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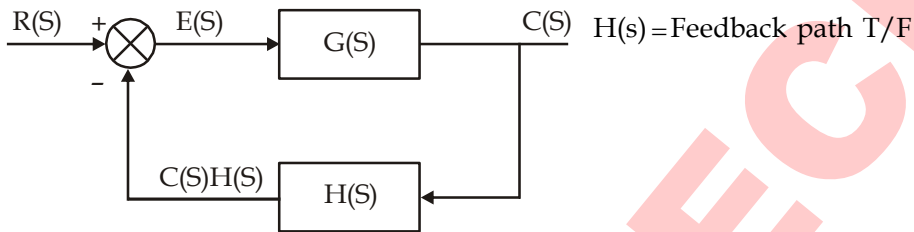
PART - A

2 Marks Questions

1.
Sol. For a linear, time invariant system transfer function is defined as laplace transform of output to the laplace transform of input. Assume initial condition zero
- Stability and output of the system can be determined easily
 - It is applicable for SISO (Single input single output) system.
 - Internal state can not be determined
 - Controllability and Observability can not be determined.

2.

Sol.



$G(s)$ = Forward path T/F
For Negative feed back

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S)H(S)}$$

For positive feed back

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1 - G(S)H(S)}$$

Transfer function does not depend on input but depend on the system parameters.
Transfer function is defined as impulse response of the system.

3.
Sol. The transient response is present in the short period of time immediately after the system is turned on. If the system is asymptotically stable, the transient response disappears whenever, if the system is unstable, the transient response will increase very quickly (exponentially). It gives us information about the system behaviour either it is oscillatory or not and the speed of the response.
4.
Sol. It is important that steady state response values are as close as possible to desired ones (specified ones) so that we have to study the corresponding errors, which represent the difference between the actual and desired system outputs at steady state, and examine conditions under which these errors can be reduced. It gives us information about the steady state error and accuracy of the response.
5.
Sol. The magnitude and phase relationship between the sinusoidal input and steady state output of a system is termed as the frequency response.
6.
Sol. It is the locus of roots of a characteristic equation when gain parameter 'k' varying from 0 to infinity

Characteristic equation

$$1 + G(s)H(s) = 0$$

$$G(s)H(s) = -1$$

Magnitude condition = $|G(s)H(s)| = 1$, magnitude of $|G(s)H(s)|$ at any point of root locus should be 1

Angle condition = $\angle G(s)H(s) = \pm 180^\circ (2q + 1)$

where $q = 0, 1, 2, \dots$

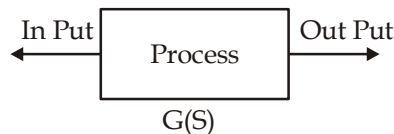


7.
Sol. Gain margin and Phase margin are positive.
Gain cross over frequency < phase cross over frequency.
8.
Sol. Transient response improves
 ξ (Damping ratio) increases
 M_p decreases
Bandwidth increases
Noise level increases
9.
Sol. Improves the steady state response
Bandwidth decreases
Noise level decreases
10.
Sol. $\dot{X}(t) = AX(t) + BU(t)$ state equation
 $Y(t) = CX(t) + DU(t)$ output equation
A = System Matrix
B = Input Matrix
C = Output Matrix
D = Transmission Matrix
11.
Sol. It is applicable for both linear time invariant and linear time variant systems.
It takes initial condition into account.
All the internal states of the system can be determined.
Applicable for multiple input multiple output.
Controllability and observability can be determined easily.
12.
Sol. $|SI - A| = 0$
Eigen value of system matrix A = Pole of the T.F.

PART - B

5 Marks Questions

13.
Sol. Open loop

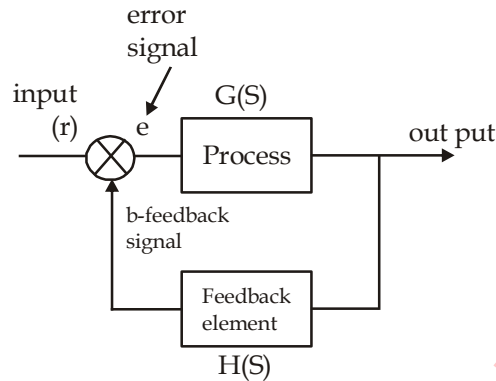


Simple , No Feedback
Inaccurate
Unreliable
Always stable
Sensitivity -

$$(S_G^T)_{OL} = 1 \quad (S_G^T)_{OL} > (S_G^T)_{CL}$$

More prone to disturbance
Gain -
Cheaper

Close Loop



Complex

Accurate

Reliable

Conditionally stable, Improve stability

Sensitivity⁻

$$(S_G^T)_{C.L} = \frac{1}{1+GH}$$

Less prone to disturbance

Gain⁻ for negative feedback Gain⁺ for positive feedback

Costly

14.

Sol. Sensitivity is defined as % change in the transfer function to the % change in system parameter.

For close loop system**Sensitivity with respect to G**

$$(S_G^T)_{C.L} = \frac{1}{1+GH}$$

Sensitivity with respect to. H

$$S_H^T = \frac{-GH}{1+GH}$$

For open loop system

$$(S_G^T)_{O.L} = 1$$

$(S_G^T)_{C.L} < (S_G^T)_{O.L}$ Sensitivity in case of open loop system is more than the close loop system.

Close loop system is more sensible to the feedback element so shielding is provided.

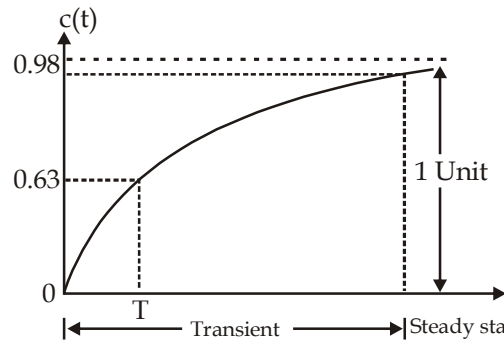
15.

Sol. • Transfer Function of first order system

$$\frac{C(s)}{R(s)} = \frac{1}{sT + 1}$$

• Unit step response





$C(t) = 1 - e^{-t/T}$, where T = time constant

Time constant = |Reciprocal of negative real root|

Rise time $t_r = 2.2T$

Settling time

(a) For 2% criteria = $4T$

(b) For 5% criteria = $3T$

16.

Sol. Transfer Function of second order system

$$\text{Transfer function} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

ω_n = Undamped natural frequency

ξ = Damping ratio

$\xi \omega_n$ = Damping factor

Critically damped system

(a) $\xi = 1$

(b) Roots pure real and equal

(c) Unit step response

$$c(t) = 1 - e^{-\omega_n t} (1 + \omega_n t)$$

17.

Sol. The gain margin is defined as the value of gain to be added to system, in order to bring the system to the verge of instability

The gain margin is given by the Reciprocal of the magnitude of open loop transfer function at phase cross over frequency. The frequency at which the phase of open loop Transfer function is 180° is called the phase cross-over frequency

$$\text{G.M.} = \left| \frac{1}{G(j\omega)H(j\omega)} \right|_{\omega=\omega_{pc}}$$

ω_{pc} = Phase cross over frequency

$$\angle G(j\omega)H(j\omega) = -180^\circ \text{ (to find } \omega_{pc}\text{)}$$

$$\text{G.M. (dB)} = 20 \log_{10} \left[\frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}} \right]$$

18.

Sol. The phase margin is defined as the additional phase lag to be added at the gain cross over frequency in order to bring the system to the verge of instability.

The gain cross over frequency is the frequency at which the magnitude of the loop transfer function is unity.

The phase margin is obtained by adding 180 to the phase angle ϕ of the open loop transfer function at the gain cross over frequency,

The phase margin indicates the additional phase lag that can be provided to the system without affecting stability

$$\text{P.M.} = 180^\circ + \phi_{gc}$$

$$\phi_{gc} = \angle G(j\omega)H(j\omega)_{\omega=\omega_{gc}}$$

ω_{gc} = gain cross over frequency

$$|G(j\omega)H(j\omega)| = 1 \text{ (to find } \omega_{gc}\text{)}$$

19.

Sol. **Concept of enclosed**

- All the right hand side points of the given path direction are said to be enclosed.
- A closed loop system is said to be stable if $(-1 + j0)$ point is not enclosed by the given polar plot

Nyquist stability criteria

$$Z = P + N$$

N = Number of clockwise encirclement of $(-1 + j0)$ point

P = Number of open loop pole on R.H.S of s plane.

Z = Number of closed loop pole on R.H.S of s plane.

For system to be stable $Z = 0$

If $P = 0$ For system to be stable, there is no encirclement of $(-1 + j0)$ point.

If $P \neq 0$ For system to be stable, there is P counter clockwise encirclement of $(-1 + j0)$ point.

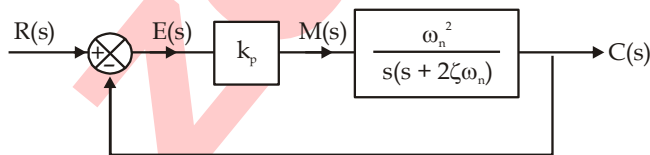
If there is no encirclement of $(-1 + j0)$ point means $N = 0$, For system to be stable no. of open loop pole in right hand side of s plane should be zero.

If there is clockwise encirclement of $(-1 + j0)$ point then system is always unstable.

If there is counter clockwise encirclement of $(-1 + j0)$ point. For system to be stable there is N no. of open loop pole in right hand side of s plane.

20.

Sol.



$$\omega'_n = \omega_n \sqrt{K_p}$$

$$\zeta' = \frac{\zeta}{\sqrt{K_p}}$$

ω_n (undamped natural frequency) increases by $\sqrt{K_p}$

ζ (Damping ratio) decreases

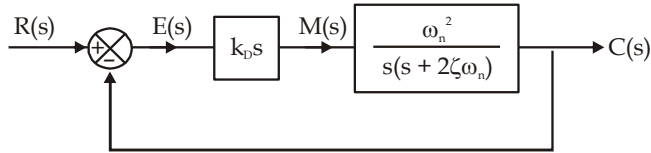
M_p (peak overshoot) increases

Steady state error reduces



21.

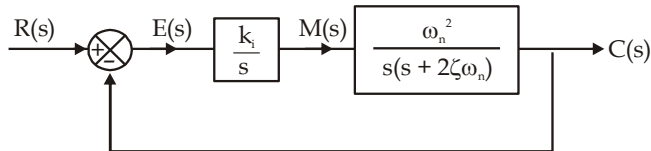
Sol.



Type and order of the system reduces by one
Oscillations has died out hence transient response improves
Not used in isolation because it's response is anticipatory in nature.

22.

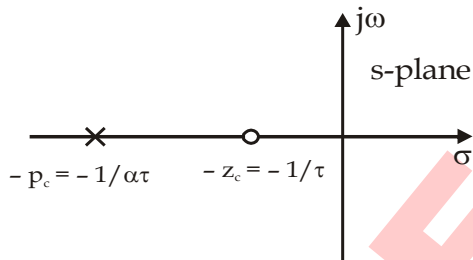
Sol.



Increases type and order by '1'
Makes the system lesser stable
Steady state error reduces
Improves the steady state response

23.

Sol.



