



ZONE TECH
Best Institute For Assistant & Junior Engineer

RPSC-AEn Mains

Work Book Solution

for

TOS



Visit Our Website
www.zonotech.in



DOWNLOAD NOW

<https://zonotech.in/downloads>

PART - A

2 Marks Questions

Q.1

Sol. The degree of freedom (D.O.F.) of a structure is the number of independent displacements or rotations required to define its configuration or movement.

In a 2D structure, each joint may have up to three degrees of freedom (two translations and one rotation).

A structure is kinematically determinate when all degrees of freedom are restrained; otherwise, it is kinematically indeterminate.

Q.2

Sol. For Plane Trusses:

1. Static Indeterminacy (D_s):

$$D_s = m + r - 2j$$

where,

- m = number of members
- r = number of support reactions
- j = number of joints

2. Kinematic Indeterminacy (D_k):

$$D_k = 2j - r$$

where,

- j = number of joints
- r = number of support reactions

Q.3

Sol. For a Plane Frame:

1. Static Indeterminacy (D_s):

$$D_s = 3m + r - 3j$$

where,

- m = number of members
- r = number of support reactions
- j = number of joints

2. Kinematic Indeterminacy (D_k):

$$D_k = 3j - r$$

Each joint in a plane frame has three degree of freedom (Two translation + one rotation)

Q.4

Sol. A rigid joint is a connection in a structure where the connected members do not undergo any relative rotation with respect to each other.

In a rigid joint, the angle between connected members remains constant even under loading, and both moment and shear forces are transferred between members.

Example: Beam-column connection in a rigid frame.

Q.5

Sol. Lack of fit is the condition that occurs in a structure when its members are not fabricated to the exact required length or dimensions.

Because of this small error in fabrication, initial stresses or strains are developed in the members even before any external load is applied.

Example: A truss member slightly longer or shorter than designed causes internal stresses due to lack of fit.

Q.6.

Sol. The following assumptions are made in the analysis of a plane truss:

1. All members are connected by frictionless pins (perfect joints).
2. Loads and reactions act only at the joints.
3. All members are straight and have uniform cross-section.

4. The self-weight of members is negligible compared to applied loads.
5. Each member carries only axial force (tension or compression).

Q.7.

Sol. The Mohr-Williot diagram is used to determine the deflection of joints in a truss due to applied loads, temperature changes, or lack of fit.

It helps in finding both the magnitude and direction of joint displacements graphically.

Application:

- Used in truss analysis to find joint deflections and member elongations or contractions accurately.

Q.8.

Sol. The ordinate of an influence line diagram (ILD) represents the magnitude of a function (reaction, shear force, or bending moment) at a specific point on a structure due to a unit moving load placed at different positions on the span.

- For support reaction → ordinate = value of reaction at that support due to unit load.
- For shear force → ordinate = shear at that section due to unit load.
- For bending moment → ordinate = bending moment at that section due to unit load.

Thus, ordinates show how these quantities vary across the span under a unit moving load.

Q.9.

Sol.

- To determine the maximum value of reaction, shear force, or bending moment at any section of a structure due to moving loads.
- To find the position of moving load which produces the maximum effect (reaction, shear, or moment) in a structure.

Q.10

Sol. The Distribution Factor (D.F.) for a member at a joint is given by:

$$\text{D.F.} = \frac{K}{\sum K}$$

where,

- $K = \frac{I}{L}$ = stiffness of the member,
- $\sum K$ = sum of stiffnesses of all members meeting at the joint.

Q.11

Sol. The relative stiffness (K) of a member depends on the end conditions.

- For both ends fixed: $K = \frac{4EI}{L}$
- For one end fixed and other pinned: $K = \frac{3EI}{L}$

Hence, the ratio of relative stiffness is:

$$K_{\text{fixed-pinned}} : K_{\text{fixed-fixed}} = 3 : 4$$

Therefore, ratio of relative stiffness = 3:4.

Q.12

Sol. When a fixed beam is subjected to a moment (M) applied at its centre, the fixed end moments at both supports are:

$$M_A = +\frac{M}{2} \text{ and } M_B = -\frac{M}{2}$$

Explanation:

- The applied moment at mid-span is shared equally by both fixed supports.
- The directions of the moments are opposite, maintaining equilibrium of the beam.

Q.13

Sol. The Portal Method is an approximate method used for analyzing multistory frames subjected to lateral loads such as wind or earthquake.

Its main assumptions are:

1. The points of contraflexure in beams and columns occur at their midpoints.
2. The shear at each storey is divided equally among all the columns of that storey.
3. The interior columns carry twice the shear of exterior columns.
4. Axial deformations of beams and columns are negligible.

Q.14

Sol. The Cantilever Method is an approximate method used for analyzing tall multistory frames subjected to lateral loads such as wind or earthquake.

Its main assumptions are:

1. The points of contraflexure in beams and columns occur at their midpoints.
2. The axial force in each column is proportional to its distance from the centroidal axis of the frame.
3. The shear in each storey is resisted by these axial forces in the columns.
4. The deformation of the frame is similar to that of a vertical cantilever fixed at the base.

Q.15

Sol. For a three-hinged arch,

$$H = \frac{M_c}{h}$$

Where,

H = horizontal thrust,

M_c = bending moment at crown for the corresponding simply supported beam,

h = rise of arch.

- Here the arch is semicircular, so span $L = 2R$ and rise $h = R$.
- For a simply supported beam of span L with UDL w per unit length:

$$M_c = \frac{wL^2}{8}$$

So,

$$M_c = \frac{w(2R)^2}{8} = \frac{w \cdot 4R^2}{8} = \frac{wR^2}{2}$$

Hence,

$$H = \frac{M_c}{h} = \frac{\frac{wR^2}{2}}{R} = \boxed{\frac{wR}{2}}$$

Horizontal thrust $H = \frac{wR}{2}$.

PART - B

5 Marks Questions

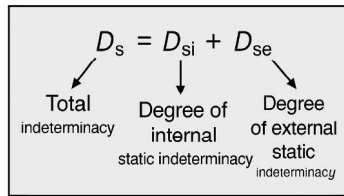
Q.16

Sol. Static indeterminacy (D_s):

It is defined as the number of equilibrium conditions, also referred as compatibility conditions, required in excess of static equilibrium equations to fully analyze a statically indeterminate structure.

$$D_s = \left[\begin{array}{c} \text{No. of unknown forces} \\ \text{in member or at support reactions} \end{array} \right] - \left[\begin{array}{c} \text{Equations of equilibrium} \\ \text{available} \end{array} \right]$$





For a 2D-framed structure :

Frames are rigid jointed structures. Restraining of deformation at support or any joint gives rise to three internal reactions. (R_x, R_y, M_z) in each member.

At every joint, three equilibrium equations are available.

$$\sum F_x = 0, \sum F_y = 0, \sum M_z = 0$$

Let

m = No. of members in a 2D framed structure

R = Total no of reaction

R_e = Total no of external reaction from support

j = Total no. of joints in the structure

Then,

$$R = 3m + R_e$$

$$D_s = 3m + R_e - 3j$$

Ex. A beam fixed at both ends



FBD



Here

$$R_e = 3 + 3 = 6$$

$$m = 1$$

$$j = 2$$

$$\therefore D_s = 3m + R_e - 3j = 3 \times 1 + 6 - 3 \times 2 = 3$$

Kinematic indeterminacy (D_K):

It is defined as the total no. of unknown joint displacements at all joints or the total no. of available degree of freedom at all joints.

$$D_K = \left[\begin{array}{c} \text{Total possible degree} \\ \text{of freedom} \end{array} \right] - \left[\begin{array}{c} \text{No. of available support reactions which} \\ \text{generate as a result of restrained joint displacements} \end{array} \right]$$

For a 2D-framed structure:

Each joint has three possible degrees of freedom, $(\Delta_x, \Delta_y, \theta_z)$.

