sigma'_1 = \sigma_y$$

$$\tau_{\max} = \frac{\sigma'_1}{2} = \frac{\sigma_y}{2}$$

$$\tau_{\max} = \max \text{ of } \begin{cases} \sigma_1 - \sigma_2 \\ \sigma_2 - \sigma_3 \\ \sigma_1 - \sigma_3 \end{cases} \Rightarrow \tau_{\max} = \frac{\sigma_1 - \sigma_3}{2}$$

Acc. to this theory :-

At failure

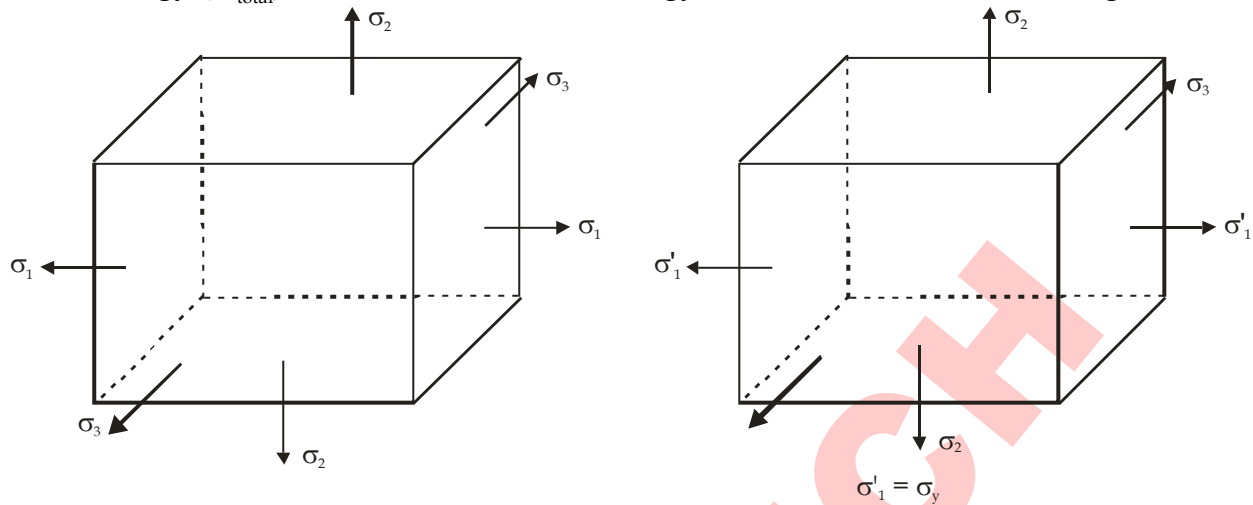
$$\tau_{\max} = \tau'_{\max}$$

$$(\sigma_1 - \sigma_3) = \sigma_y$$

Hence

$$\sigma_1 - \sigma_3 \leq \left(\frac{\sigma_y}{\text{fOS}} \right)$$

(IV) Max strain energy theory (Haigh's Theory) :- Acc to this theory a material will fail when total strain energy (U_{total}) reaches to total strain energy at failure in uni-axial loading.



$$\sigma_1 > \sigma_2 > \sigma_3$$

$$\sigma'_1 = \sigma_y$$

$$U'_{\text{total}} = \frac{1}{2E} (\sigma'_1)^2$$

$$U'_{\text{total}} = \frac{1}{2} \left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3) \right] = \frac{\sigma_y^2}{2E}$$

Acc. to this theory :-

At failure :-

$$U_{\text{total}} = U'_{\text{total}}$$

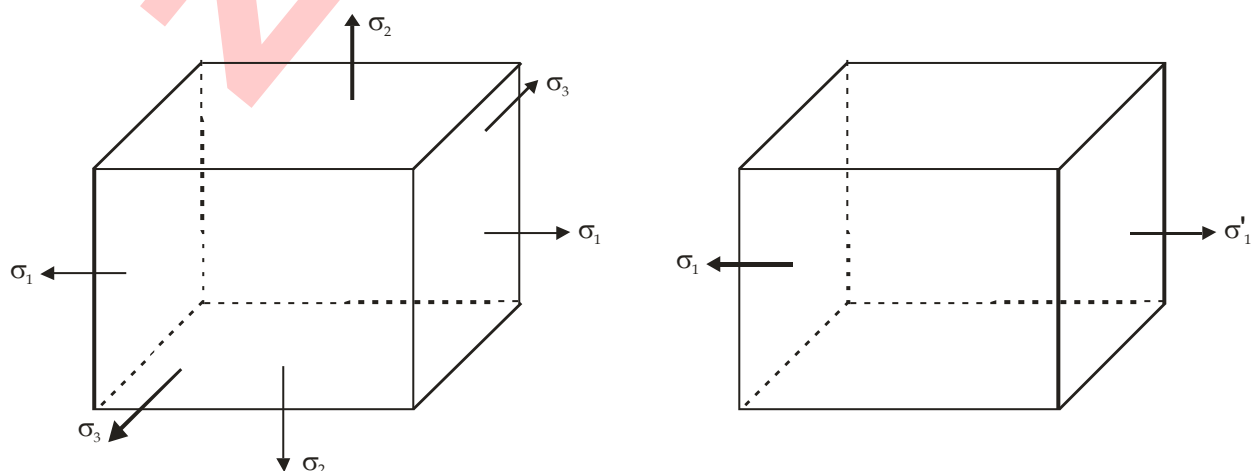
$$\frac{1}{2E} \left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) \right] = \frac{\sigma_y^2}{2E}$$