The s-plane representation of lead compensator

It has a zero at $s = -1/\tau$ and a pole at $s = -1/\alpha\tau$ with the zero closer to the origin than the pole.

$$G_c(s) = \frac{(s + z_c)}{(s + p_c)} = \frac{(s + 1/\tau)}{(s + 1/\alpha\tau)}$$

$$\alpha < 1, \tau > 0$$

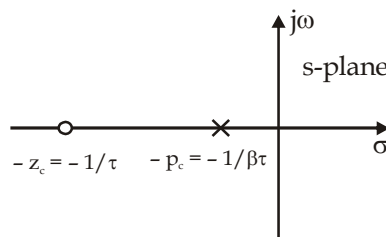
$$= \alpha \left(\frac{\tau s + 1}{\alpha \tau s + 1} \right)$$

Effects of Lead Compensator

Improves the transient response.
Increases the margin of stability that means improves stability
Increases the bandwidth
It helps to increase error constant up to some extent

24.

Sol.



$$G_c(s) = \frac{(s+z_c)}{(s+p_c)} = \frac{s+\frac{1}{\tau}}{s+\frac{1}{\beta\tau}}$$

$$\beta > 1$$

Effects of Lag Compensator

Improves the steady state response, it increases the error constant.

Decreases the bandwidth.

Reduces the effect of noise.

Reduces the stability margin.

Does not affect the transient response.

System become lesser stable.

25.

Sol. The state equation is said to be completely state controllable or simply state controllable if for any initial state $x(0)$ and any final state $x(N)$, there exists an input sequence $u(k)$, $k = 0, 1, 2, \dots, N$, which transfers $x(0)$ to $x(N)$ for some finite N . Otherwise the state equation is state uncontrollable

$$Q_c = [B : AB : A^2B : \dots : A^{n-1}B]$$

where Q_c = Controllability test matrix

System is completely controllable if the rank of matrix Q_c is equal to order of the system

Condition for state controllability

$|Q_c| \neq 0$ (Matrix be non singular)

26.

Sol. The state model (or the pair (A, C)) is said to be observable if any initial state $x(0)$ can be uniquely determined from the knowledge of output $y(k)$ and input sequence $u(k)$, for $k = 0, 1, 2, \dots, N$, where N is some finite time. Otherwise the state model is unobservable.

$$Q_o = [C^T : A^T C^T : (A^T)^2 C^T : \dots : (A^T)^{n-1} C^T]$$

where Q_o = Observability test matrix

System is completely observable if the rank of matrix Q_o is equal to order of the system

Condition for state Observability

$|Q_o| \neq 0$ (Matrix be non singular)

27.

Sol. All the systems can be described by differential equations of various orders. While the output of such systems for any input can be obtained by solving such differential equations. Mathematically it is very difficult to solve such equations in time domain.

The laplace transform of such differential equations converts them into simple algebraic equations. All the complicated computations then can be easily performed in s-domain as the equations to be handled are algebraic in nature.

Then by eliminating unwanted variable, the required variable in s-domain can be obtained. Then by using technique of laplace inverse, time domain function for the required variable can be obtained. Hence making the computations easy.

28.

Sol. A linear system satisfies following two properties:

(i) **Superposition:** Consider a system that is initially at rest. Let the system be subjected to an input $x_1(t)$, producing an output $y_1(t)$. Suppose next that the same system is subjected to a different input $x_2(t)$, producing a corresponding output $y_2(t)$. Then for the system to be linear, it is necessary that the composite input $x(t) = x_1(t) + x_2(t)$ produce the corresponding output $y(t) = y_1(t) + y_2(t)$.

(ii) **Homogeneity:** Consider again a system that is initially at rest and suppose an input $x(t)$ results in an output $y(t)$. Then the system is said to exhibit the property of homogeneity. When if the input $x(t)$ is scaled by a constant factor a , the output $y(t)$ is scaled by exactly the same constant factor a .

i.e.,

$$x_1(t) \rightarrow y_1(t)$$

$$ax_1(t) \rightarrow ay_1(t)$$

When a system violates either the principle of superposition or the property of homogeneity the system is said to be non-linear.

Stability of linear system determined by poles of the system.

29.

Sol. The characteristic equation is

$$s^3 + 20ks^2 + 5s^2 + 10s + 15 = 0$$

$$s^3 + (5 + 20k)s^2 + 10s + 15 = 0$$

The Routh array is formed below

$$\begin{array}{c|cc} s^3 & 1 & 10 \\ s^2 & (20k+5) & 15 \\ s^1 & \left(10 - \frac{15}{20k+5}\right) & 0 \\ s^0 & 15 & \end{array}$$

For system to be stable

$$10 - \frac{15}{20k+5} > 0 \text{ and } 20k + 5 > 0$$

$$\frac{3}{4k+1} < 10 \text{ and } k > -\frac{1}{4}$$

$\Rightarrow$

$$3 < 40k + 10$$

$\Rightarrow$

$$40k > -7$$

$$k > -\frac{7}{40}$$

$\therefore$ The system is stable for $k \in \left(-\frac{7}{40}, \infty\right)$

30.

Sol. Characteristic equation $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$$s_{1,2} = \xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

Given roots are

$$s_{1,2} = -3 \pm j7$$

$\therefore$

$$\xi\omega_n = +3 \quad \dots(1)$$

and

$$\omega_n\sqrt{1-\xi^2} = 7 \quad \dots(2)$$

(1) $\div$ (2)

$$\frac{\xi}{\sqrt{1-\xi^2}} = \frac{3}{7}$$

$$\text{Peak time} = t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{7} = 0.448 \text{ sec.}$$

$$\% \text{overshoot} = e^{-\xi\pi/\sqrt{1-\xi^2}} = e^{-\left(\frac{3}{7}\pi\right)} = 0.26 = 26\%$$

$$\text{Setting time for } \pm 2\% \text{ range} = t_s = \frac{4}{\xi\omega_n} = \frac{4}{3} = 1.33 \text{ sec.}$$

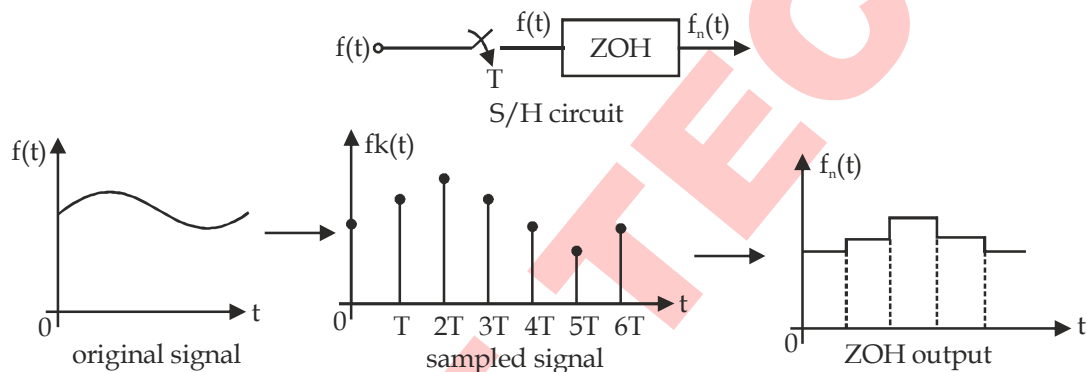
31.

Sol. **Relative stability:** Relative stability is quantitative measure of how fast the transient die out in the system. Relative stability may be measured by relative settling times of each root or pair of roots. As a root/or a pair of roots, moves farther away from the imaginary axis in left half of s plane the relative stability of system improves.

- (i) Relative stability interms of closed loop pole location: If the closed loop poles moves away from $j\omega$ axis in left half of s plane the relative stability of system improves.
- (ii) Relative stability interms of gain margin and phase margin: Gain margin and phase margin gives the relative stability. For system to be stable both gain margin and phase margin of system to be positive. Larger values of gain margin and phase yield a relatively more stable system.

32.

Sol. The zero-order sample and hold circuit holds the output signal at a fixed level between two consecutive sampling instants such that the slope of the hold circuit output signal is zero or in other words hold signal is the zeroth derivative of an impulse.



The output of s/H circuit is a unit step function appearing upto time T,

$$h(t) = u(t) - u(t - T)$$

$$\Rightarrow H(s) = \frac{1}{s} - \frac{1}{s} e^{-sT}$$

as the input to ZOH is $\delta(t)$ the laplace transform of the input is 1

$$\text{Transfer function} = \frac{\angle(\text{output of ZOH})}{\angle(\text{input of ZOH})} = \frac{(1 - e^{-sT})/s}{1} = \frac{1 - e^{-sT}}{s}$$

PART - C

20 Marks Questions

33.

Sol. **Open loop system:** A system in which output is dependent on input but controlling action or input is totally independent of the output or changes in output of the system is called on open loop system.

Reference input $r(t) \rightarrow$ controller $\xrightarrow[\text{Actuating signal}]{} \text{Process}$ $\rightarrow$ Controlled output $c(t)$

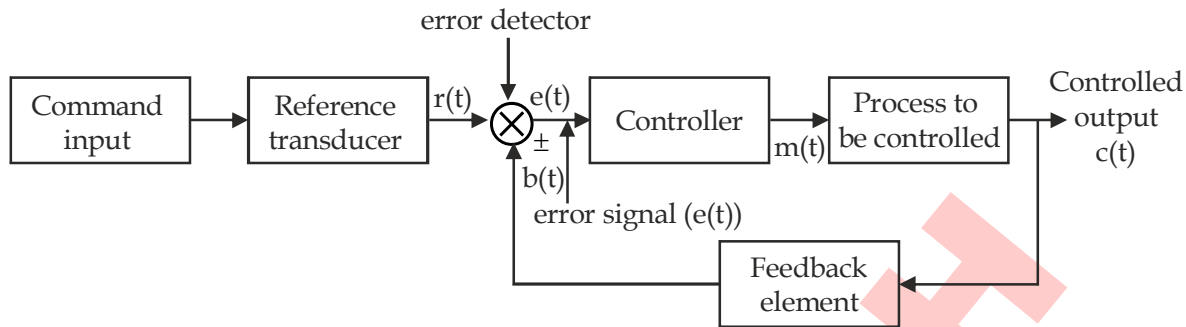
The good example on open loop system is on electric switch. This is open loop because output is light and switch is controller of lamp.

Closed loop system: A system in which the controlling action or input is somehow dependent on the output or changes in the output is called closed loop system.

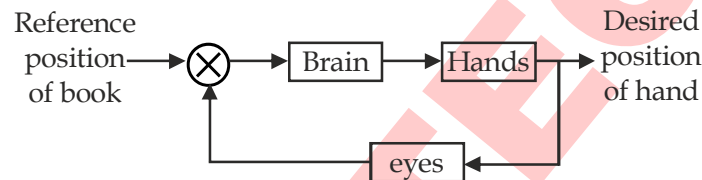


In such system, output or part of the output is feedback to the input for comparison with the reference input applied to it.

Closed loop system can be represented as shown in figure below:



The best example of closed loop system is human being. If a person wants to reach for a book on the table closed loop system can be represented as shown below



Manual control system: In manual control system, operators play an important role as sensors and decision makers. They compare the inputs with the status needed, then provide feedback and make decisions (their brains). Afterwards, they adopt some procedures to stabilize the systems and minimize the errors. Therefore, manual operation in the system is a kind of closed-loop control system.

Automatic control system: Under the conditions of clear and repeated procedures, and those procedures which are operated by an automatic adjustment system instead, automatic control machinery is more suitable for use. Therefore, those automatic controlled machines are suitable for boring and repeated works.

For example, it is better for a temperature sensor involved in the control of the switching On or OFF the compressor of the air conditioner.

34.

Sol.