For a frame having j no. of joints

Total possible degrees of freedom = $3j$

But some displacements will be constrained by the supports provided

Let R_e be the total no. of external support reactions

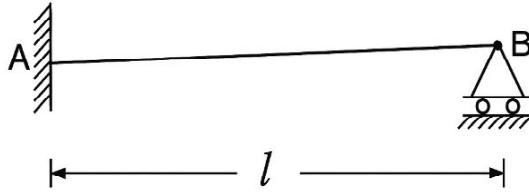
$$\therefore D_K = 3j - R_e$$

If the members are considered inextensible then there will be no axial deformations and

$$D_K = 3j - R_e - m$$

m = no. of members

Ex:



$$j = 2$$

$$R_e = 3 + 1 = 4$$

$$m = 1$$

$$D_K = 3j - R_e = 3 \times 2 - 4 = 2 \quad (\Delta_x, \theta_z, \text{ at B})$$

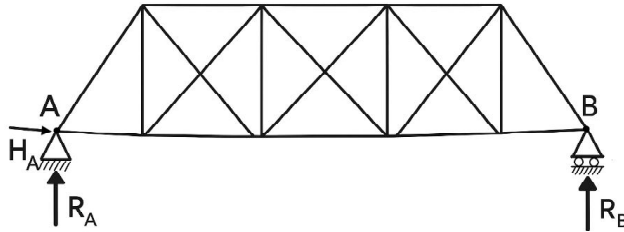
Q.17

Sol. {Total degree of static indeterminacy} = {Degree of external indeterminacy} + {Degree of internal indeterminacy}

$$D_{s, \text{tot}} = D_{s, e} + D_{s, i}$$

$$D_{s, e} = (\text{Total no. of support reactions}) - (\text{Total no. of equations of static equilibrium})$$

$$= 3 - 3 = 0$$



($\therefore$ Support reactions = H_A, R_A, R_B ; equilibrium equations = $\sum F_H = 0; \sum F_V = 0; \sum M = 0$)

$$D_{s, \text{tot}} = m + r - 2j$$

where, m = No. of members of the truss = 20

r = No. of support reactions = 3

j = No. of joints of the truss = 10

$$\Rightarrow D_{s, \text{tot}} = 20 + 3 - 2 \times 10 = 3$$

$$\Rightarrow D_{s, i} = D_{s, \text{tot}} - D_{s, e} = 3 - 0 = 3$$

$$\Rightarrow D_{s, i} = 3$$

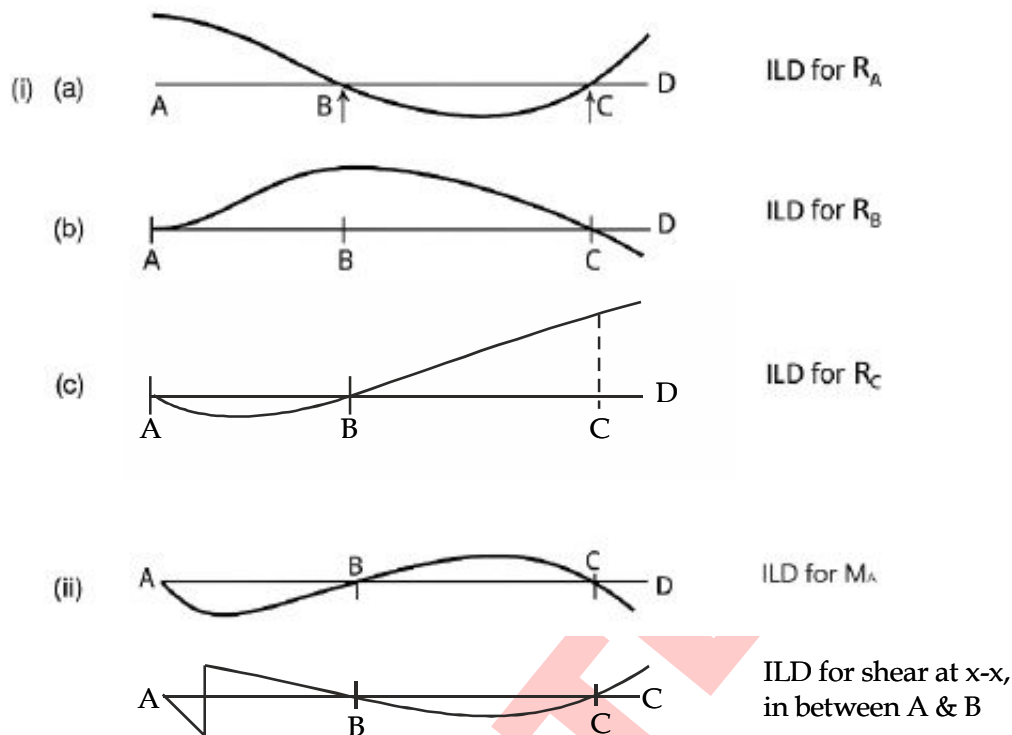
$$D_{s, e} = 0$$

$$D_{s, \text{tot}} = 3$$

Q.18

Sol. Muller-Breslau Principle

Influence line for any stress function may be obtained by removing the restraint offered by that function and introducing a directly related generalized unit displacement at the location and in the direction of the function.



Q.19
Sol.

Force Method		Displacement Method	
1	Also called Compatibility method or Method of consistent deformation or Flexibility method	1	Also called Stiffness method or Equilibrium method
2	Unknowns in this case are Forces (Reaction, BM, SF)	2	Unknowns in this case are Displacement (Δ)
3	Force-displacement equation are written and solution for unknown Forces is obtained from Compatibility equations.	3	Force-displacement equations are written and solution for unknown Displacement is obtained from Equilibrium equations.
4	Once unknown forces are known. The reactive forces like BM, SF etc. are found using equilibrium equation.	4	Once unknown displacements are known, internal forces are found using compatibility and load-displacement equation.
5	Force method of analysis of indeterminate structure is suitable when degree of static indeterminacy is less than degree of kinematic indeterminacy.	5	Displacement method is suitable when degree of kinematic indeterminacy is less than degree of static indeterminacy.

Q.20

Sol. As per Betti's theorem, virtual work done by P-force system in going through the deformation caused by Q-force system is equal to the virtual work done by Q-force system in going through the

deformation caused by P-force system.

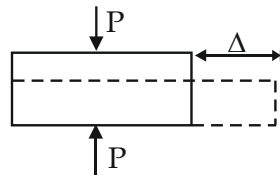


Fig A

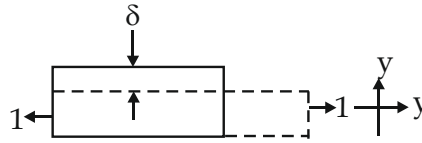


Fig B

⇒

$$P\delta = 1\Delta$$

⇒

$$\Delta = P\delta$$

We have to calculate Δ and for this we need to calculate δ .

$$\delta = \epsilon_y \cdot d$$

$$\delta = \left[\frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} \right] d$$

$$\delta = 0 - \frac{\mu\sigma_x}{E} d$$

$$\delta = \frac{-\mu d}{E} \left[\frac{1}{A} \right], \text{ A = cross sectional area of bar}$$

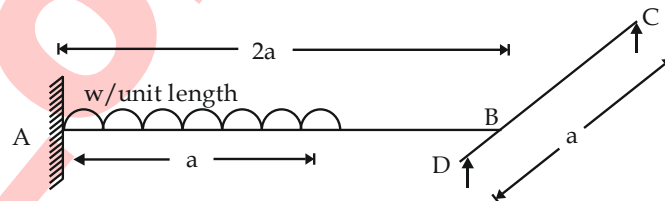
$$\delta = \frac{-\mu d}{E} \left[\frac{1}{\pi d^2 / 4} \right]$$

⇒

$$\Delta = P\delta = \frac{P\mu d \times 4}{\pi d^2 \times E}$$

$$\Delta = \frac{4\mu P}{\pi d E} = \frac{4 \times 0.3 \times 10 \times 10^3 \text{ N}}{3.14 \times 10 \text{ mm} \times 2 \times 10^5 \text{ N/mm}^2} = 1.911 \times 10^{-3} \text{ mm}$$

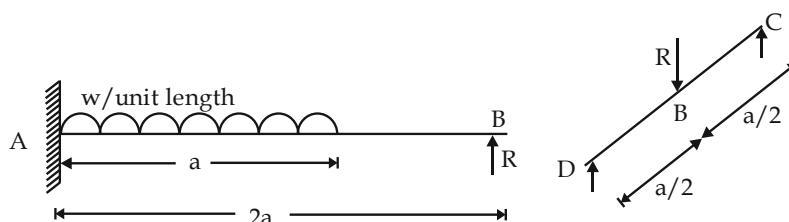
Q.21
Sol.



As the cantilever beam is supported freely at its free end, there will be one unknown in the analysis of cantilever portion.

