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# RPSC-AEn Mains

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*for*

**EMFT**



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## PART - A

## 2 Marks Questions

Q.1

**Sol.** The region around a magnet within which the influence of the magnet can be experienced is called magnetic field. Such a field is represented by imaginary lines around the magnet which are called magnetic lines of force. These lines of force are also called magnetic lines of flux or magnetic flux lines.

Q.2

**Sol.** The magnetic field intensity at any point in the magnetic field is defined as the force experienced by a unit north pole of one weber strength, when placed at that point. Magnetic field intensity is measured in newtons/weber (N/Wb) or amperes per metre (A/m) or ampere-tums/metre (AT/m). It is denoted as  $\vec{H}$ .

Q.3

**Sol.** Permeability is the ability or ease with which the current carrying conductor forces the magnetic flux through the region around it. For a free space, the permeability is denoted as  $\mu_0$  and its value is  $4\pi \times 10^{-7}$ . The permeability  $\mu$  is measured in henries per metre (H/m). For any other region, a relative permeability is specified as  $\mu = \mu_0 \mu_r$ .

Q.4

**Sol.** The Biot-Savart law states that,  
The magnetic field intensity  $d\vec{H}$  produced at a point P due to a differential current element  $I d\vec{L}$  is,  
1. Proportional to the product of the current  $I$  and differential length  $dL$ .  
2. The sine of the angle between the element and the line joining point P to the element.  
3. And inversely proportional to the square of the distance  $R$  between point P and the element.  
Mathematically it is represented as,

$$d\vec{H} = \frac{I d\vec{L} \times \vec{R}}{4\pi R^3} \text{ A/m}$$

Q.5

**Sol.** The expression for  $\vec{H}$  on the axis of a circular loop placed in x-y plane is,

$$\vec{H} = \frac{I r^2}{2(r^2 + z^2)^{3/2}} \vec{a}_z \text{ A/m}$$

where

$r$  = Radius of the circular loop

and

$z$  = Distance of point along the axis

If point is shifted at the centre of the circular loop i.e.  $z = 0$ ,

$$\vec{H} = \frac{I r^2}{2(r^2)^{3/2}} \vec{a}_z = \frac{1}{2r} \vec{a}_z \text{ A/m}$$

where  $\vec{a}_z$  is the unit vector normal to xy plane in which the circular loop is lying.

Q.6

**Sol.** The Ampere's circuital law states that,

The line integral of magnetic field intensity  $\vec{H}$  around a closed path is exactly equal to the direct current enclosed by that path.

The mathematical representation of Ampere's circuital law is,

$$\oint \vec{H} \cdot d\vec{L} = I$$

Q.7

Sol. For an infinite sheet of current density point is

$$\bar{K} \text{ A / m, } \bar{H}$$

$$\bar{H} = \frac{1}{2} \bar{K} \times \bar{a}_N$$

where

$\bar{a}_N$  = Unit vector normal from the current sheet to the point  
at which  $\bar{H}$  is to be obtained.

Q.8

Sol. For a cylindrical conductor carrying a direct current I with radius 'R'm,

$$\bar{H} = \frac{Ir}{2\pi R^2} \bar{a}_\phi \text{ A / m} \quad \dots \text{ for } r < R$$

$$\bar{H} = \frac{I}{2\pi R} \bar{a}_\phi$$

on the surface of conductor

$$\bar{H} = \frac{I}{2\pi r} \bar{a}_\phi \quad \dots \text{ for } r > R$$

Q.9

Sol. The integral  $\oint_S \bar{B} \cdot d\bar{S}$  evaluated over a closed surface is always zero. $\therefore$ 

$$\oint_S \bar{B} \cdot d\bar{S} = 0$$

This is called law of conservation of magnetic flux or Gauss's law in integral form for magnetic fields. The Gauss's law in differential form for magnetic fields is given by,

$$\nabla \cdot \bar{B} = 0$$

The divergence of magnetic flux density is always zero.