Transfer Function of second order system

$$\text{Transfer function} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

ω_n = Undamped natural frequency

ξ = Damping ratio

$\xi \omega_n$ = Damping factor

Under damped system

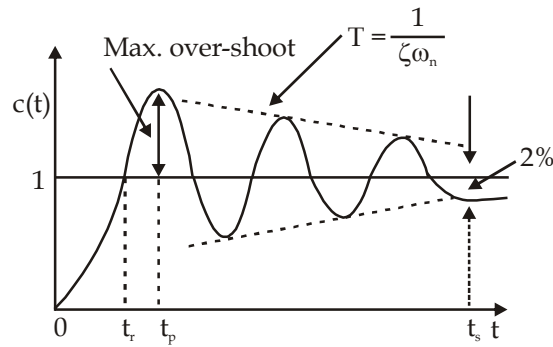
(a) $0 < \xi < 1$

(b) Roots complex conjugate

(c) $\omega_d = \omega_n \sqrt{1 - \xi^2}$ (Damped frequency of oscillation)

(d) Unit step response





$$C(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1 - \xi^2}} \sin(\omega_d t + \phi)$$

$$\phi = \tan^{-1} \left(\frac{\sqrt{1 - \xi^2}}{\xi} \right)$$

Time constant, $T = 1 / \xi \omega_n$

Settling time

(a) For 2% criteria $= 4T = 4 / \xi \omega_n$

(b) For 5% criteria $= 3T = 3 / \xi \omega_n$

Delay time (t_d)

$$t_d = \frac{1 + 0.7\xi}{\omega_n}$$

Rise Time

$$t_r = \frac{\pi - \phi}{\omega_n \sqrt{1 - \xi^2}}$$

$$t_r = \frac{0.8 + 2.5\xi}{\omega_n}$$

Peak time

$$t_p = \frac{\pi}{\omega_n \sqrt{1 - \xi^2}}$$

$$t_p = \frac{n\pi}{\omega_d}$$

$n=1, 3, 5, \dots$ over shoot

$n=2, 4, 6, \dots$ under shoot

Maximum overshoot

$$M_p = e^{-\left(\frac{\pi\xi}{\sqrt{1-\xi^2}}\right)}$$

35.

Sol.

$$\frac{C(s)}{R(s)} = \frac{s^2 + 3s + 4}{s^2 + 4s + 4}$$

Ans

$$R(s) = \frac{1}{s} (\text{unit step})$$

$$\begin{aligned} C(s) &= \frac{1}{s} \cdot \frac{s^2 + 3s + 4}{s^2 + 4s + 4} = \frac{1}{s} \left[1 - \frac{s}{s^2 + 4s + 4} \right] \\ &= \left[\frac{1}{s} - \frac{1}{s^2 + 4s + 4} \right] = \left[\frac{1}{s} - \frac{1}{(s+2)^2} \right] \end{aligned}$$

$$\therefore c(t) = \mathcal{Z}^{-1}[C(s)] = 1 - te^{-2t}$$

Comparing the denominator of $\frac{C(s)}{R(s)}$ with $s^2 + 2\xi\omega_n s + \omega_n^2$, we get

$$2\xi\omega_n = 4$$

$$\text{and } \omega_n = 2$$

$$\text{then } \xi = 1$$

Hence, the circuit is critically damped.

36.

Sol.

n^{th} order characteristic equation

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n = 0$$

Necessary condition:- (For the system stability)

1. All the coefficient of the given characteristic equation should be positive.
2. No missing coefficient in the given equation.

Sufficient condition:- (R-H criteria)

A closed loop system is said to be stable if the first column element of a Routh array table of the given characteristic equation are positive.

If there is sign change the system is said to be unstable

No. of sign change = No. of root lie in the RHS of the S plane

s^n	a_0	a_2	a_4
s^{n-1}	a_1	a_3	a_5
s^{n-2}	b_1	b_2	b_3
s^{n-3}	c_1	c_2	c_3
$\vdots$			
$\vdots$			
s^0	a_n	0	0
$b_1 =$	$\frac{a_1 a_2 - a_0 a_3}{a_1}$	$b_2 =$	$\frac{a_1 a_4 - a_5 a_0}{a_1}$
$c_1 =$	$\frac{b_1 a_3 - b_2 a_1}{b_1}$	$c_2 =$	$\frac{b_1 a_5 - b_3 a_1}{b_1}$

Case-1

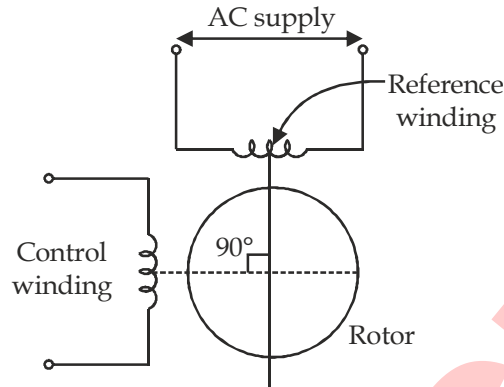
In the Routh Array if first element of any row is zero and remaining terms non zero then it is replaced by a small positive quantity ϵ which tends to zero

Case-2

When all the elements in any one row of the Routh Array are zero. This indicates that there are symmetrical located roots in the S-plane (pair of real roots with opposite sign and pair of conjugate roots on the imaginary axis and complex conjugate roots forming quadrates in the S plane).

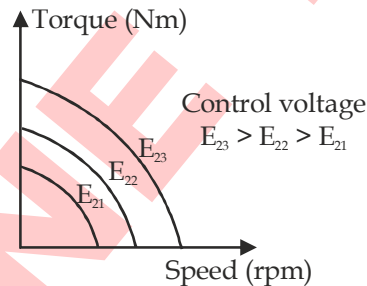
37.

Sol. Two phase servomotor :



The stator of two-phase servomotor carries two windings. Uniformly distributed and displaced by 90° in space. One winding is called main winding or field winding or reference winding. This is excited by constant voltage AC supply. The other winding is called control winding. It is excited by variable control voltage. This voltage is 90° out of phase with respect to the voltage applied to the reference winding. This is necessary to obtain rotating magnetic field.

Torque-speed characteristics: The torque-speed characteristics mainly depends on the ratio of reactance to resistance. For small X to R ratio, the characteristics is much more linear while it is non-linear for large X to R ratio.



Transfer function of two-phase servomotor:

Torque at any speed 'N' is

$$T_m = k_{tm} E_{2t} + m \frac{d\theta_m}{dt}$$

where, E_{2t} control voltage

If load consists inertia J_m and friction B_m then load torque

$$T_L = J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt}$$

at equilibrium

$$T_m = T_L$$

$\therefore$

$$K_{tm} E_{2t} + m \frac{d\theta_m}{dt} = J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt}$$

taking laplace transform of above equation

$$K_{tm} E_{2t}(s) + sm\theta_m(s) = s^2 J_m \theta_m(s) + B_m s \theta_m(s)$$

or

$$\frac{\theta_m(s)}{E_{2t}(s)} = \frac{K_{tm}}{s[sJ_m + (B_m - m)]} = \frac{\frac{K_{tm}}{(B_m - m)}}{s\left[\frac{J_m}{(B_m - m)}s + 1\right]}$$

Let

$$\frac{K_{tm}}{B_m - m} = K \quad \text{and} \quad \frac{J_m}{B_m - m} = \tau$$

then

$$\frac{\theta_m(s)}{E_{2t}(s)} = \frac{K}{s(1 + s\tau)}$$

38.

Sol. Mason's gain formula is a mathematical tool to find transfer function of given control system with the help of signal flow graph (SFG).

Mason's gain formul : $T(s) = \sum_{k=1}^k \frac{P_k \Delta_k}{\Delta}$

Where,

$T(s)$ = transfer function of the control system or the overall gain of the system

P_k = forward path gain of k^{th} path from a specified input node to an output node

Δ = graph determinant which involves closed loop gain and mutual interaction between non-touching loops

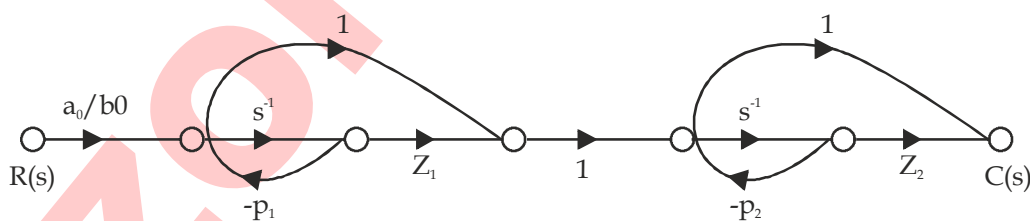
= 1 - [sum of all individual loop gains]

+ [sum of loop gain products of all possible pairs of non-touching loops]

- [sum of loop gain products of all possible triplet of non-touching loops]

Δ_k = The value of Δ which is not touching the k^{th} forward path.

Thw two closed loops are



The four forward paths are

$$P_1 = \frac{a_0}{b_0} \cdot s^{-1} \cdot Z_1 \cdot 1 \cdot s^{-1} \cdot Z_2 \cdot 1 = \frac{a_0}{b_0} s^{-2} Z_1 Z_2$$

$$P_2 = \frac{a_0}{b_0} \cdot 1 \cdot 1 \cdot s^{-1} \cdot Z_2 \cdot 1 = \frac{a_0}{b_0} s^{-1} Z_2$$

$$P_3 = \frac{a_0}{b_0} \cdot 1 \cdot 1 \cdot 1 \cdot 1 = \frac{a_0}{b_0}$$

$$P_4 = \frac{a_0}{b_0} \cdot s^{-1} \cdot Z_1 \cdot 1 \cdot 1 \cdot 1 = \frac{a_0}{b_0} s^{-1} Z_1$$



$$L_1 = -s^{-1}p_1 \text{ and } L_2 = -s^{-1}p_2$$

As both the loops L_1 and L_2 are touching all the forward paths

$$\therefore \Delta_1 = 1, \Delta_2 = 1, \Delta_3 = 1 \text{ and } \Delta_4 = 1$$

Number of two non touching loops = 1

$$L_1 L_2 = \frac{p_1 p_2}{s^2}$$

The graph determinants is $\Delta = 1 - (L_1 + L_2) = L_1 L_2$

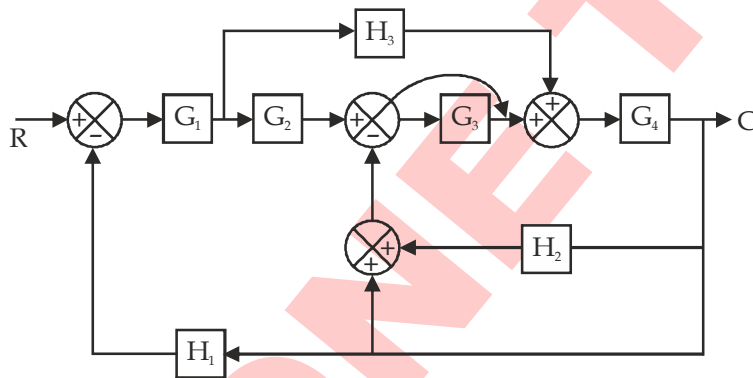
Applying Mason's gain formula, the transfer function of the control system,

$$T(s) = \frac{C(s)}{R(s)} = \frac{\frac{a_0}{b_0} (1 + s^{-1})(z_1 + z_2) + s^{-2} z_1 z_2}{1 + s^{-1} p_1 + s^{-1} p_2 + \frac{p_1 p_2}{s^2}}$$

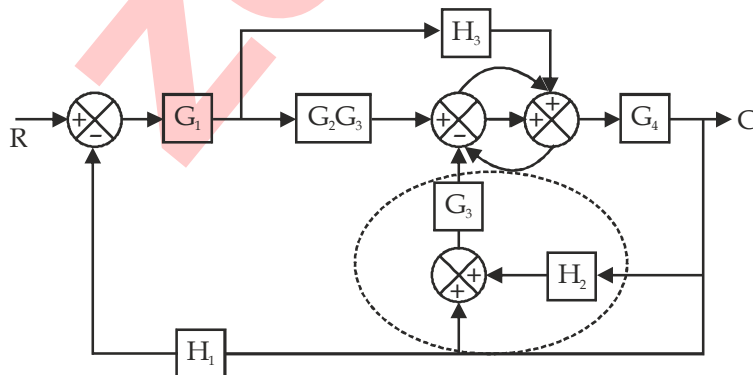
$$= \frac{a_0 [s^2 + s(z_1 + z_2) + z_1 z_2]}{b_0 [s^2 + s(p_1 + p_2) + p_1 p_2]}$$

39.