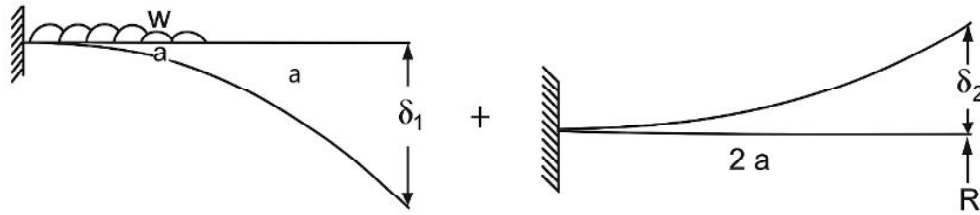
Let the reaction at common point D be 'R'.

Hence the FBD will be as shown below;



To determine R, we will use the compatibility equation. The compatibility equation is that deflection of end B in span AD will be same as deflection of point B in span DC.

Deflection at point B, for the cantilever, (Δ_B)



$$\Delta_B = \left[\frac{wa^4}{8EI} + \frac{wa^3}{6EI} \times a \right] - \frac{R(2a)^3}{3EI} (\downarrow)$$

$$\Delta_B = \frac{wa^4}{8EI} + \frac{wa^4}{6EI} - \frac{8Ra^3}{3EI}$$

$$\Rightarrow \Delta_B = \frac{7}{24} \frac{wa^4}{EI} - \frac{8Ra^3}{3EI}$$

Deflection at B in span DC = Δ_{BSSB}

$$\Delta_{BSSB} = \frac{Ra^3}{48EI}$$

Thus from compatibility condition

$$\Rightarrow \frac{7}{24} \frac{wa^4}{EI} - \frac{8Ra^3}{3EI} = \frac{Ra^3}{48EI}$$

$$\Rightarrow \frac{7}{24} \frac{wa^4}{EI} = \frac{Ra^3}{48EI} + \frac{8Ra^3}{3EI}$$

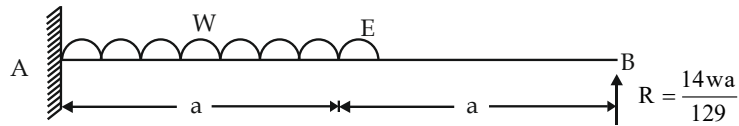
$$\frac{7}{24} \frac{wa^4}{EI} = \frac{Ra^3 + 128Ra^3}{48EI} = \frac{129Ra^3}{48EI}$$

$$\Rightarrow \frac{7}{24} \frac{wa^4}{EI} = \frac{129Ra^3}{48EI}$$

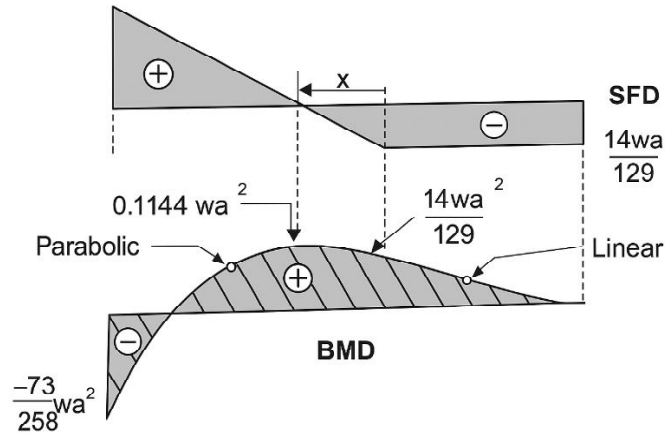
$$\Rightarrow \frac{7 \times 48}{24 \times 129} wa = R$$

$$\Rightarrow \boxed{R = \frac{14}{129} wa}$$

Here, F.B.D for cantilever beam,



Bending moment, diagram



$$SF_x = 0$$

$$\Rightarrow -\frac{14wa}{129} + wx = 0$$

$$\Rightarrow x = \frac{14a}{129}$$

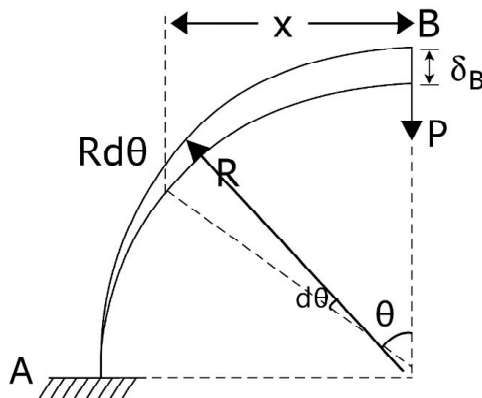
$$BM_{\max} = \frac{14wa}{129} \times (a + x) - \frac{wx^2}{2} = \frac{14wa}{129} \times \left(a + \frac{14a}{129}\right) - \frac{w}{2} \left(\frac{14a}{129}\right)^2$$

$$BM_{\max} = 0.1144 wa^2 \quad (\text{sagging})$$

$$BM_A = \frac{14wa}{129} \times 2a - \frac{wa^2}{2} = \frac{28wa^2}{129} - \frac{wa^2}{2}$$

$$BM_A = -\frac{73}{258} wa^2 \quad (\text{hogging})$$

Q.22
Sol.



By strain energy method, we know that deflection is given by

$$\delta_B = \int \frac{M}{EI} \cdot \frac{\partial M}{\partial P} \cdot dx \quad \dots(i)$$

Bending moment at any distance 'x' from point B is

$$M = P \cdot x \text{ (consider hogging as +ve)}$$

From figure, we can write

$$x = R \sin \theta$$

$$M = P \cdot R \sin \theta$$

$$\therefore \dots(ii)$$

Partially differentiating equation (ii), with respect to P, we get

$$\frac{\partial M}{\partial P} = R \sin \theta \quad \dots(iii)$$

we can also write $dx = R \cdot d\theta$

$$\dots(iv)$$

Putting values of M, $\frac{\partial M}{\partial P}$ and dx in equation (i), we get

$$\delta_B = \int_0^{\pi/2} \frac{P \cdot R \sin \theta \cdot R \sin \theta \cdot R d\theta}{EI}$$

$$\therefore \delta_B = \frac{PR^3}{EI} \int_0^{\pi/2} \sin^2 \theta d\theta$$

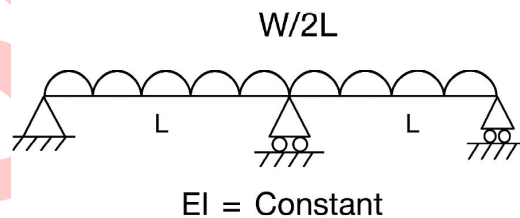
$$\delta_B = \frac{PR^3}{EI} \int_0^{\pi/2} \left(\frac{1 - \cos 2\theta}{2} \right) d\theta = \frac{PR^3}{2EI} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{\pi/2}$$

After integrating, we get

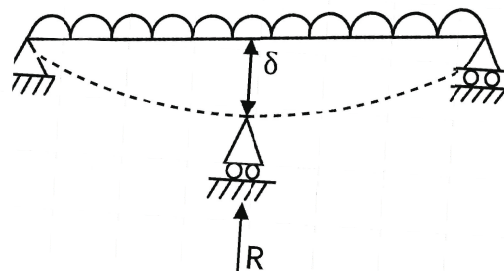
$$\delta_B = \frac{\pi PR^3}{4EI} \downarrow$$

Q.23

Sol. Given:



Total uniformly distributed load = W



Let the deflection at elastic prop be δ , then according to question,

$$\delta = \lambda.R$$

From compatibility condition:

$$\frac{5 \cdot \frac{W}{2L} \times (2L)^4}{384EI} - \lambda.R = \frac{R.(2L)^3}{48EI}$$

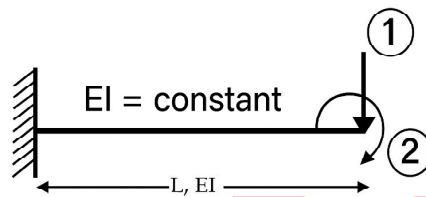
or,

$$\frac{5WL^3}{48EI} = R \left(\frac{L^3}{6EI} + \lambda \right)$$

$$R = \frac{\frac{5WL^3}{48EI}}{\left(\frac{L^3}{6EI} + \lambda \right)} = \frac{5WL^3}{8(L^3 + 6\lambda EI)}$$

Q.24

Sol.