Q.10

Sol. The Laplace's equation for scalar magnetic potential is,

$$\nabla^2 V_m = 0 \text{ for } \bar{J} = 0$$

Q.11

Sol. The vector magnetic potential  $\bar{A}$  is defined such that curl of vector magnetic potential is the flux density.

$$\bar{B} = \nabla \times \bar{A}$$

The definition requires that  $\nabla \cdot \bar{B} = 0$ . The vector magnetic potential  $\bar{A}$  serves as an intermediate quantity from which  $\bar{B}$  and hence  $\bar{H}$  can be obtained. It is measured in Wb/m.

Q.12

Sol.  $\nabla \times \bar{H} = \bar{J}$  hence  $\nabla \times \frac{\bar{B}}{\mu_0} = \bar{J} \dots \bar{B} = \mu_0 \bar{H}$ 

$$\therefore \nabla \times \bar{B} = \mu_0 \bar{J} \quad \text{but } \bar{B} = \nabla \times \bar{A}$$

$$\therefore \nabla \times \nabla \times \bar{A} = \mu_0 \bar{J}$$

Using vector identity to express left hand side we can write,

$$\nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A} = \mu_0 \bar{J}$$

$$\therefore \bar{J} = \frac{1}{\mu_0} [\nabla \times \nabla \times \bar{A}] = \frac{1}{\mu_0} [\nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A}]$$



Q.13

**Sol.** A static magnetic field can exist in a good conductor. When conductor carries current, it produces the flux which can exist inside the conductor. Due to this flux, magnetic field and intensity exists at a point inside the good conductor.

Q.14

**Sol.** For an infinite current sheet,

$$\vec{H} = \frac{1}{2} \vec{K} \times \vec{a}_N \quad \text{where}$$

$$\vec{K} = 30\vec{a}_z \text{ mA / m}$$

For plane  $y = 0$ ,  $\vec{H}$  cannot have any component in  $z$  direction and  $\vec{a}_N = \vec{a}_y$ .

$$\therefore \vec{H} = \frac{1}{2} [30\vec{a}_z \times \vec{a}_y] \quad \text{But}$$

$$\vec{a}_z \times \vec{a}_y = -\vec{a}_x$$

$$\therefore \vec{H} = -15\vec{a}_x \text{ mA / m}$$

Q.15

**Sol.** Ampere's circuital law can be used to obtain  $\vec{H}$  due to,

- (a) Infinitely long straight conductor
- (b) Co-axial cable
- (c) Infinite sheet of current
- (d) Circular conductor carrying current  $I$ .

## PART - B

### 5 Marks Questions

Q.16

**Sol.** Maxwell's equations in free space (no free charges, no free currents) are:

- $\nabla \cdot \vec{E} = 0$
- $\nabla \cdot \vec{B} = 0$
- $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
- $\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

These equations describe the relationship between the electric field ( $\vec{E}$ ), magnetic field ( $\vec{B}$ ), and how they propagate as electromagnetic waves in free space.

Q.17

**Sol.** At the interface:

- The tangential component of electric field ( $\vec{E}_t$ ) is continuous.
- The normal component of the electric displacement ( $\vec{D}_n$ ) satisfies:

$$D_{1n} - D_{2n} = \sigma_s$$

where  $\sigma_s$ , is the surface charge density at the boundary. For no surface charge,  $D_{1n} = D_{2n}$ , ensuring field continuity across the media.



Q.18

**Sol.** A time-varying current produces an electromagnetic field (EM field), meaning both time-varying electric and magnetic fields coexist and propagate as electromagnetic waves. The changing magnetic field produces an electric field (per Faraday's law), and vice versa (per Maxwell's correction to Ampère's law).

Q.19

**Sol.** A transmission line is distortionless when the following condition holds:

$$\frac{R}{L} = \frac{G}{C}$$

Where, R = resistance/length,

L = inductance/length,

G = conductance/length,

C = capacitance/length.