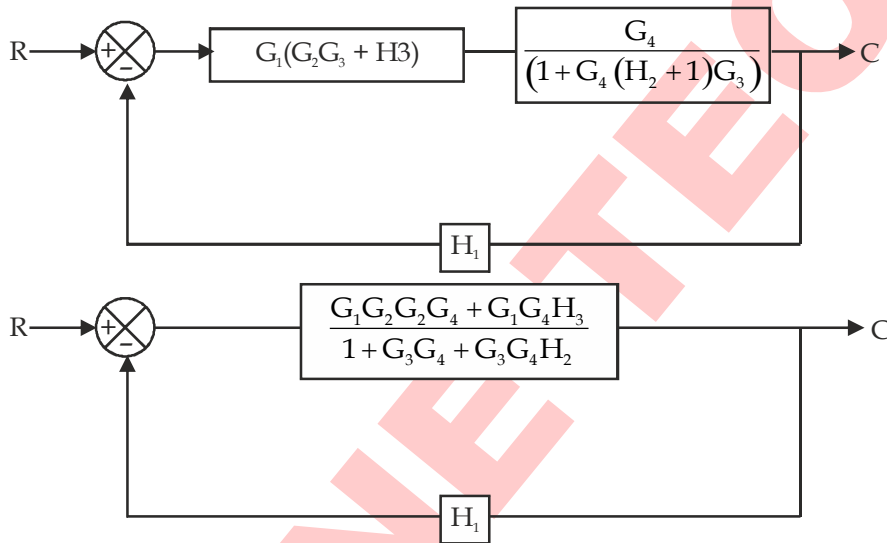
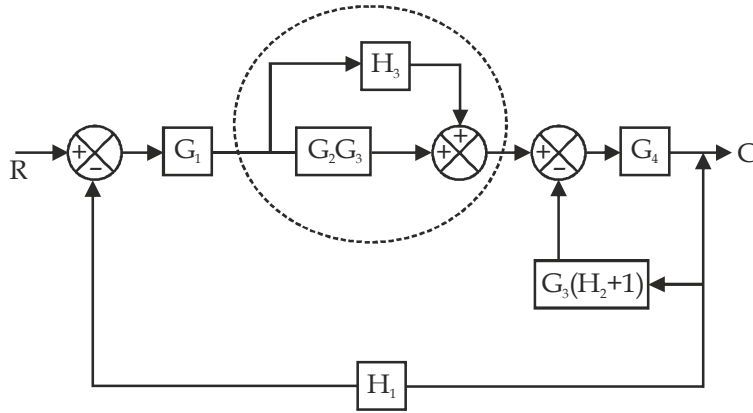
Sol.



Moving summing point ahead of G_3 we get



By combining parallel path, and alternating the summing point



Thus,

$$\frac{C}{R} = \frac{(G_1G_2G_3G_4 + G_1G_4H_3)(1 + G_3G_4 + G_3G_4H_2)}{1 + \frac{G_1G_2G_3G_4 + G_1G_4H_3}{1 + G_3G_4 + G_3G_4H_2} \times H_1}$$

$$\left[\frac{C}{R} \right] = \frac{G_1G_2G_3G_4 + G_1G_4H_3}{1 + G_3G_4 + G_3G_4H_2 + G_1G_2G_3G_4H_1 + G_1G_4H_1H_3}$$

40.

Sol. (i) The characteristic equation of given system is

$$1 + G(s) = 0$$

$$\Rightarrow s(1 + s\tau_1)(1 + s\tau_2) + K = 0$$

$$\Rightarrow \tau_1\tau_2s^3 + (\tau_1 + \tau_2)s^2 + s + K = 0$$

The routh array is formed below

$$\begin{array}{c|cc} s^3 & \tau_1 \tau_2 & 1 \\ s^2 & (\tau_1 + \tau_2) & K \\ s^1 & \left(1 - \frac{K\tau_1\tau_2}{(\tau_1 + \tau_2)}\right) & 0 \\ s^0 & K & \end{array}$$

For system to be stable

$$K > 0, \tau_1 \tau_2 > 0, (\tau_1 + \tau_2) > 0 \text{ and } 1 - \frac{K\tau_1\tau_2}{(\tau_1 + \tau_2)} > 0$$

$$\Rightarrow K\tau_1\tau_2 < (\tau_1 + \tau_2)$$

$$\Rightarrow 0 < K < \frac{(\tau_1 + \tau_2)}{\tau_1 \tau_2}$$

$$(ii) \angle G(j\omega) = -90^\circ - \tan^{-1}(\omega\tau_1) - \tan^{-1}\omega\tau_2$$

at phase crossover frequency $\angle G(j\omega)|_{\omega=\omega_{pc}} = -180^\circ$

$$\Rightarrow -\tan^{-1}\left(\frac{\omega_{pc}(\tau_1 + \tau_2)}{1 - \tau_1\tau_2\omega_{pc}^2}\right) = -90^\circ$$

$$\Rightarrow \omega_{pc} = \frac{1}{\sqrt{\tau_1\tau_2}}$$

$$\begin{aligned} \text{Gain margin} &= -20\log\left[|G(j\omega_{pc})|\right] \\ &= \frac{K}{\omega_{pc}\sqrt{1 + (\omega_{pc}\tau_1)^2}\sqrt{1 + (\omega_{pc}\tau_2)^2}} \\ &= \frac{K\sqrt{\tau_1\tau_2}}{\sqrt{1 + \left(\frac{\tau_1}{\tau_2}\right)}\sqrt{1 + \left(\frac{\tau_2}{\tau_1}\right)}} = \frac{K\tau_1\tau_2}{(\tau_1 + \tau_2)} \end{aligned}$$

$$\Rightarrow \text{G.M.} = -20\log\left(\frac{K\tau_1\tau_2}{\tau_1 + \tau_2}\right)$$

For system to be stable, $\frac{K\tau_1\tau_2}{\tau_1 + \tau_2} < 1$

41.

Sol. Stability: A linear time invariant system is said to be stable if following conditions are satisfied:

1. When the system is excited by a bounded input, output is also bounded and controllable.
2. In the absence of the input, output must tend to zero irrespective of the initial conditions.

Methods of finding stability of a linear system:

1. Nyquist stability criteria: Nyquist stability criterion states that for absolute stability of the system the number of encirclements of new origin of GH(s) plane by Nyquist plot must be equal to the number of poles of $1+G(s)H(s)$ i.e. poles of $G(s)H(s)$ which are in the right half of s-plane and in clockwise direction.

If N = number of encirclements of origin of $GH(s)$ plane by nyquist plot

P = number of open loop poles of $G(s) H(s)$ in right half s -plane

Z = number of closed loop zeros of $1+G(s) H(s)$ in right half of s -plane

then for stability, $Z = 0$ and

$$\boxed{Z = N + P}$$

2. Routh's stability criteria: Routh suggested a method of tabulating the coefficient of characteristic equation in particular form consider the general characteristic equation as

$$F(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n = 0$$

Routh's array

$$\begin{array}{c|cccc} s^n & a_0 & a_2 & a_4 & a_6 \dots \\ s^{n-1} & a_1 & a_3 & a_5 & a_7 \dots \\ s^{n-2} & b_1 & b_2 & b_3 & \dots \\ s^{n-3} & c_1 & c_2 & c_3 & \dots \\ \vdots & \vdots & & & \\ s^0 & a_n & & & \end{array}$$

where,

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}, \quad b_2 = \frac{a_1 a_4 - a_5 a_0}{a_1}, \quad b_3 = \dots$$

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}, \quad c_2 = \frac{b_1 a_5 - b_3 a_1}{b_1}, \quad c_3 = \dots$$

This process is to be continued till the coefficient for s^0 is obtained which be a_n .

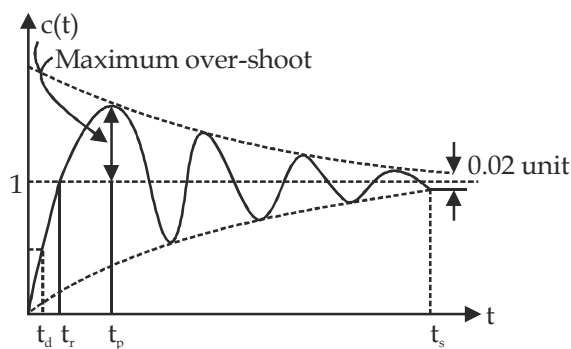
The necessary and sufficient condition for system to be stable is "All the terms in the first column of Routh's array must have same sign. There should not be any sign change in the first column of Routh's array.

Advantages of Routh's criteria: Stability of the system can be judged without actually solving the characteristic equation

- For unstable system it gives number of roots of characteristic equation having positive real part.
- By using this criteria, critical value of system gain can be determined easily and frequency of sustained oscillations can be determined.

42.

Sol.



Delay time: The delay time is the time needed for the response to reach from 0 to 50% of the desired value of the output at very first instant.

Empirical formula for delay time t_d is

$$t_d = \frac{1 + 0.7\xi}{\omega_n}$$

where ω_n is the undamped natural frequency and ξ is damping ratio.

Rise time: It is the time needed for the response to reach from 10 to 90% or 0 to 100% of the desired value of output at very first instant. Usually 0 to 100% basis is used for underdamped systems and 10 to 90% for overdamped system.

rise time,
$$t_r = \frac{\pi - \phi}{\omega_d}$$

where, $\phi = \tan^{-1} \left(\frac{\sqrt{1 - \xi^2}}{\xi} \right)$ and $\omega_d = \omega_n \sqrt{1 - \xi^2}$

Peak time: The time needed to reach the maximum overshoot is called peak time.

peak time,
$$t_p = \frac{\pi}{\omega_n \sqrt{1 - \xi^2}}$$

Peak overshoot: The maximum positive deviation of the output with respect to its desired value is known as maximum overshoot.

peak overshoot,
$$M_p = e^{\frac{-\xi\pi}{\sqrt{1 - \xi^2}}}$$

Settling time: The time needed to settle down a fore said oscillations within 2% of desired value of the output is known as settling time. The setting time for a second order control system is approximately four times the time constant of the system.

settling time,
$$t_s = 4 \cdot \frac{1}{\xi\omega_n}$$

Note: On 5% basis, the settling time for a second order control system is approximately three times the time constant.

$$t_s = 3 \cdot \frac{1}{\xi\omega_n}$$

Steady state error: It is defined as the difference between actual output and desired output as time tends to infinity.

$$\begin{aligned} e_{ss} &= \lim_{s \rightarrow 0} \frac{sR(s)}{1 + G(s)H(s)} = \lim_{s \rightarrow 0} \frac{s \cdot \frac{1}{s}}{1 + \frac{\omega_n^2}{s(s + 2\xi\omega_n)}} \\ &= \lim_{s \rightarrow 0} \frac{s(s + 2\xi\omega_n)}{s^2 + 2\xi\omega_n s + \omega_n^2} = 0 \end{aligned}$$

43.