As the no. of co-ordinates chosen are 2, the flexibility matrix will be a 2×2 matrix. To find out 1st column of flexibility matrix, apply unit load in the direction ① and find out displacements in all co-ordinate directions.

f_{ij} = Displacement in the direction i due to unit load applied in the direction j
 = Element of i th row and j th column of flexibility matrix

$$\Rightarrow f_{11} = \frac{1 \times L^3}{3EI}$$

$$f_{21} = \frac{1 \times L^2}{2EI}$$

$$f_{12} = \frac{1 \times L^2}{2EI}$$

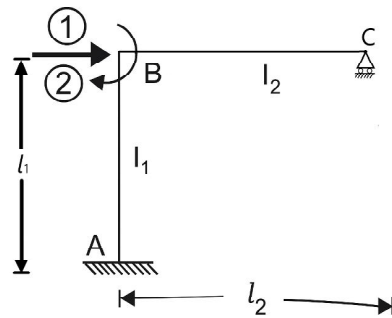
$$f_{22} = \frac{1 \times L}{EI}$$

Hence the flexibility matrix is

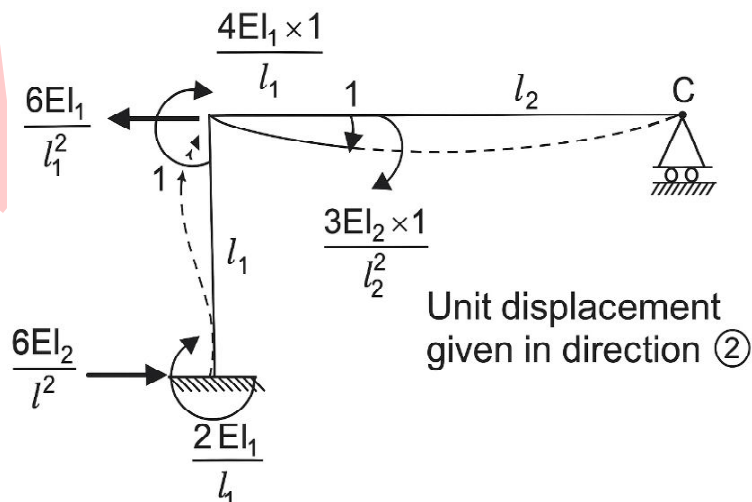
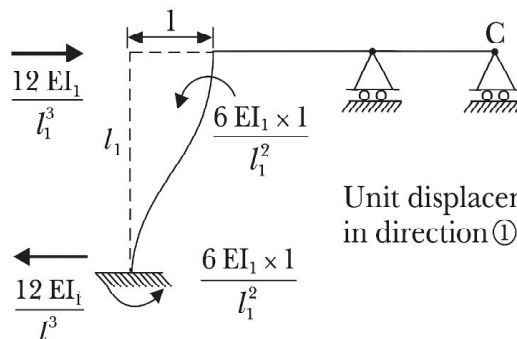
$$[f] = \begin{bmatrix} L^3 / 3EI & L^2 / 2EI \\ L^2 / 2EI & L / EI \end{bmatrix}$$

Q.25

Sol.



- ∴ As the no. of co-ordinate directions are two, the stiffness matrix will be a 2×2 matrix.
- To determine the 1st column of stiffness matrix we give unit displacement in the direction ① without giving displacement in any other co-ordinate direction and find out the forces developed in all co-ordinate directions. The elements obtained are K_{11} and K_{21}
- K_{11} = Force developed in the direction ① when unit displacement was given in direction ① without giving displacement in any other co-ordinate direction.
- K_{21} = Force developed in the direction ② when unit displacement was given in direction ① without giving displacement in any other co-ordinate direction.



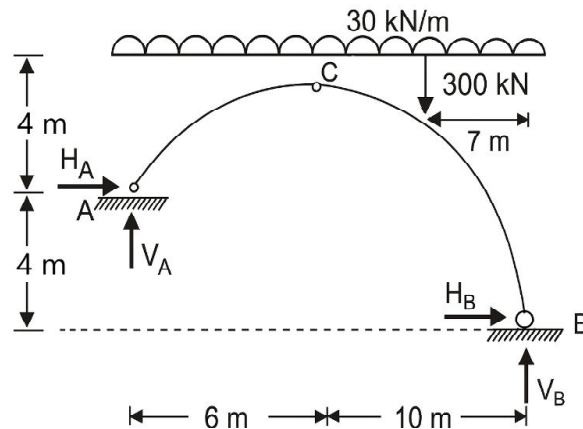
$$\Rightarrow K_{11} = \frac{12EI_1}{l_1^3}; K_{21} = \frac{-6EI_1}{l_1^2}$$

$$\Rightarrow K_{12} = \frac{-6EI_1}{l_1^2}; K_{22} = \frac{4EI_1}{l_1^2} + \frac{3EI_2}{l_2^2}$$

$$\Rightarrow \text{Stiffness matrix is } [K] = \begin{bmatrix} \frac{12EI_1}{l_1^3} & \frac{-6EI_1}{l_1^2} \\ \frac{-6EI_1}{l_1^2} & \frac{4EI_1}{l_1^2} + \frac{3EI_2}{l_2^2} \end{bmatrix}$$

Q.26

Sol.



V_A = Vertical reaction at A

V_B = Vertical reaction at B

H_A = Horizontal reaction at A

H_B = Horizontal reaction at B

For equilibrium

$$\sum F_x = 0; \sum F_y = 0$$

$$\sum F_x = 0$$

$$\therefore H_A + H_B = 0 \quad \dots(i)$$

$$\sum F_y = 0$$

$$\therefore V_A + V_B = 30 \times 16 + 300$$

$$\Rightarrow V_A + V_B = 780 \quad \dots(ii)$$

Moment about any point should be zero. Let us choose point B and equate moment about point B equal to zero.

$$\therefore H_A \times 4 + V_A \times 16 - 30 \times 16 \times 8 - 300 \times 7 = 0$$

$$\therefore 16V_A + 4H_A = 5940 \quad \dots(iii)$$

As point C is hinged. Moment at point C from both sides should be zero. Writing total moment from left side of C and equate it to zero we get

$$\therefore H_A \times 4 + 30 \times 6 \times 3 - V_A \times 6 = 0$$

$$\Rightarrow 6V_A - 4H_A = 540 \quad \dots(iv)$$

Solving equation (i), (ii), (iii) & (iv), we get

$$V_A = 294.54 \text{ kN}$$

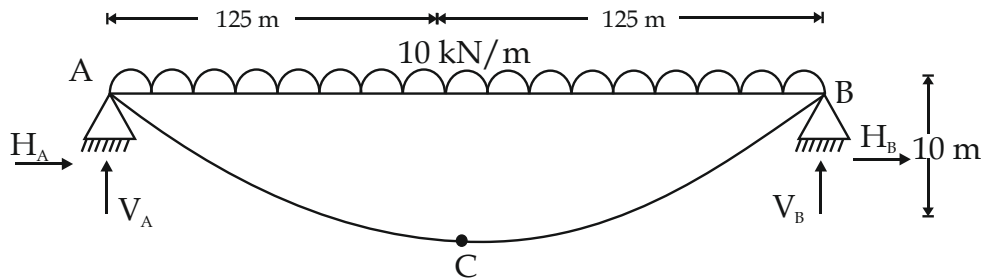
$$V_B = 485.46 \text{ kN}$$

$$H_A = 306.82 \text{ kN}$$

$$H_B = -306.82 \text{ kN}$$

Q.27

Sol. As it is mentioned clearly whether both support are at same level or not. We will assume that both supports are at same level.



Maximum sag will be at mid span, which is equal to $\frac{L}{25} = \frac{250}{25} = 10\text{m}$

V_A = Vertical reaction at support A

V_B = Vertical reaction at support B

H_A = Horizontal reaction at support A

H_B = Horizontal reaction at support B

Now,

$\Rightarrow$

$$\sum F_x = 0$$

$$H_A + H_B = 0$$

$\Rightarrow$

$$\sum F_y = 0$$

$$V_A + V_B = 10 \times 250$$

$$V_A + V_B = 2500$$

Equating moment about B to zero, we get

$$V_A \times 250 - 10 \times 250 \times 125 = 0$$

$\Rightarrow$

$$V_A = 1250 \text{ kN}$$

Putting value of V_A in equation (ii), we get

$$1250 + V_B = 2500$$

$\Rightarrow$

$$V_B = 1250 \text{ kN}$$

As cable is not able to transfer moment, bending moment at every point in a cable must be zero, Therefore, bending moment at C is also zero.