Hence -

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) \leq \left(\frac{\sigma_y}{\text{FOS}} \right)^2$$

(V) Max shear strain energy theory (von-mises theory) :-

→ This theory is also known as distortion energy theory, Acc. to this theory a material will fail when shear strain energy (U_{shear}) reaches to shear strain energy at failure in uni-axial loading



$$\sigma_1 > \sigma_2 > \sigma_3$$

$$\sigma'_1 = \sigma_y$$

$$U_{\text{shear}} = \frac{1}{12G} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \quad U'_{\text{shear}} = \frac{\sigma_y^2}{6G}$$

Acc. to this theory :-

At failure -

$$U_{\text{shear}} = U'_{\text{shear}}$$

$$\frac{1}{12G} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]_r = \frac{\sigma_y^2}{6G}$$

Hence

$$\frac{1}{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \leq \left(\frac{\sigma_y}{\text{FOS}} \right)^2$$

Note :-

If $\sigma_1 \gg \sigma_2 > \sigma_3$ then all theories of failure give same result.

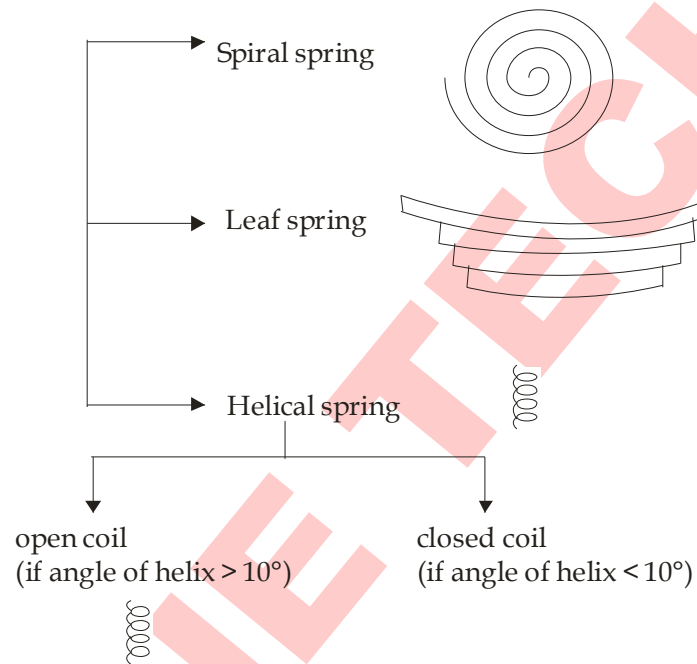
- (ii) In case of hydrostatic loading max shear stress theory (Tresca's theory) is not suitable.
- (iii) Max distortion energy theory give most accurate result in case of pure shear.
- (iv) Max shear stress theory give over safe result in case of pure shear.
- (v) Other three theories are unsafe for pure shear.

Theory	Result	Graphical representation	Suitability
1. max principal stress theory (rankines)	$\sigma_1 \leq \frac{\sigma_y}{\text{fos}}$	Square	suitable for brittle
2. max principal strain theory (st venants theory)	$(\sigma_1 - \mu\sigma_2 - \mu\sigma_3) \leq \frac{\sigma_y}{\text{fos}}$	Rhombus	theory) Neither for brittle nor ductile
3. max shear stress theory (trescas theory)	$(\sigma_1 - \sigma_3) \leq \left(\frac{\sigma_y}{\text{fos}} \right)$	Hexagon	suitable for ductile material
4. max strain energy (Haigh's energy)	$(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)) \leq \left(\frac{\sigma_y}{\text{fos}} \right)^2$	Ellipse	
5. Distortion energ theory (van mises theory)	$[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \leq \left(\frac{\sigma_y}{\text{fos}} \right)^2$	Ellipse	suitable for ductile material

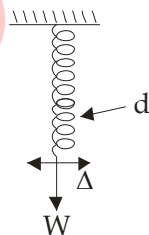
CHAPTER - 13

(SPRING)