Sol.

$$G(s) = \frac{25}{s(s + 2)}$$

closed loop transfer function

$$\frac{C(s)}{R(s)} = T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{25}{s^2 + 2s + 25}$$

comparing with

$$T(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\omega_n^2 = 25, 2\xi\omega_n = 2$$

$\Rightarrow$

$$\omega_n = 5 \text{ and } \xi = \frac{1}{5}$$

damping factor,

$$\xi\omega_n = 1$$

undamped natural frequency,

$$\omega_n = 5 \text{ rad / sec}$$

damped natural frequency,

$$\omega_d = \omega_n \sqrt{1 - \xi^2} = \frac{5\sqrt{24}}{5} = \sqrt{24} \text{ rad / sec}$$

$$R(s) = \frac{1}{s}; C(s) = \frac{25R(s)}{s^2 + 2s + 25} = \frac{25}{s(s^2 + 2s + 25)}$$

$$= \frac{1}{s} - \frac{s+1}{(s+1)^2 + 24} - \frac{1}{(s+1)^2 + 24}$$

$\Rightarrow$

$$C(t) = \mathcal{Z}^{-1}[C(s)]$$

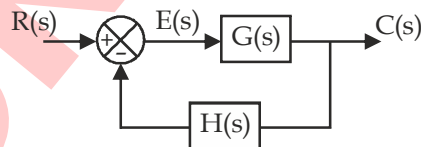
$$= 1 - \cos(\sqrt{24}t) \cdot e^{-t} - \frac{1}{\sqrt{24}} \sin(\sqrt{24}t) e^{-t}$$

$$= 1 - e^{-t} \left[\cos \sqrt{24}t + \frac{1}{\sqrt{24}} \sin \sqrt{24}t \right]$$

$$C(t) = \left[1 - \frac{5e^{-t}}{\sqrt{24}} \sin \left\{ \sqrt{24}t + \cos^{-1} \left(\frac{1}{5} \right) \right\} \right]$$

44.

Sol. Steady state error: It is the deviation of output from desired output at steady state



$$E(s) = R(s) - C(s)H(s)$$

$$E(s) = R(s) - E(s)G(s)H(s)$$

$$E(s)[1 + G(s)] = R(s)$$

$$E(s) = \frac{R(s)}{1 + G(s)H(s)}$$

From above equation it is clear that error depends on input as well as system parameter i.e. $[G(s)$ and $H(s)]$

Steady state error

$$e_{ss} = \lim_{s \rightarrow 0} s \cdot E(s) = \lim_{s \rightarrow 0} \frac{s \cdot R(s)}{1 + G(s)H(s)}$$

Case-1 : For unit step input

$$R(s) = \frac{1}{s}$$

$$e_{ss} = \lim_{s \rightarrow 0} \frac{s \cdot \frac{1}{s}}{1 + G(s)H(s)}$$

$$e_{ss} = \frac{1}{1 + \lim_{s \rightarrow 0} G(s)H(s)} = \frac{1}{1 + K_p}$$

$$K_p = \lim_{s \rightarrow 0} G(s)H(s)$$

$\Rightarrow$ Positional error constant

(i) Type '0':

$$G(s)H(s) = \frac{K}{1 + sT}$$

$$K_p = \lim_{s \rightarrow 0} G(s)H(s) = K \Rightarrow \text{Finite}$$

$$e_{ss} = \frac{1}{1 + K_p} = \frac{1}{1 + K} \Rightarrow \text{Finite}$$

(ii) Type '1':

$$G(s)H(s) = \frac{K}{s(1 + sT)}$$

$$K_p = \lim_{s \rightarrow 0} G(s)H(s) = \lim_{s \rightarrow 0} \frac{K}{s(1 + sT)} = \infty$$

$$e_{ss} = \frac{1}{1 + K_p} = \frac{1}{\infty} = 0$$

$$e_{ss} = 0$$

(iii) Type '2':

$$G(s)H(s) = \frac{K}{s^2(1 + sT)}$$

$$K_p = \lim_{s \rightarrow 0} G(s)H(s) = \lim_{s \rightarrow 0} \frac{K}{s^2(1 + sT)} = \infty$$

$$e_{ss} = \frac{1}{1 + K_p} = 0$$

$$e_{ss} = 0$$

Case-II: For unit ramp input

$$R(s) = \frac{1}{s^2}$$

$$e_{ss} = \lim_{s \rightarrow 0} \frac{s \left(\frac{1}{s^2} \right)}{1 + G(s)H(s)} = \lim_{s \rightarrow 0} \frac{1}{s + sG(s)H(s)}$$

$$= \lim_{s \rightarrow 0} \frac{1}{sG(s)H(s)} = \frac{1}{K_v}$$

$$K_v = \lim_{s \rightarrow 0} sG(s)H(s) \Rightarrow \text{velocity error constant.}$$



(i) Type '0':

$$G(s)H(s) = \frac{K}{1+sT}$$

$$K_v = \lim_{s \rightarrow 0} sG(s)H(s) = \lim_{s \rightarrow 0} \frac{s.K}{1+sT} = 0$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{0} = \infty$$

$$e_{ss} = \infty$$

(ii) Type '1':

$$G(s)H(s) = \frac{K}{1+sT}$$

$$K_v = \lim_{s \rightarrow 0} sG(s)H(s) = K \Rightarrow \text{finite}$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{K} \Rightarrow \text{Finite}$$

(iii) Type '2':

$$G(s)H(s) = \frac{K}{s^2(1+sT)}$$

$$K_v = \lim_{s \rightarrow 0} s.G(s)H(s) = \infty$$

$$e_{ss} = \frac{1}{K_v} = 0$$

Case-III: For unit parabola input

$$R(s) = \frac{1}{s^3}$$

$$e_{ss} = \lim_{s \rightarrow 0} \frac{s \frac{1}{s^3}}{1+G(s)H(s)} = \lim_{s \rightarrow 0} \frac{1}{s^2 + s^2 G(s)H(s)}$$

$$e_{ss} = \frac{1}{K_a} = \lim_{s \rightarrow 0} s^2 G(s)H(s) \Rightarrow \text{Acceleration error constant.}$$

(i) Type '0':

$$G(s)H(s) = \frac{K}{1+sT}$$

$$K_a = \lim_{s \rightarrow 0} s^2 G(s)H(s) = 0$$

$$e_{ss} = \frac{1}{K_a} = \infty$$

(ii) Type '1':

$$G(s)H(s) = \frac{K}{s(1+sT)}$$

$$K_a = \lim_{s \rightarrow 0} s^2 G(s)H(s) = 0$$

$$e_{ss} = \frac{1}{K_a} = \infty$$

(iii) Type 2:

$$G(s)H(s) = \frac{K}{s^2(1+sT)}$$

$$K_a = \lim_{s \rightarrow 0} s^2 G(s)H(s) = K \Rightarrow \text{Finite}$$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{K} \Rightarrow \text{Finite}$$

Note: Steady state error

Input ↓ Type No. ⇒	Type 0	Type 1	Type 2
Unit step	$\frac{1}{1+K_p}$	0	0
Unit ramp	∞	$\frac{1}{K_v}$	0
Unit parabola	∞	∞	$\frac{1}{K_a}$

45.

Sol. Given,

$$M_p = 16.2\% \text{ at } t = \frac{\pi}{5\sqrt{3}}$$

$$\therefore M_p = -\frac{\xi\pi}{e\sqrt{1-\xi^2}} \Rightarrow M_p = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}$$

$$e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} = 0.162$$

$$\therefore \xi = 0.5$$

$$t_p = \frac{\pi}{5\sqrt{3}} \text{ (as } M_p \text{ occurs at } t_p \text{)}$$

$$\therefore t_p = \frac{\pi}{\omega_d} = \frac{\pi}{5\sqrt{3}}$$

$$\Rightarrow \omega_d = 5\sqrt{3}$$

$$\Rightarrow \omega_n \sqrt{1-\xi^2} = 5\sqrt{3}$$

$$\Rightarrow \omega_n^2 (1-\xi^2) = 75$$

$$\Rightarrow \omega_n^2 (1-0.5^2) = 75$$

$$\omega_n = 10 \text{ rad / sec}$$

For sinusoidal input, the frequency response is the steady state response (output).

$\therefore$ Frequency of input at which amplitude of steady state response will have maximum value = resonant frequency.

$$\text{Resonant frequency } (\omega_r) = \omega_n \sqrt{1-2\xi^2} = 10\sqrt{1-2 \times 0.5^2} = 7.07 \text{ rad / sec.}$$

$$\text{Maximum value of steady state output, } M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = 1.155$$



46.

Sol.

$$G(s)H(s) = \frac{K(s+2)}{s(s+1)}$$

- Number of open loop poles, $P = 2$ at $s = 0, -1$
- Number open loop zeros, $Z = 1$ at $s = -2$
- Angle of asymptotes,

$$\theta = \frac{(2q+1)180^\circ}{P-Z} = 180^\circ, q = 0$$

$$\theta = 180^\circ$$

Centroid,

$$\sigma = \frac{\sum (\text{real part of open loop poles}) - \sum (\text{real part of open loop zeros})}{P-Z}$$

$$= \frac{0 - 1 - (-2)}{2-1} = 1$$

- Break away point :

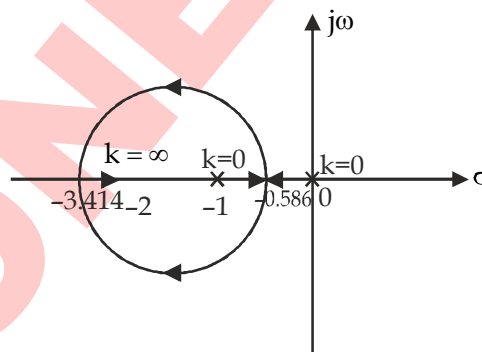
$$s(s+1) + K(s+2) = 0$$

$$\frac{dK}{ds} = 0 \Rightarrow (s+2)(2s+1) - s(s+1) = 0$$

$$s^2 + 4s + 2 = 0$$

 $\Rightarrow$

$$s = -2 \pm \sqrt{2} = -3.414, -0.586$$



47.