This ensures that all frequency components of the signal travel at the same velocity, preventing distortion during transmission.

Q.20

**Sol.** The intrinsic impedance ( $\eta$ ) of a dielectric is given by:

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

where,  $\mu$  = permeability,

$\epsilon$  = permittivity of the medium. For free space,

$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\Omega$  It quantifies the ratio of electric to magnetic field strengths in a plane electromagnetic wave in the medium.

## PART - C

### 20 Marks Questions

Q.21

**Sol.** Relationship between E & H :

Intrinsic impedance of any medium is given by -

$$\eta = \frac{\bar{E}}{H} \text{ or } \eta = \frac{|E|}{|H|}$$

Case I : Perfect Dielectric :

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}}$$

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} \cdot \sqrt{\frac{\mu_r}{\epsilon_r}}$$

$$\eta = \eta_0 \sqrt{\frac{\mu_r}{\epsilon_r}}$$

where,

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$\eta_0$  = intrinsic impedance of free space,

$$\begin{aligned}\therefore \eta_0 &= \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7} \text{ H / m}}{\left(\frac{1}{36\pi} \times 10^{-9}\right) \text{ V / m}}} \\ &= \sqrt{4\pi \times 36\pi \times 10^2}\end{aligned}$$

$$\Rightarrow \eta_0 = 120\pi\Omega = 377\Omega$$

$$\therefore \eta = 120\pi \sqrt{\frac{\mu_r}{\epsilon_r}}$$

For all dielectrics,  $\mu_r = 1$

$$\eta = \frac{120\pi}{\sqrt{\epsilon_r}} = \frac{\eta_0}{\sqrt{\epsilon_r}}$$

**Case II :** Intrinsic impedance for free space,

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi\Omega = 377\Omega$$

**Case III :** Intrinsic impedance for Lossy dielectrics

$$\bar{\Delta}^2 \bar{E} = j\omega\mu(\sigma + j\omega\epsilon)$$

$$\therefore \eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

**A. For good dielectrics,  $\omega\epsilon \gg \sigma$**

$$\begin{aligned}\eta &= \sqrt{\frac{j\omega\mu}{j\omega\epsilon \left(1 + \frac{\sigma}{j\omega\epsilon}\right)}} \\ &= \sqrt{\frac{j\omega\mu}{j\omega\epsilon}} \frac{1}{\sqrt{\left(1 + \frac{\sigma}{j\omega\epsilon}\right)}}\end{aligned}$$

$$\eta = \sqrt{\frac{\pi}{\epsilon}} \left[1 - j \frac{\sigma}{\omega\epsilon}\right]^{-1/2}$$

Since for a good dielectric,  $\sigma \ll \omega\epsilon$

$$\text{therefore, } \left[1 - j \frac{\sigma}{\omega\epsilon}\right]^{-1/2} = \left(1 + j \frac{\sigma}{2\omega\epsilon}\right)$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} \left[1 + j \frac{\sigma}{2\omega\epsilon}\right]$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} + j \frac{\sigma}{2\omega\epsilon} \sqrt{\frac{\mu}{\epsilon}}$$

B. For good conductors,  $\omega\epsilon \ll \sigma$

$$\begin{aligned}\eta &= \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{\sigma}} \left[ \frac{1}{\sqrt{1 + \frac{j\omega\epsilon}{\sigma}}} \right] \\ &= \sqrt{\frac{j\omega\mu}{\sigma}} \left[ 1 + \frac{j\omega\epsilon}{\sigma} \right]^{-1/2}\end{aligned}$$