Spring may be classified in following ways



Closed coil helical spring :-



Δ = deflection of helix

d = dia of wire

G = shear modulus

Δ = elongation

n = no of helix

deflection in spring $\Delta = \frac{8WD^3n}{Gd^4}$

If spring constant for (stiffness) is 'k' then

$$W = k\Delta$$

$$\text{stiffness } k = \frac{W}{\Delta}$$

$$\text{stiffness } \kappa = \left(\frac{W}{\frac{8WD^3n}{Gd^4}} \right)$$

$$\text{stiffness } \kappa = \frac{Gd^4}{8D^3n}$$

Spring Index = $\frac{\Delta}{d}$ (spring available in market on the basis of this index)

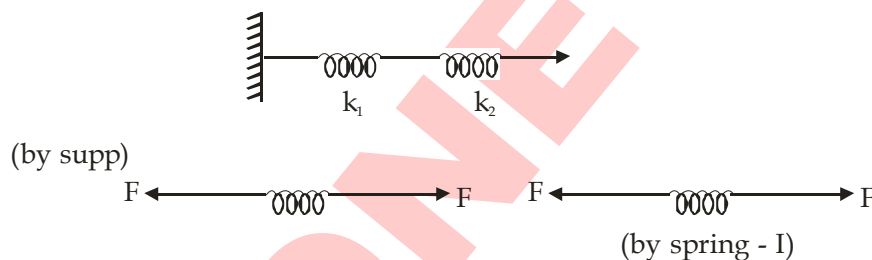
Energy Stored In Spring :-

$$U = \frac{1}{2} W \times \Delta = \frac{1}{2} \times k\Delta \times \Delta$$

$$U = \frac{1}{2} K\Delta^2$$

Connections Of Spring :-

(I) Series Connection :-



$$\Delta_1 = \frac{F}{k_1}$$

$$\Delta_2 = \frac{F}{k_2}$$

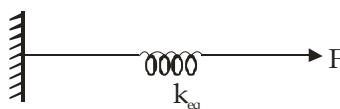
Total elongation = Δ

$$\Delta = \Delta_1 + \Delta_2$$

$$\Delta = \frac{F}{k_1} + \frac{F}{k_2} \dots (i)$$

if spring constant of equivalent spring is k_{eq} then :-

$$\Delta_{eq} = \frac{F}{k_{eq}}$$

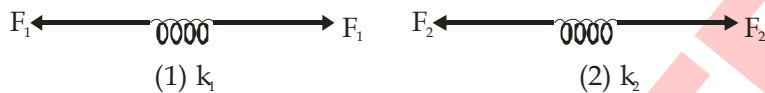
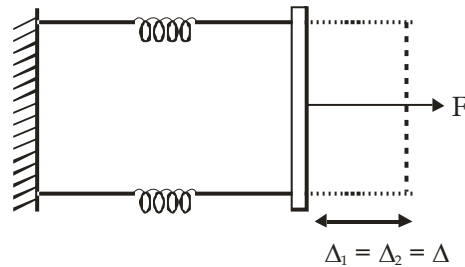


For $\Delta_{eq} = \Delta$

$$\frac{F}{k_{eq}} = \frac{F}{k_1} + \frac{F}{k_2}$$

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

(II) Parallel Connection :-



$$\Delta_1 = \frac{F_1}{k_1}$$

$$\Delta_2 = \frac{F_2}{k_2}$$

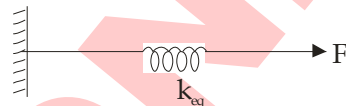
$$F_1 = k_1 \Delta_1$$

$$F_2 = k_2 \Delta_2$$

$$F_1 + F_2 = F$$

$$k_1 \Delta_1 + k_2 \Delta_2 \dots (1)$$

If spring constant of equivalent spring is k_{eq} then



$$\Delta_{eq} = \frac{F}{k_{eq}}$$

$$F = (\Delta_{eq}) (k_{eq}) \dots (ii)$$

from (i) & (ii)

$$k_{eq} \Delta_{eq} = k_1 \Delta_1 + k_2 \Delta_2$$

$$\Delta_1 = \Delta_2 = \Delta_{eq}$$

$$k_{eq} = k_1 + k_2$$

Note :-

- (i) When axial force (along axis of spring) is applied on helical spring then bending and twisting both occurs for closed coil helical spring effect of twisting is much more in comparison of bending. Hence due to twisting shear stresses are generated.
- (ii) When a moment is applied about axis of helical spring (Torque) then bending stresses are generated

in wire $\left[\text{radius reduce of helix } \frac{\uparrow \sigma}{y} = \frac{E}{R \downarrow} \right]$