$\Rightarrow$

$$(B.M)_c = 0$$

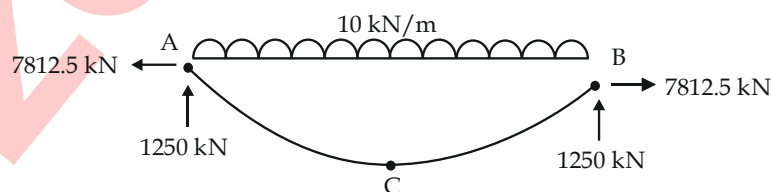
$\therefore$

$$H_A \times 10 + V_A \times 125 - 10 \times 125 \times \frac{125}{2} = 0$$

$\therefore$

$$H_A = \frac{10 \times 125 \times \frac{125}{2} - 1250 \times 125}{10}$$

$$H_A = -7812.5 \text{ kN}$$



Maximum tension in cable is at A or B, therefore

$$T_{\max} = \sqrt{H_A^2 + V_A^2}$$

$$T_{\max} = \sqrt{(7812.5)^2 + (1250)^2}$$

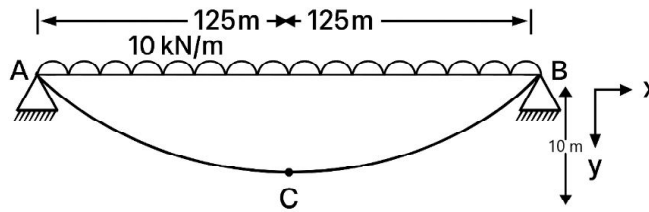
$$T_{\max} = 7911.87 \text{ kN}$$

Let the equation of cable profile be

$$y = ax^2 + bx + c$$

...(i)

with A as origin and axes as shown below



Boundary conditions are

(i) at $x = 0$; $y = 0$

$$\Rightarrow y = ax^2 + bx + c$$

$$0 = a.(0)^2 + b.(0) + c$$

...(ii)

$$\therefore c = 0$$

(ii) at $x = 250$; $y = 0$

$$y = ax^2 + bx$$

$$\Rightarrow 0 = a.(250)^2 + b.(250) [\because c = 0]$$

...(iii)

(iii) at $x = 125$; $y = 10$

$$y = ax^2 + bx$$

$$\Rightarrow 10 = a.(125)^2 + b.(125)$$

...(iv)

Solving equation (iii) and (iv), we get

$$a = -6.4 \times 10^{-4}$$

$$b = 0.16$$

$\therefore$ Cable profile is

$$y = (-6.4 \times 10^{-4})x^2 + 0.16x$$

...(v)

To find sag at 50m from one end, we have to put $x = 50$ in equation (v)

$$\therefore y = (-6.4 \times 10^{-4}).50^2 + 0.16 \times 50 = 6.4 \text{ m}$$

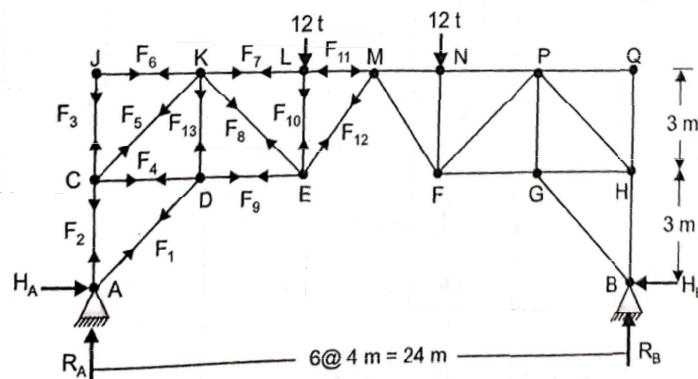
$\therefore$ Sag at 50m from one end will be 6.4m.

PART - C

20 Marks Questions

Q.28

Sol.



Due to symmetry in loading

$$R_A = R_B = 12 \text{ t}$$

Moment of forces about hinge M will be zero for forces on either side of M.

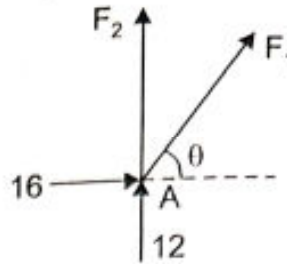
$$\Rightarrow R_A \times 12 - H_A \times 6 - 12 \times 4 = 0$$

$$12 \times 12 - H_A \times 6 - 12 \times 4 = 0$$

$$H_A = \frac{12 \times 12 - 12 \times 4}{6} = \frac{96}{6} = 16t$$

$$H_A = H_B \quad [\text{From the equilibrium of horizontal forces}]$$

Analysis of joint A



$$\tan \theta = 3/4$$

$$\sin \theta = 3/5$$

$$\cos \theta = 4/5$$

$$F_1 \cos \theta + 16 = 0$$

$$F_1 = \frac{-16 \times 5}{4} = -20t \Rightarrow \boxed{F_1 = -20t}$$

$$F_1 \sin \theta + F_2 + 12 = 0$$

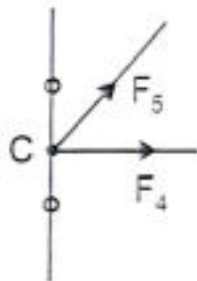
$$-20 \times \frac{3}{5} + F_2 + 12 = 0 \Rightarrow \boxed{F_2 = 0}$$

Member JK and JC are zero force members because two non-concurrent members meet at joint 'J' and no external force is acting at J

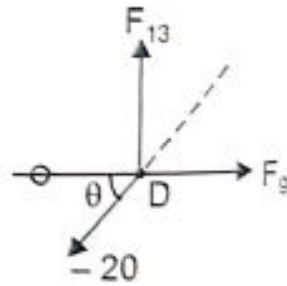
$$\Rightarrow \boxed{F_6 = 0}$$

$$\boxed{F_3 = 0}$$

Once $F_3 = 0$ and $F_2 = 0$, F_4 and F_5 will also be zero force member because F_4 and F_5 are two non-concurrent members meeting at joint C and no external force act at C.



At joint D



$$F_{13} + 20 \sin \theta = 0$$

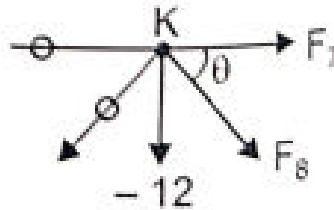
 $\Rightarrow$

$$F_{13} = -20 \times \frac{3}{5} = -12t$$

$$F_9 + 20 \cos \theta = 0$$

$$F_9 = -20 \times \frac{4}{5} = -16t$$

At joint K

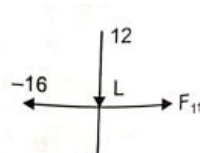


$$F_8 \sin \theta - 12 = 0$$

$$F_8 = 12 \times \frac{5}{3} = 20t$$

$$F_8 \cos \theta + F_7 = 0 \Rightarrow F_7 = -20 \times \frac{4}{5} = -16t$$

At joint L



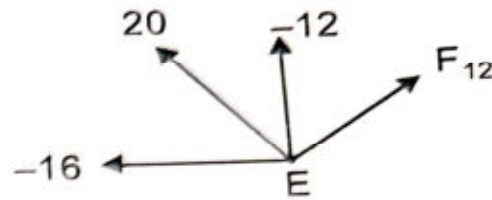
$$12 + F_{10} = 0 \Rightarrow F_{10} = -12t$$