For good conductors,  $\frac{\omega\epsilon}{\sigma} \ll 1$

$$\Rightarrow 1 + j\frac{\omega\epsilon}{\sigma} = 1$$

$$\therefore \eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 90^\circ$$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$

$$\eta = \frac{|E|}{|H|} = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$

$$\bar{E} = \bar{H} \left[ \frac{\omega\mu}{\sigma} \angle 45^\circ \right]$$

**Note:-** Electric field in a good conductor leads the magnetic field by  $45^\circ$ .

$$\begin{aligned}\text{Also, } \eta &= \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ \\ &= \sqrt{\frac{\omega\mu}{\sigma}} (\cos 45^\circ + j \sin 45^\circ) \\ &= \sqrt{\frac{\omega\mu}{\sigma}} \left( \frac{1}{\sqrt{2}} + j \frac{1}{\sqrt{2}} \right)\end{aligned}$$

$$\text{Thus, } \eta = \sqrt{\frac{\omega\mu}{2\sigma}} + j \sqrt{\frac{\omega\mu}{2\sigma}}$$

Intrinsic impedance ( $\eta$ ) can be represented as -

$$\eta = R_s + jX_s$$

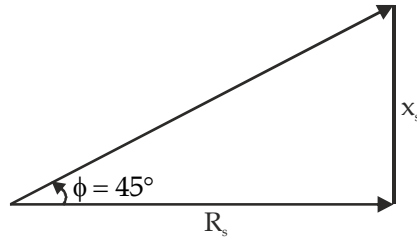
$$\text{where, } R_s = \sqrt{\frac{\epsilon\mu}{2\sigma}} \quad \& \quad X_s = \sqrt{\frac{\omega\mu}{2\sigma}}$$

( $R_s$  is known as Surface Resistance).

Analysis :

- (i) A lossy dielectric behaves like an inductive medium for EM waves.
- (ii) A good conductor behaves like an inductive medium with impedance angle of  $45^\circ$ .

(iii)



From the above figure,

$$\text{Power factor} = \cos \phi = \cos 45^\circ = \frac{1}{\sqrt{2}} = 0.707$$

Hence, a good conductor has a power factor of  $\frac{1}{\sqrt{2}}$  or 0.707.

Q.22

**Sol. Reflection and Transmission of Electromagnetic Waves**

Whenever a wave travelling in a medium finds a change in the impedance of the medium, some part of the wave is reflected back and the remaining part is transmitted in other medium.

**Snell's Law**

Snell's law gives relation between velocities in mediums before and after the refraction of wave at the boundary separating two mediums. According to this law,

$$\begin{aligned} \frac{\sin \theta_i}{\sin \theta_r} &= \frac{v_1}{v_2} \\ \theta_i &= \theta_r \\ v_1 &= \frac{1}{\sqrt{\mu_i \epsilon_i}} = \frac{1}{\sqrt{\mu_0 \mu_{r1} \epsilon_0 \epsilon_{r1}}} \\ &= \frac{1}{\sqrt{\mu_0 \epsilon_0}} \cdot \frac{1}{\sqrt{\mu_{r1} \epsilon_{r1}}} = \frac{C}{\sqrt{\mu_{r1} \epsilon_{r1}}} \\ v_1 &= \frac{C}{\sqrt{\mu_{r1} \epsilon_{r1}}} \end{aligned}$$

where,  $C = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/sec.} = \text{Velocity of EM wave in free space.}$

Similarly,  $v_2 = \frac{C}{\sqrt{\mu_{r2} \epsilon_{r2}}}$

So, according to Snell's Law, we have

$$\frac{\sin \theta_i}{\sin \theta_r} = \frac{v_1}{v_2} = \frac{\sqrt{\mu_{r2} \epsilon_{r2}}}{\sqrt{\mu_{r1} \epsilon_{r1}}}$$

since, for dielectric mediums,  $\mu_{r1} = \mu_{r2} = 1$

$$\frac{\sin \theta_i}{\sin \theta_r} = \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}}$$