Sol.

$$G(s) = \frac{K}{s(1+s\tau_1)(1+s\tau_2)}$$

- Number of open loop poles, $P = 3$ at $s = 0, \frac{1}{\tau_1}, \frac{1}{\tau_2}$
- Number of open loop zeros, $z = 0$
- Angle of asymptotes,

$$\theta = \frac{(2q+1)180^\circ}{p-z}; q = 0, 1, 2$$



$$= \frac{(2q+1)180^\circ}{3-0} = 60^\circ, 180^\circ, 300^\circ$$

- Centroid,

$$\sigma = \frac{\sum \left(\text{real part of open loop poles} \right) - \sum \left(\text{real part of open loop zeros} \right)}{P - Z}$$

$$= \frac{0 - \frac{1}{\tau_1} - \frac{1}{\tau_2} - 0}{3} = -\frac{1}{3} \left[\frac{1}{\tau_1} + \frac{1}{\tau_2} \right]$$

- The characteristic equation is

$$s(\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2)s + 1) + k = 0$$

$$k = -s[1 + (\tau_1 + \tau_2)s + \tau_1 \tau_2 s^2]$$

For break away point, $\frac{dk}{ds} = 0$

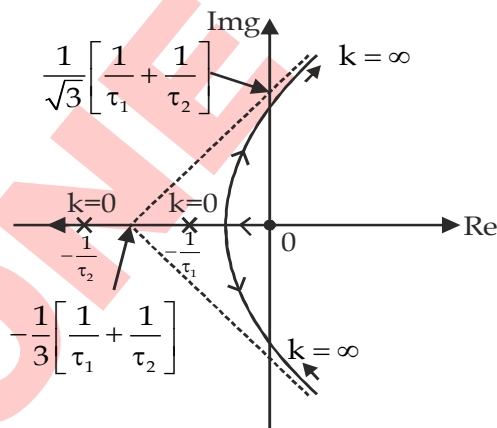
$\Rightarrow$

$$-1 - 2(\tau_1 + \tau_2)s - 3\tau_1 \tau_2 s^2 = 0$$

$$s^2 + \frac{2}{3} \left[\frac{1}{\tau_1} + \frac{1}{\tau_2} \right] s + \frac{1}{3\tau_1 \tau_2} = 0$$

From the above analysis the rough sketch of root locus is

Let $\tau_1 > \tau_2$



R - H table

$$\begin{array}{c|cc} s^3 & \tau_1 \tau_2 & 1 \\ s^2 & (\tau_1 + \tau_2) & k \\ s^1 & 1 - k \left[\frac{\tau_1 \tau_2}{\tau_1 + \tau_2} \right] & 0 \\ s^0 & k & \end{array}$$

for system to be stable

$$1 - \frac{k\tau_1 \tau_2}{(\tau_1 + \tau_2)} > 0 \text{ and } k > 0$$

$$\Rightarrow 0 < k < \left[\frac{1}{\tau_1} + \frac{1}{\tau_2} \right]$$

Effect of adding a pole $G(s) = \frac{k}{s(1+s\tau_1)(1+s\tau_2)(1+s\tau_3)}$

- Number of open loop poles, $P = 4$ at $s = 0, -\frac{1}{\tau_1}, -\frac{1}{\tau_2}, -\frac{1}{\tau_3}$
- Number of open loop zeros, $z = 0$

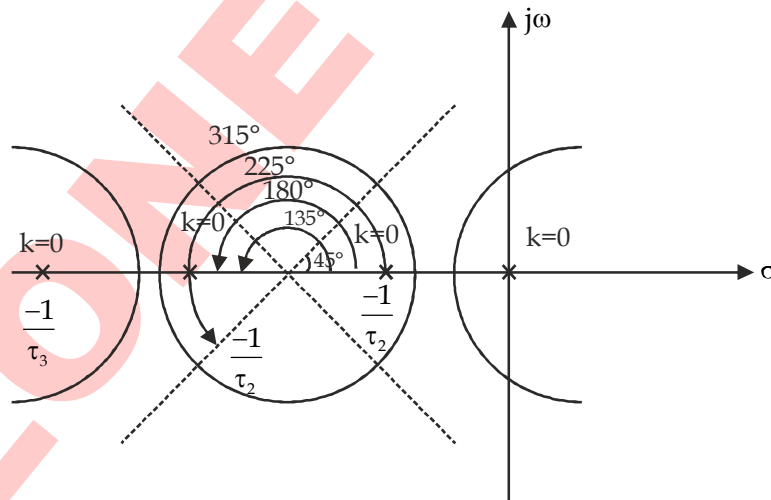
- Angle of asymptotes $\theta = \frac{(2q+1)180^\circ}{p-z}; q = 0, 1, 2, 3$

$$\theta = \frac{(2q+1)180^\circ}{4-0}; 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

- Centroid, $\sigma = \frac{0 - \frac{1}{\tau_1} - \frac{1}{\tau_2} - \frac{1}{\tau_3}}{4} = \frac{1}{4} \left[\frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3} \right]$

- The characteristic equation is $s(1+s\tau_1)(1+s\tau_2)(1+s\tau_3) + K = 0$

Effect of adding of pole: $G(s) = \frac{K}{s(1+s\tau_1)(1+s\tau_2)(1+s\tau_3)}$



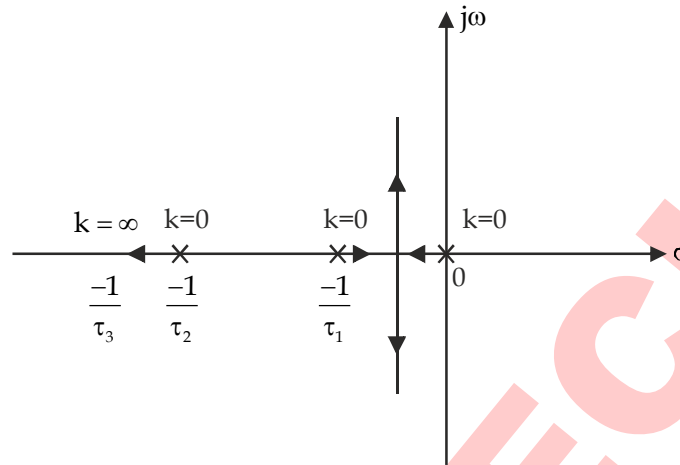
Effect of adding of zero. $G(s) = \frac{K(1+s\tau_3)}{s(1+s\tau_1)(1+s\tau_2)}$

- No. of open loop poles, $P = 3$ at $s = 0, -\frac{1}{\tau_1}, -\frac{1}{\tau_2}$
- No. of open loop zero, $z = 1$ at $s = -\frac{1}{\tau_3}$
- No. of asymptotes = $P - Z = 2$

- Angle of asymptotes

$$Q_A = \frac{(2q+1)180^\circ}{P-z}, q=0,1$$

$$Q_A = 90^\circ, 270^\circ$$



Effect of Adding of open loop poles on Root Locus

- (i) Root locus bends towards real axis
- (ii) Root Locus shifts towards right half of s plane.
- (iii) Range of gain parameter for stability reduces.
- (iv) Stability of system reduces.
- (v) $j\omega$ cross over reduces so frequency of oscillation reduces.
- (vi) Improve steady state response.

Effect of Adding of Open loop zeros on root Locus

- (i) Range of K for stability of system increases.
- (ii) Stability of system increases. Root Locus moves away from $j\omega$ axis
- (iii) Frequency of oscillation increases
- (iv) Size of oscillation decreases
- (v) Improve transient response

48.

Sol.

$$G(s) = \frac{K}{s(s+2)(s+6)}$$

- Number of open loop poles, $p = 3$ at $s = 0, -2, -6$
- Number of open loop zeros, $z = 0$

- Angle of asymptotes, $\theta = \frac{(2q+1)180^\circ}{p-z} = 60^\circ, 180^\circ, 300^\circ$

- Centroid, $\sigma = \frac{0-2-6-0}{3-0} = -\frac{8}{3}$

- Break away points:

Characteristic equation is: $s(s+2)(s+6) + K = 0$

$$\frac{dK}{ds} = 0 \Rightarrow (s^2 + 2s) + (s+6)(2s+2) = 0$$

$$3s^2 + 16s + 12 = 0$$

$$s = -0.9, -4.43$$

for $s = -0.9 \Rightarrow K > 0$

for $s = -4.43 \Rightarrow K < 0$

$\therefore s = -0.9$ is a valid break away point.

Routh's away

$$\begin{array}{c|cc} s^3 & 1 & 12 \\ s^2 & 8 & K \\ s^1 & 12 - \frac{K}{8} & 0 \\ s^0 & K & \end{array}$$

For marginal stability,

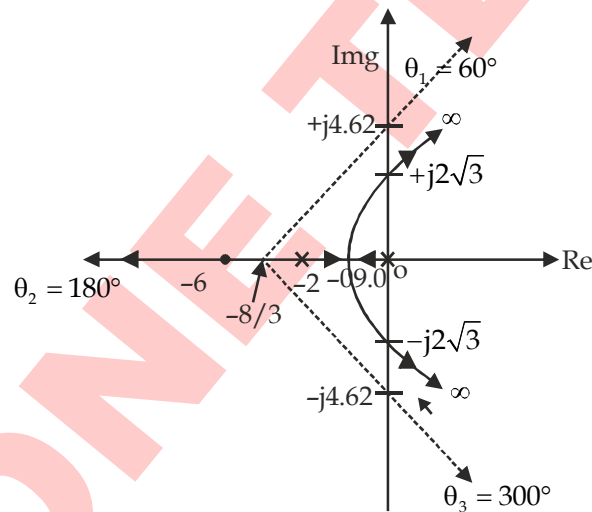
$$K = 96$$

Auxiliary equation is

$$8s^2 + 96 = 0$$

$$s = \pm j2\sqrt{3}$$

$S = \pm j2\sqrt{3}$ are the points of intersection of root locus with imaginary axis.



Effect of adding zero ($s + 3$) to the transfer function

$$G(s)H(s) = \frac{K(s+3)}{s(s+2)(s+6)}$$

- $p = 3, z = 1, p - z = 2$
- $\theta_1 = 90^\circ, \theta_2 = 270^\circ$

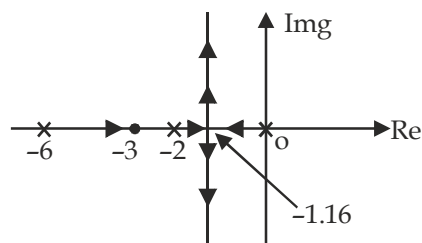
- Centroid,

$$\sigma = \frac{0 - 2 - 6 - (-3)}{3 - 1} = \frac{5}{2} = -2.5$$

- Break away point

$$s(s+2)(s+6) - (s+3)(3s^2 + 16s + 12) = 0$$

$$\Rightarrow s = -1.16$$



49.

Sol. (i) Gain margin : It is defined as the margin in gain allowable by which gain can be increased till system reaches on the verge of instability.

Mathematically,
$$G.M. = \frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}}$$

in decibels,
$$G.M. = 20 \log \left[\frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}} \right]$$

$$G.M. = -20 \log [|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}]$$

Where, ω_{pc} is the phase cross over frequency.

(ii) Phase margin : The amount of additional phase lag which can be introduced in the system till system reaches on the verge of instability is called phase margin (P.M.)

Mathematically,
$$P.M. = 180^\circ + \angle G(j\omega)H(j\omega) \Big|_{\omega=\omega_{gc}}$$

Where, ω_{gc} is the gain cross over frequency.

(iii) Gain crossover frequency: The frequency at which the magnitude of $G(j\omega)H(j\omega)$ (i.e., $\angle G(j\omega)H(j\omega)$) is unity is called gain crossover frequency (ω_{gc})

$$\angle G(j\omega)H(j\omega) \Big|_{\omega=\omega_{gc}} = 1$$

(iv) Phase crossover frequency: The frequency at which the phase angle of $G(j\omega)H(j\omega)$ (i.e., $\angle G(j\omega)H(j\omega)$) is -180° is called phase crossover frequency (ω_{pc})

$$\angle G(j\omega)H(j\omega) \Big|_{\omega=\omega_{pc}} = 180^\circ$$

50.

Sol.

$$G(s)H(s) = \frac{100(0.1s + 1)}{s(0.2s + 1)(0.5s + 1)}$$

$$G(j\omega)H(j\omega) = \frac{100(j0.1\omega + 1)}{j\omega(j0.2\omega + 1)(j0.5\omega + 1)}$$

The system is type-1, hence the initial slope of bode plot = -20 dB/dec and intersects 0 dB axis at $\omega = k = 100$.