 $\Rightarrow$

$$F_{10} = -12t$$

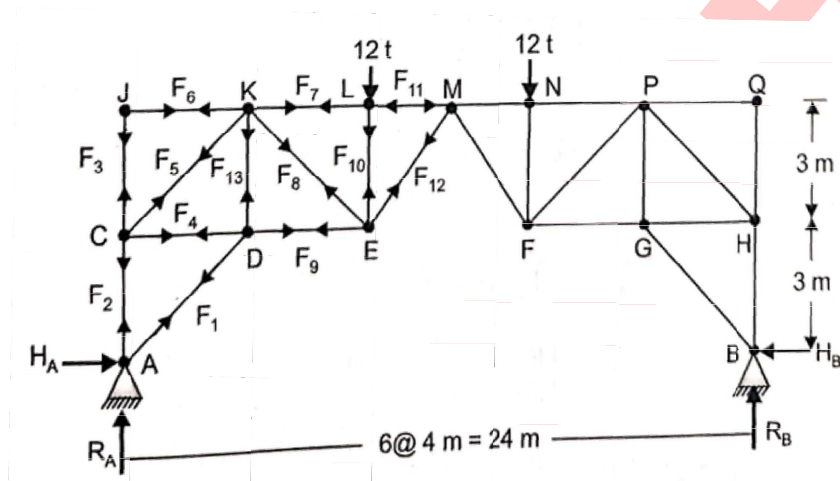
$$F_{11} = -16t$$

At joint E



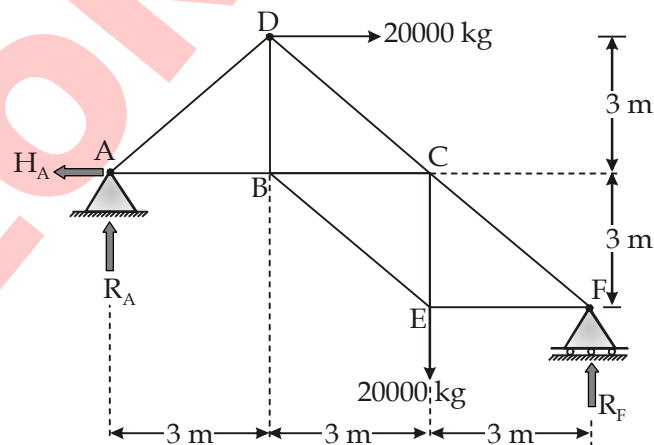
Resultant of $-12t$ & $-16t$ force will balance $20t$ load hence force in $F_{12} = 0$:

As there is symmetry about M, forces in corresponding members will be same. Hence the forces in various members are as shown below.



Q.29

Sol.



Degree of static indeterminacy of given truss (D_s) :

$$D_s = m + r_e - 2j$$

$$m = 9$$

$$r_e = 3$$

$$j = 6$$

$$D_s = 9 + 3 - 2 \times 6 = 0$$

$$\begin{aligned}\text{Degree of external indeterminacy} &= r_e - 3 \\ &= 3 - 3 \\ &= 0\end{aligned}$$

Hence, the given truss is statically determinate.

Reaction Calculation :

$$\text{From } \Sigma F_y = 0$$

$$R_A + R_F = 20000 \text{ kg}$$

$$\text{From } \Sigma F_x = 0$$

$$H_A = 20000 \text{ kg}$$

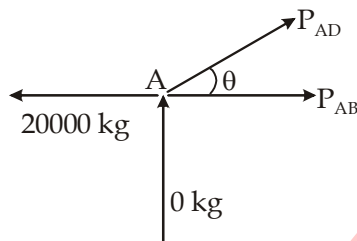
$$\text{From } \Sigma M_A = 0 \quad (\curvearrowright)$$

$$20000 \times 3 + 20000 \times 6 - R_F \times 9 = 0$$

$$R_F = 20000 \text{ kg}$$

$$R_A = 0 \text{ kg}$$

Joint A :



$$\begin{aligned}\tan \theta &= 1 \\ \sin \theta &= \frac{1}{\sqrt{2}} \\ \cos \theta &= \frac{1}{\sqrt{2}}\end{aligned}$$

$$\begin{aligned}& \left[\begin{array}{l} +\text{ve sign} = \text{Tension} \\ -\text{ve sign} = \text{Compression} \end{array} \right]\end{aligned}$$

$$\Sigma F_y = 0$$

$$\Sigma F_x = 0$$

$$P_{AD} \sin \theta + 0 = 0$$

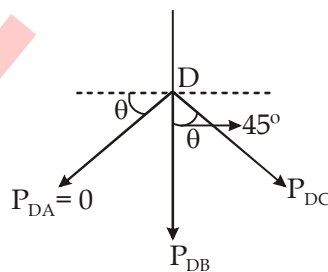
$$P_{AB} + P_{AD} \cos \theta - 20000 = 0$$

$$P_{AD} = 0$$

$$P_{AB} + 0 - 20000 = 0$$

$$P_{AB} = 20000 \text{ kg (Tension)}$$

Joint D :



$$\begin{aligned}\tan \theta &= 1 \\ \cos \theta &= \frac{1}{\sqrt{2}} \\ \sin \theta &= \frac{1}{\sqrt{2}}\end{aligned}$$

$$\Sigma F_y = 0$$

$$\Sigma F_x = 0$$

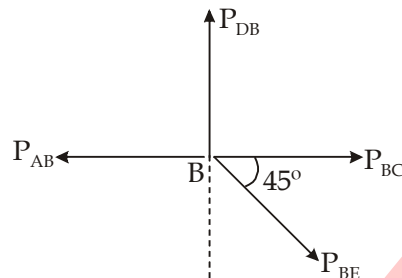
$$P_{AD} \sin 45^\circ + P_{DC} \cos 45^\circ + P_{DB} = 0$$

$$P_{DC} \sin 45^\circ = P_{DA} \cos 45^\circ$$

$$0 + \frac{P_{DC}}{\sqrt{2}} + P_{DB} = 0$$

$$P_{DB} = 0$$

Joint B :



$$\tan 45^\circ = 1$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\Sigma F_y = 0$$

$$\Sigma F_x = 0$$

$$P_{DB} - P_{BE} \sin 45^\circ = 0$$

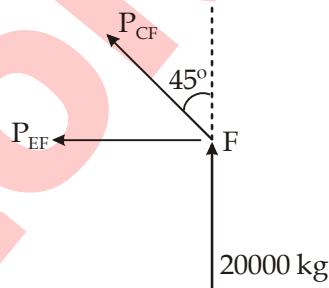
$$P_{BC} + P_{BE} \cos 45^\circ - P_{AB} = 0$$

$$P_{BE} = 0$$

$$P_{BC} = P_{AE}$$

$$P_{BC} = 20000 \text{ kg (Tension)}$$

Joint F :



$$\tan 45^\circ = 1$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\Sigma F_y = 0$$

$$\Sigma F_x = 0$$

$$P_{CF} \sin 45^\circ + 20000 = 0$$

$$P_{CF} \cos 45^\circ + P_{EF} = 0$$

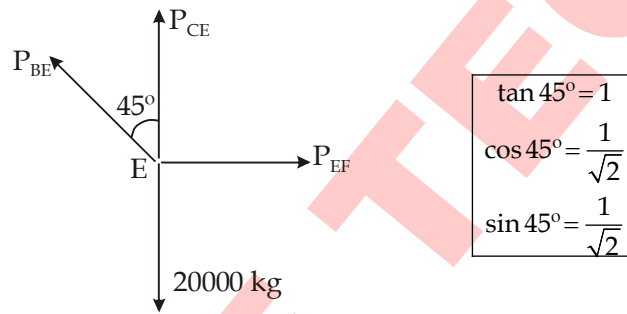
$$P_{CF} = -\frac{20000}{\sin 45^\circ}$$

$$-\frac{20000\sqrt{2}}{\sqrt{2}} + P_{EF} = 0$$

$$P_{CF} = -20000\sqrt{2} \text{ kg} \\ \text{(Compression)}$$

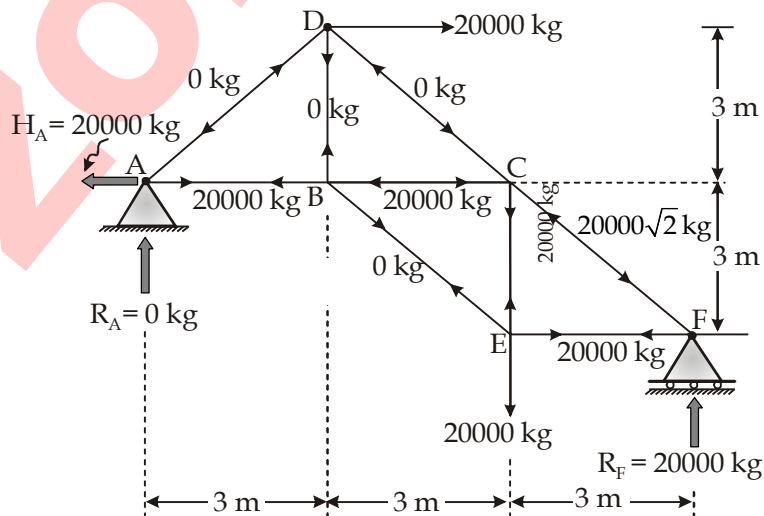
$$P_{EF} = 20000 \text{ kg (Tension)}$$

Joint E :



$$\begin{aligned} \Sigma F_y &= 0 \\ P_{CE} + P_{BE} \cos 45^\circ - 20000 &= 0 \\ P_{CE} + 0 - 20000 &= 0 \\ P_{CE} &= 20000 \text{ kg (Tension)} \end{aligned}$$