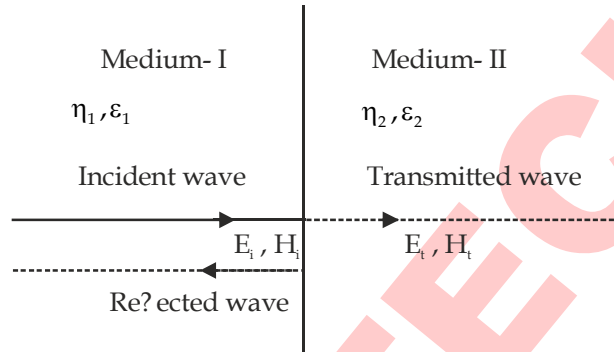
We know that

$$\eta_1 = \frac{\eta_0}{\sqrt{\epsilon_{r1}}} \text{ \& } \eta_2 = \frac{\eta_0}{\sqrt{\epsilon_{r2}}}$$

$$\frac{\eta_1}{\eta_2} = \frac{\eta_0 / \sqrt{\epsilon_{r1}}}{\eta_0 / \sqrt{\epsilon_{r2}}} = \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}}$$

$$\frac{\sin \theta_i}{\sin \theta_t} = \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} = \frac{\eta_1}{\eta_2}$$

### Reflection at the boundary of two dielectrics with normal incidence



At the boundary of two mediums,

$$E_i + E_r = E_t$$

At the boundary separating the two dielectrics, the tangential component of the electric field is same in both the mediums.

i.e.

$$E_{\tan 1} = E_{\tan 2}$$

At the boundary of two mediums,

$$E_i + E_r = E_t$$

Similarly,

$$H_{\tan 1} = H_{\tan 2}$$

The relationship between incident, transmitted & Reflected fields in medium 1 & medium 2 can be given by -

$$E_i + E_r = E_t \quad \dots(i)$$

$$H_i + H_r = H_t \quad \dots(ii)$$

for incident wave,

$$\frac{E_i}{H_i} = \eta_1 \quad \dots(iii)$$

for reflected wave,

$$\frac{E_r}{H_r} = \eta_1 \quad \dots(iv)$$

for transmitted wave,

$$\frac{E_t}{H_t} = \eta_2 \quad \dots(v)$$

Putting the expression of  $H_i$ ,  $H_r$  &  $H_t$  from equation (iii) (iv) & (v) in equation (ii), we get

$$\frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = \frac{E_t}{\eta_2}$$

$$E_i - E_r = \frac{\eta_1}{\eta_2} E_t \quad \dots(vi)$$

Subtracting equation (vi) from equation (i)

$$2E_r = \left(1 - \frac{\eta_1}{\eta_2}\right)E_t$$

$$E_r = \frac{1}{2} \left[ \frac{\eta_2 - \eta_1}{\eta_2} \right] E_t$$

$$E_r = \frac{\eta_2 - \eta_1}{2\eta_2} \times \frac{2\eta_2}{\eta_1 + \eta_2} \cdot E_t$$

Therefore, Reflection co-efficient for electric component of EM wave is given by

$$\alpha_E = \frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}$$

From equation (i) & (vi), we have

$$2E_i = \left[1 + \frac{\eta_1}{\eta_2}\right] E_t$$

$$\therefore \frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1 + \eta_2}$$

Therefore, Transmission co-efficient for electric component of EM wave is given by

$$\beta_E = \frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1 + \eta_2}$$

Similarly,

Reflection co-efficient of the magnetic component of EM wave is,

$$\alpha_H = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$$

&

Transmission co-efficient of the magnetic component of EM wave is,

$$\beta_H = \frac{2\eta_1}{\eta_1 + \eta_2}$$

Relationship between  $\alpha_E, \alpha_H, \beta_E$  &  $\beta_H$

$$\alpha_H = -\alpha_E$$

$$\beta_H = 1 + \alpha_H$$

$$\beta_E = 1 + \alpha_E$$

Expression for Reflected and transmission power :

$$E_r = \alpha_E E_i$$

Normalized power  $= V^2 = E^2$

$$P_r = \alpha_E^2 P_i$$

$\therefore$  Transmitted power  $P_t = P_i - P_r$

or,

$$P_t = P_i - \alpha_E^2 P_i$$

$$P_t = (1 - \alpha_E^2) P_i$$