The corner frequencies due to first order terms are

$$\omega = \frac{1}{0.5} = 2 \text{ rad/sec due to denominator term}$$

$$\omega = \frac{1}{0.2} = 5 \text{ rad/sec due to denominator term}$$

$$\omega = \frac{1}{0.1} = 10 \text{ rad/sec due to numerator term}$$

As the initial slope of the plot is -20 db/decade and corner frequency $\omega = 2$ is due to denominator first order term, the slope of the plot after $\omega = 2$ rad/sec is $(-20 - 20) = -40$ db/decade. Similarly

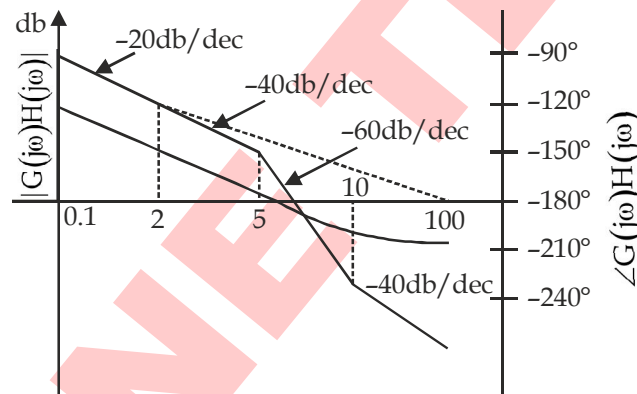
After = 5 rad/sec, $(-40 - 20) = -60$ db/decade

After = 10 rad/sec, $(-60 + 20) = -40$ db/decade

The phase angle,

$$\angle G(j\omega)H(j\omega) = 90^\circ \tan^{-1} \frac{\omega}{2} - \tan^{-1} \frac{\omega}{5} + \tan^{-1} \frac{\omega}{10}$$

ω rad / sec	=	0.1	2	5	10	100
$\angle G(j\omega)H(j\omega)$	=	-93.43°	-145.5°	-176.63°	-187°	181.7°



51.

Sol. Gain margin: Gain margin is defined as the margin in gain allowable by which gain can be increased till control system reaches on the verge of instability.

The positive gain margin means such increase in gain 'k' is possible before system becomes unstable, hence system is stable and negative gain margin means k is greater than $K_{\max}$ and system is unstable so 'k' is required to be reduced to make the system stable.

$$\text{Gain margin} = \frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}}$$

where,

ω_{pc} = phase cross-over frequency

Phase margin: The amount of additional phase lag which can be introduced in the system till system reaches on the verge of instability is called phase margin.

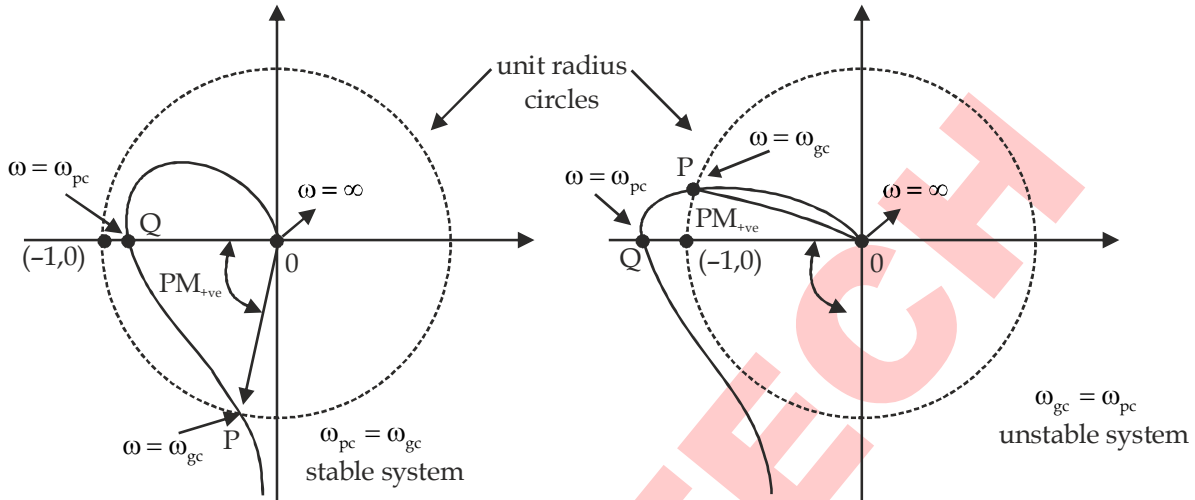
The positive phase margin means such negative angle introduction in system is possible before system becomes unstable. Such system is stable. While negative phase margin means present negative phase lag should be changed by adding positive angle hence phase margin is said to be negative and system is unstable.

$$\text{Phase margin} = 180^\circ + \left| \angle G(j\omega)H(j\omega) \right|_{\omega=\omega_{gc}}$$

where,

ω_{gc} = gain cross-over frequency

(i) Calculation of G.M. and P.M. from polar plot:



in polar plot $|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}$ is length of OQ where Q is the intersection of polar plot with negative real axis

Q is the point corresponding to $\omega = \omega_{pc}$

$$GM = \frac{1}{l(OQ)}$$

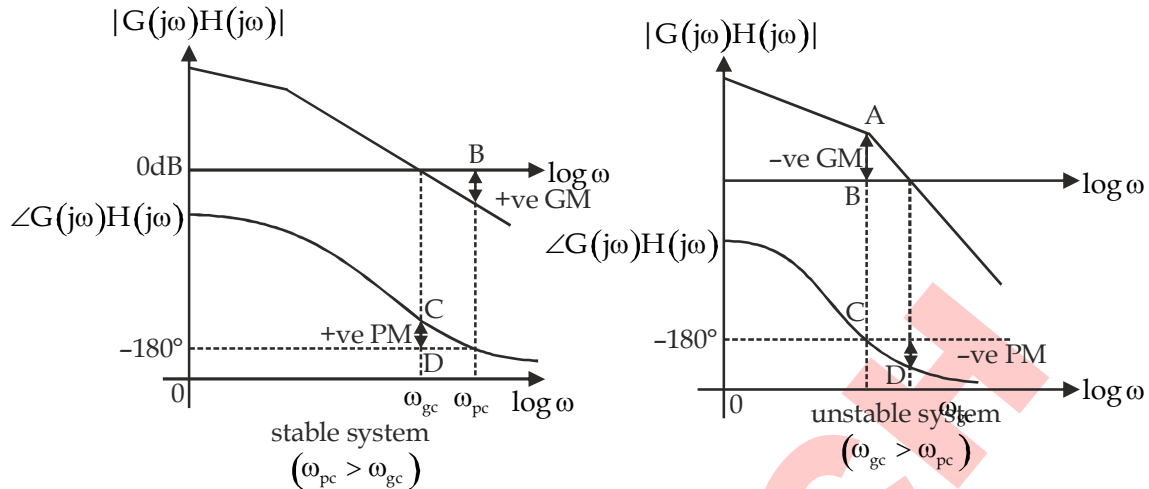
The point P is the point on polar plot corresponding to $\omega = \omega_{gc}$ obtained by drawing unit radius circle.

So if ϕ is the angle of point P corresponding to $\omega = \omega_{gc}$. The P.M. can be calculated as shown in figure above.

(ii) Determination of G.M. and P.M. from bode plot:

According to definition of G.M. $GM = -20 \log |G(j\omega)H(j\omega)|_{\omega=\omega_{pc}} \text{ dB}$

Now $20 \log |G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}$ in dB can be directly read out from magnitude plot. Extend $\omega = \omega_{pc}$ line upwards till it intersects resultant magnitude plot say point A as shown in figure below. The magnitude corresponding to point A is $|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}$



Similarly.

$$\text{P.M.} = 180^\circ + \angle G(j\omega)H(j\omega)|_{\omega=\omega_{gc}}$$

Now $\angle G(j\omega)H(j\omega)|_{\omega=\omega_{gc}}$ can be read out from phase angle plot. Extend $\omega = \omega_{gc}$ line downwards till it intersects phase angle plot. This point of intersection is point C shown in figure above.

52.

Sol. Relative advantages and disadvantages of the Bode and Nyquist plots: Advantages of the Nyquist plot:

- (i) The Nyquist plot can be used for the study of stability of systems with nonminimum-phase transfer functions.
- (ii) The stability analysis of a closed-loop system can be easily investigated by examining the Nyquist plot of the loop transfer function with reference to the $(-1, j0)$ point once the plot is made.

Disadvantage of the Nyquist Plot:

- (i) It's not so easy to carry out the design of the controller by referring to the Nyquist plot.

Advantages of the Bode plot:

- (i) In the absence of a computer, a Bode diagram can be sketched by approximating the magnitude and phase with straight-line segments.
- (ii) Gain crossover, phase crossover, gain margin and phase margin are more easily determined on the Bode plot than from the Nyquist plot.
- (iii) For design purposes, the effects of adding controllers and their parameters are more easily visualized on the Bode plot than on the Nyquist plot.

Disadvantage of the Bode plot:

- (i) Stability of only minimum-phase systems can be determined from the Bode plot. For instance, there is no way of telling what the stability criterion is on the Bode plot.

Relative Advantages and Disadvantages of the Bode and Nyquist plots

The Bode plot provides more information than the Nyquist plot because the frequency is shown explicitly. In addition, it facilitates analysis over a wide range of frequencies due to its logarithmic frequency scale. Another advantage of the Bode Plot is that it allows the open-loop frequency response characteristic to be graphically constructed from the characteristics for the individual transfer function.

53.

Sol.

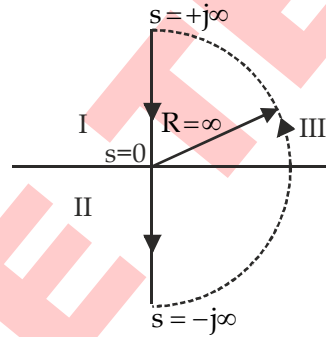
$$G(s)H(s) = \frac{K}{(s+2)(s+5)}$$

$$G(j\omega)H(j\omega) = \frac{K}{(j\omega+2)(j\omega+5)}$$

$$|G(j\omega)H(j\omega)| = \frac{K}{(\sqrt{\omega^2+4})(\sqrt{\omega^2+25})}$$

$$\angle G(j\omega)H(j\omega) = -\tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{5}\right)$$

- No open loop pole in right hand side of s-plane hence $P=0$
- No open loop pole at origin hence nyquist path is as shown in figure below



- Section I : $s = +j\omega$ to $s = 0$

$\omega \rightarrow +\infty$ start	$0 \angle -180^\circ$	$\Rightarrow$	$0^\circ - (-180^\circ) = +180^\circ$ Anticlockwise
$\omega = 0$ end	$\frac{k}{10} \angle 0^\circ$		

- Section II: $s = j0$ to $s = -j\infty$

This is mirror image of section-I about real axis

- Section III: $s = -j\infty$ to $s = +j\infty$

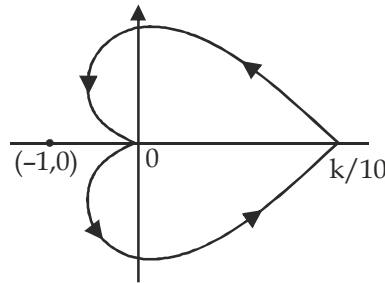
$\omega \rightarrow +\infty$ start	$0 \angle 180^\circ$	$\Rightarrow$	$-180^\circ - 180^\circ = -360^\circ$ Clockwise
$\omega = \infty$ end	$0 \angle -180^\circ$		

The mapping of section III has radius 0 hence it is around origin with 360° clockwise rotation with radius tending to zero. This is generally not required for the closed loop stability as it does not affect the encirclements of critical point $-1 + j0$

$$G(j\omega)H(j\omega) = \frac{k(2-j\omega)(5-j\omega)}{(\omega^2+4)(\omega^2+25)} = \frac{k(10-\omega^2)}{(\omega^2+4)(\omega^2+25)} - j \frac{7k\omega}{(\omega^2+4)(\omega^2+25)}$$

The imaginary part is zero for $\omega = 0$

$$G(j\omega)H(j\omega)|_{\omega=0} = \frac{10k}{4 \times 25} = \frac{k}{10}$$



- Critical point $(-1+j0)$ is not encircled by the Nyquist plot hence $N = 0$, $Z = N + P$, $Z = 0$. Hence the system is absolutely stable.