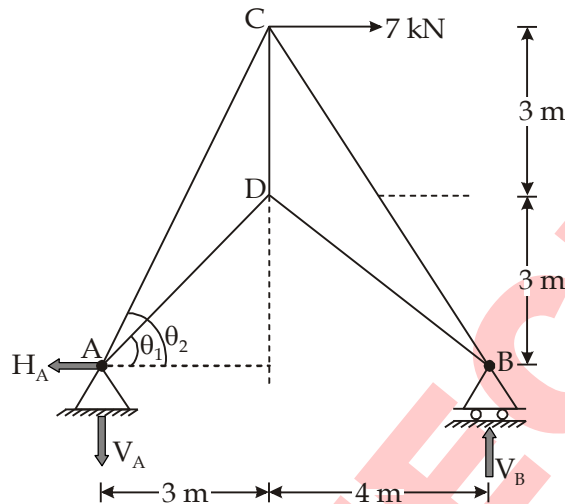
Hence, the forces in the numbers of truss are as shown below :



[All values in kg]

Q.30

Sol. Let us take $100 \text{ kgf} = 1 \text{ kN}$ and analysing the **truss by method of joints**.
 Let the vertical reactions at A and B be V_A and V_B respectively.
 Let the horizontal reaction at A be H_A .



$$(i) \sum F_y = 0$$

$$\Rightarrow$$

$$-V_A + V_B = 0$$

$$V_A = V_B$$

$$(ii) \sum M_A = 0$$

$$\Rightarrow$$

$$V_B \times 7 - 7 \times 6 = 0$$

$$\Rightarrow$$

$$V_B = 6 \text{ kN}$$

$$\therefore$$

$$V_A = 6 \text{ kN}$$

$$(iii) \sum F_x = 0$$

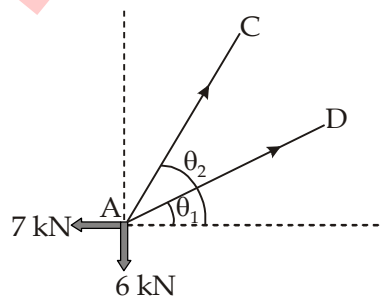
$$\Rightarrow$$

$$-H_A + 7 = 0$$

$$\Rightarrow$$

$$H_A = 7 \text{ kN}$$

Considering joint A and assuming tensile force in both members AD and AC
 Assuming tensile force as positive and compressive force as negative.



$$\tan \theta_1 = \frac{3}{3} = 1 \text{ and } \tan \theta_2 = \frac{6}{3} = 2$$

$$\sin \theta_1 = \frac{1}{\sqrt{2}}$$

$$\sin \theta_2 = \frac{2}{\sqrt{5}}$$

$$\cos \theta_1 = \frac{1}{\sqrt{2}}$$

$$\cos \theta_2 = \frac{1}{\sqrt{5}}$$

$$\Rightarrow$$

$$\sum F_x = 0$$

$$\Rightarrow F_{AD} \cos\theta_1 + F_{AC} \cos\theta_2 - 7 = 0$$

$$\Rightarrow F_{AD} \times \frac{1}{\sqrt{2}} + \frac{F_{AC}}{\sqrt{5}} - 7 = 0 \quad \dots(i)$$

and $\Sigma F_y = 0$

$$\Rightarrow F_{AD} \sin\theta_1 + F_{AC} \sin\theta_2 - 6 = 0$$

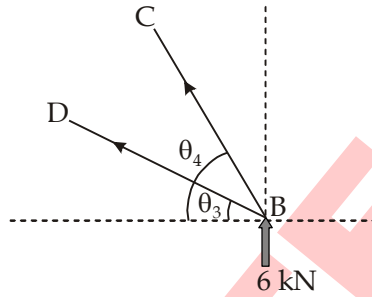
$$\Rightarrow \frac{F_{AD}}{\sqrt{2}} + \frac{2F_{AC}}{\sqrt{5}} = 6 \quad \dots(ii)$$

Solving equation (i) and (ii), we get

$$F_{AD} = 11.314 \text{ kN (Tensile)}$$

$$F_{AC} = -2.236 \text{ kN (Compressive)}$$

Considering joint B and assuming tensile force in members BD and BC.



$$\tan\theta_3 = \frac{3}{4} \text{ and } \tan\theta_4 = \frac{6}{4} = \frac{3}{2}$$

$$\sin\theta_3 = \frac{3}{5} \quad \sin\theta_4 = \frac{3}{\sqrt{13}}$$

$$\cos\theta_3 = \frac{4}{5} \quad \cos\theta_4 = \frac{2}{\sqrt{13}}$$

$$\Rightarrow \Sigma F_x = 0$$

$$\Rightarrow -F_{BD} \cos\theta_3 - F_{BC} \cos\theta_4 = 0$$

$$\Rightarrow \frac{4}{5} F_{BD} + \frac{2}{\sqrt{13}} F_{BC} = 0 \quad \dots(iii)$$

and $\Sigma F_y = 0$

$$\Rightarrow F_{BD} \sin\theta_3 + F_{BC} \sin\theta_4 + 6 = 0$$

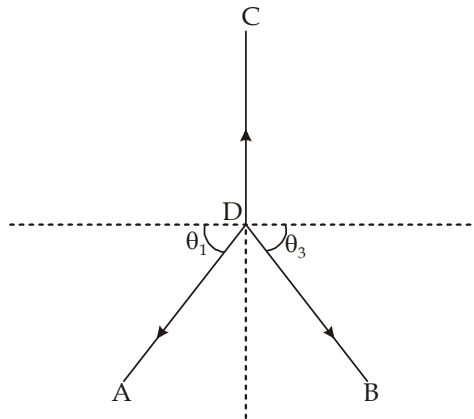
$$\Rightarrow \frac{3}{5} F_{BD} + \frac{3}{\sqrt{13}} F_{BC} = -6 \quad \dots(iv)$$

Solving equation (iii) and (iv), we get

$$F_{BD} = 10 \text{ kN (Tensile)}$$

$$F_{BC} = -14.422 \text{ kN (Compressive)}$$

Considering joint D, and assuming tensile force in CD



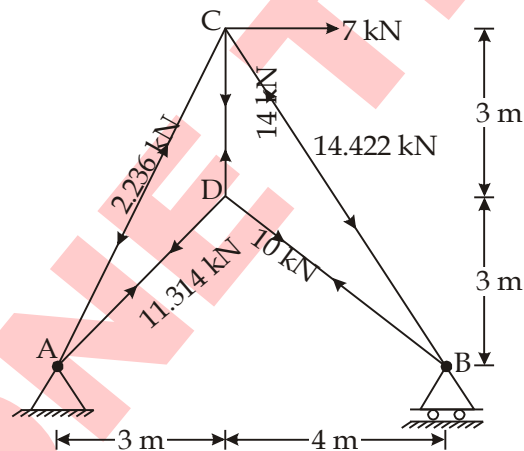
$$\Rightarrow \Sigma F_y = 0$$

$$\Rightarrow -F_{AD} \sin \theta_1 - F_{BD} \sin \theta_3 + F_{DC} = 0$$

$$\Rightarrow F_{DC} = 11.314 \times \frac{1}{\sqrt{2}} + 10 \times \frac{3}{5}$$

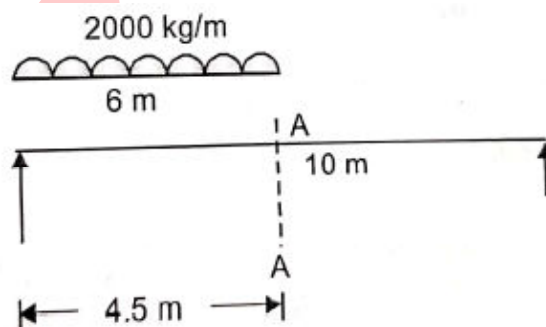
$$\Rightarrow F_{DC} = 8 + 6$$

$$\Rightarrow F_{DC} = 14 \text{ kN (Tensile)}$$

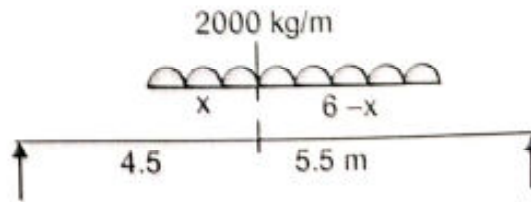


Q.31

Sol.