54.

Sol. Given:

$$G(s)H(s) = \frac{K}{(s+20)(s^2-2s+1)}$$

$$= \frac{K}{(s+20)(s-1)^2}$$

$$GH(j\omega) = \frac{K}{(j\omega+20)(j\omega-1)^2}$$

$$|GH(j\omega)| = \frac{K}{(\sqrt{\omega^2+400})(\omega^2+1)}$$

$$\text{For } \omega=0 \quad |GH(j\omega)| = \frac{K}{20}$$

$$\omega \rightarrow \infty \quad |GH(j\omega)| \rightarrow 0$$

and

$$\angle GH(j\omega) = -\tan^{-1}\left(\frac{\omega}{20}\right) - \left[\tan^{-1}\left(\frac{\omega}{-1}\right) + 180^\circ\right] \times 2$$

$$= -\tan^{-1}\left(\frac{\omega}{20}\right) + 2\tan^{-1}(\omega)$$

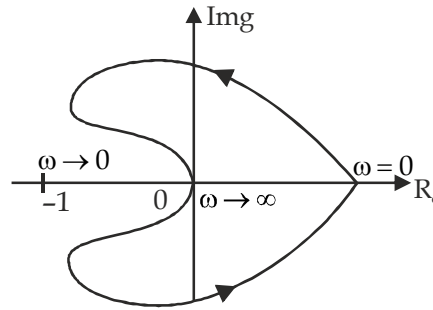
$$\text{For } \omega=0 \quad ; \quad \angle GH(j\omega) = 0^\circ$$

$$\omega=10 \quad ; \quad \angle GH(j\omega) = 142^\circ$$

$$\omega=20 \quad ; \quad \angle GH(j\omega) = 129^\circ$$

$$\omega=\infty \quad ; \quad \angle GH(j\omega) = 90^\circ$$

Therefore, Nyquist plot



Stability from Nyquist plot is given by the equation

$$Z = P - N$$

where Z = closed loop poles on RHS of s -plane

P = open loop poles on RHS of s -plane

N = anti-clock encirclements of $(-1 + 0j)$

Here, $P = 2$, $N = 0$ ($\therefore K$ is always positive)

For stability: $Z = P - N = 2 - 0 = 2$

From Nyquist plot : $N = 0$

$$Z \neq 0$$

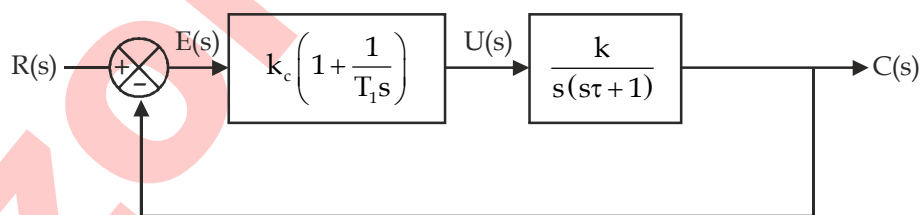
Hence, the system is always unstable irrespective of values of K .

55.

Sol. In an integral error compensation scheme, the output response depend in some manner upon the integral of the actuating signal. In the feedback control system, shown below, this type of compensation has been introduced by using a PI controller. The plant transfer function is

$$G(s) = \frac{k}{s(\tau s + 1)}$$

The controller transfer function is $D(s) = k_c \left(1 + \frac{1}{T_i s} \right) = k_c + \frac{k_i}{s}$



The control signal is $u(t) = k_c e(t) + k_i \int_0^t e(\tau) d\tau$

The open loop transfer function of the overall system is

$$D(s)G(s) = \frac{k(k_c s + k_i)}{s^2(\tau s + 1)}$$

Clearly, the PI control adds a zero at $s = -k_i/k_c$ and a pole at $s = 0$ to the open loop transfer function. It may therefore be less stable than the original second order system or even become unstable if k_c and k_i are not properly chosen.

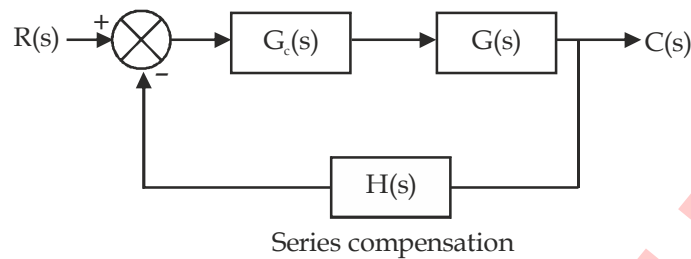
The steady state error of the compensated system is always zero for ramp input. By choosing proper value of k_c and k_i the transient response can be made satisfactory.

The Integral action is often realized in biological systems.



56.

Sol. (i) Compensators: The compensators are physical devices, which are generally used for series compensation. Compensators in control systems are used for improvement of the performance specification i.e. transient and steady state response characteristics.

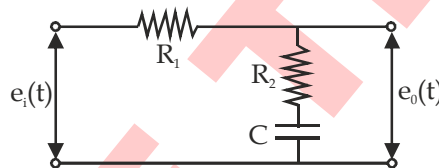


The commonly used electrical compensators are

- Lead compensator
- Lag compensator
- Lag-lead compensator

(ii) Lag compensator: If the steady state output has phase lag with respect to the input applied to the compensator then the network is called lag network or lag compensator.

Lag compensator is used for improving the steady state response characteristics of the system i.e. for elimination of steady state error between input and output.

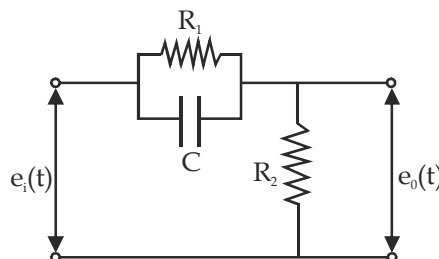


$$\frac{E_o(s)}{E_c(s)} = \beta = \frac{(1 + s\tau)}{(1 + s\tau\beta)}$$

where, $\beta = \frac{R_1 + R_2}{R_2}$ and $T = R_2 C$

Lead compensator: If the steady state output has phase lead with respect to the input applied to the compensator then the network is called lead network or lead compensator.

Lead compensator is used for improvement the transient state characteristics of the system i.e. speed of response of the system.

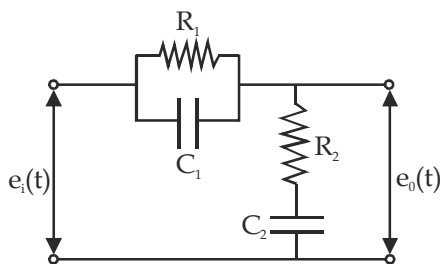


$$\frac{E_o(s)}{E_i(s)} = \frac{(s + 1/T)}{(s + 1/\alpha T)}$$

where, $\alpha = \frac{R_2}{R_1 + R_2}$ and $T = R_1 C$

Lag-Lead compensator: In lag-lead compensator, both phase lag and lead occurs but in the different frequency regions.

The lag-lead compensator improves both transient and steady state response characteristics



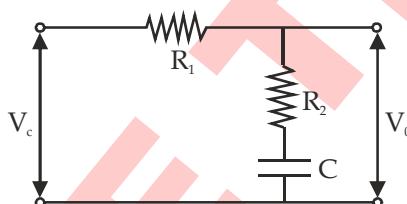
$$\frac{E_o(s)}{E_i(s)} = \frac{(1 + T_1 s)(1 + T_2 s)}{\left(1 + \frac{T_1}{\beta} s\right)(1 + T_2 \beta s)}$$

where,

$$T_1 = R_1 C_1 \text{ and } T_2 = R_2 C_2$$

57.

Sol. In phase-lag network, the phase of output voltage lags the phase of input voltage for sinusoidal inputs.



Transfer function of phase-lag network shown above is given by

$$\frac{V_o(s)}{V_i(s)} = \frac{1 + sT}{1 + s\beta T}$$

where,

$$\beta = \frac{R_1 + R_2}{R_2} \text{ and } T = R_2 C$$

The phase-lag network when introduced in cascade with forward-path of a transfer function.

- (i) The gain crossover frequency decreases, as the pole is nearer to the origin compared to zero. Hence the effect of pole is dominant, therefore, the phase shift is reduced.
- (ii) The lag-network allows to pass low frequencies and high frequencies are attenuated, hence bandwidth also decreases and
- (iii) Signal to noise ratio increases.

58.

Sol.

$$[A] = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s & -1 \\ 2 & s + 3 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{(s+1)(s+2)} \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}$$

$$= \begin{bmatrix} \left(\frac{2}{s+1} - \frac{1}{s+2} \right) \left(\frac{1}{s+1} - \frac{1}{s+2} \right) \\ \left(\frac{-2}{s+1} + \frac{2}{s+2} \right) \left(\frac{-1}{s+1} + \frac{2}{s+2} \right) \end{bmatrix}$$

State transition matrix,

$$\phi(t) = \mathfrak{I}^{-1} \{ [sI - A]^{-1} \} = \begin{bmatrix} (2e^{-t} - e^{-2t}) & (e^{-t} - e^{-2t}) \\ (-2e^{-t} + 2e^{-2t}) & (-e^{-t} + 2e^{-2t}) \end{bmatrix}$$

time response,

$$x(t) = [\mathfrak{I}^{-1} (sI - A)^{-1}] x(0) + \mathfrak{I}^{-1} [(sI - A)^{-1} B U(s)] \quad \dots(i)$$

$$(sI - A)^{-1} B U(s) = \begin{bmatrix} \frac{1}{s} \left(\frac{1}{s+1} - \frac{1}{s+2} \right) \\ \frac{1}{s} \left(\frac{1}{s+1} + \frac{1}{s+2} \right) \end{bmatrix} = \begin{bmatrix} \frac{1}{s} - \frac{1}{s+1} - \frac{1}{2} \left(\frac{1}{s} - \frac{1}{s+2} \right) \\ -\frac{1}{s} + \frac{1}{s+1} + \frac{1}{s} - \frac{1}{s+2} \end{bmatrix}$$

$$\Rightarrow \mathfrak{I}^{-1} (sI - A)^{-1} B U(s) = \begin{bmatrix} \frac{1}{2} - e^{-t} + \frac{1}{2} e^{-2t} \\ e^{-t} - e^{-2t} \end{bmatrix}$$

$$x(t) = \begin{bmatrix} 2e^{-t} - e^{-2t} - e^{-t} + e^{-2t} + \frac{1}{2} - e^{-t} + \frac{1}{2} e^{-2t} \\ -2e^{-t} + 2e^{-2t} + e^{-t} - 2e^{-t} + e^{-t} - e^{-2t} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} + \frac{1}{2} e^{2t} \\ -e^{-2t} \end{bmatrix}$$

$$\Rightarrow \begin{aligned} x_1(t) &= \frac{1}{2}(1 + e^{-t}) \\ x_2(t) &= -e^{-t} \end{aligned}$$

59.

Sol.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a & -b & -c \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a & -b & -c \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a & -b & -c \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a & -b & -c \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ -a & -b & -c \\ ac & -a+bc & -b+c^2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a & -b & -c \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -c \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 0 & 0 & 1 \\ -a & -b & -c \\ ac & -a+bc & -b+c^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -c \\ -b+c^2 \end{bmatrix}$$

Controllable matrix

$$Q_c = [B \mid AB \mid A^2B] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -c \\ 1 & -c & -b+c^2 \end{bmatrix}$$

$$|Q_c| = -1 \neq 0$$

Therefore system is controllable for all values of a, b and c.