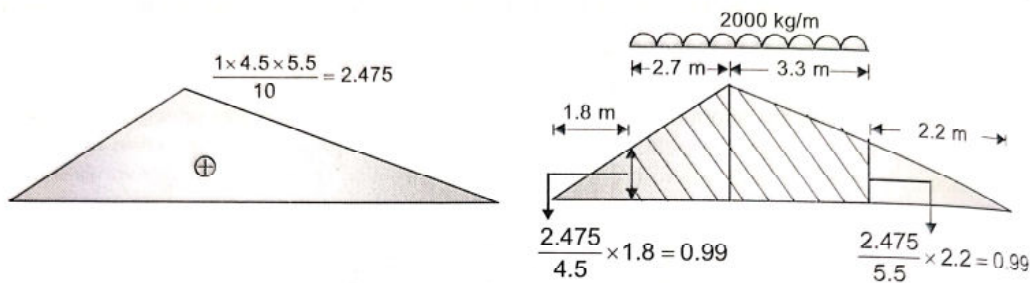


For max BM at A - A, udl should be placed in such a way that section divides the load in same ratio as it divides the span



$$\begin{aligned} \Rightarrow \quad \frac{x}{6-x} &= \frac{4.5}{5.5} \\ 5.5x &= 27 - 4.5x \\ 10x &= 27 \\ x &= 2.7 \text{ m} \\ 6-x &= 3.3 \text{ m} \end{aligned}$$

ILD for BM at section A - A is given as below



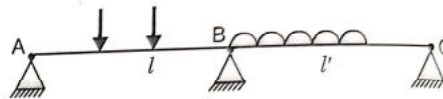
BM at section A - A = (Area under udl) $\times$ (udl)

$$\begin{aligned} &= \left[\frac{1}{2} (0.99 + 2.475) \times 2.7 + \frac{(2.475 + 0.99)}{2} \times 3.3 \right] \times 2000 \text{ kgm} \\ &= \frac{(0.99 + 2.475) \times 6}{2} \times 2000 = 20790 \text{ kgm} \end{aligned}$$

Q.32

Sol. Conceptual Background:

Clapeyron's theorem of three moments: relationship among the bending moments at three consecutive supports of a horizontal beam.



$$M_A \cdot l + 2M_B (l + l') + M_C l' = \frac{6a_1 x_1}{l} + \frac{6a_2 x_2}{l'}$$

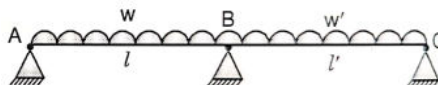
a_1 = Area of BM diagram due to vertical loads on AB

a_2 = Area of BM diagram due to vertical loads on BC

x_1 = Distance from A to c.g for the BMD for AB

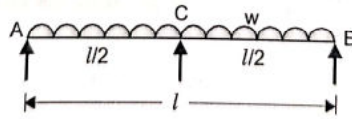
x_2 = Distance from C to c.g for the BMD for BC

If weight distributed uniformly i.e., udl on both spans, equation can also be written as,



$$M_A \cdot I + 2M_B (I + I') + M_C I' = \frac{wl^3}{4} + \frac{w'l'^3}{4}$$

3 Moments equation of clapeyron's theorem is applicable for a continuous span where we get a relationship among the bending moments at three consecutive supports of a horizontal beam. To calculate the central deflection, let us assume a support at centre of beam AB. Thus applying 3 moments equation, we get,



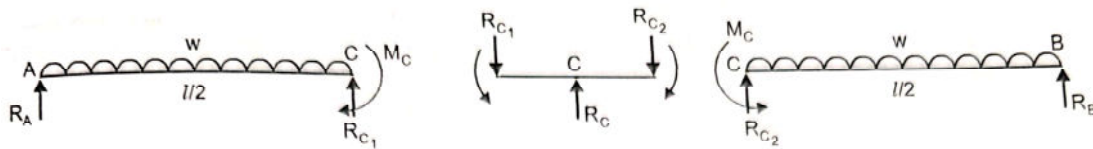
$$M_A \frac{l}{2} + 2M_C \left(\frac{l}{2} + \frac{l}{2} \right) + M_B \times \frac{l}{2} = \frac{w \left(\frac{l}{2} \right)^3}{4} + \frac{w \left(\frac{l}{2} \right)^3}{4}$$

$$\Rightarrow M_A + 4M_C + M_B = \frac{2wl^3}{4 \times 8} \times \frac{2}{l} = \frac{wl^2}{8}$$

As simply supported ends, $\Rightarrow M_A = M_B = 0$

$$\Rightarrow M_C = \frac{wl^2}{8} \times \frac{1}{4} = \frac{wl^2}{32}$$

Now, to calculate reactions at C, we use FBD of beam



$$R_{C1} = \frac{w(l/2)}{2} + \frac{M_C}{l/2} = \frac{wl}{4} + \frac{wl^2}{32 \times \frac{l}{2}}$$

$$R_{C2} = \frac{w(l/2)}{2} + \frac{M_C}{l/2}$$

$$R_{C1} = \frac{wl}{4} + \frac{wl}{16}$$

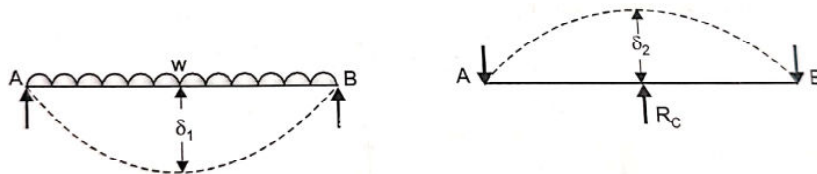
$$R_{C2} = \frac{wl}{4} + \frac{wl}{16}$$

$$\Rightarrow R_{C1} = \frac{5wl}{16}$$

$$\Rightarrow R_{C2} = \frac{5wl}{16}$$

$$\Rightarrow R_C = R_{C1} + R_{C2} = \frac{5wl}{16} + \frac{5wl}{16} = \frac{5wl}{8}$$

Now using compatibility condition that the net downward deflection due to udl on span AB is equal to net upward deflection due to R_C acting alone.

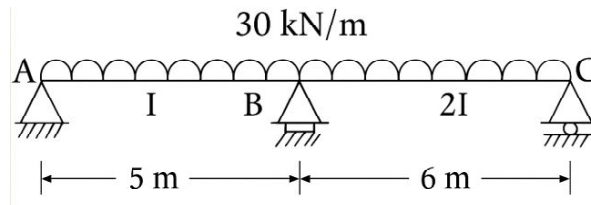


$$\Rightarrow \delta_1 = \delta_2 = \frac{R_C l^3}{48EI} = \frac{5wl}{8} \times \frac{l^3}{48EI} = \frac{5wl^4}{384EI}$$

Thus the central deflection in a simply supported beam is $\frac{5wl^4}{384EI}(\downarrow)$

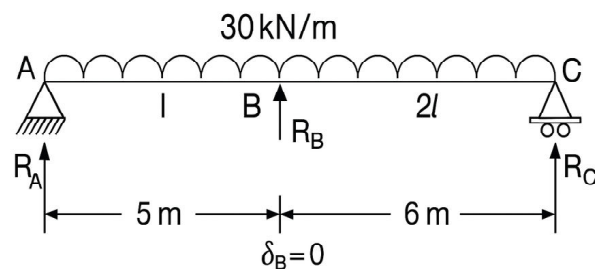
Q.33

Sol.



It is a statically indeterminate structure with degree of indeterminacy equal to 1.

Let us assume the reaction R_B at support B. Therefore, we can also draw the above structure as shown below, with the condition that deflection at B should be zero.



Support reaction at A,

$$R_A = \frac{30 \times 11}{2} - \frac{6R_B}{11}$$

$$R_A = 165 - \frac{6R_B}{11} \quad \dots(i)$$

Support reaction at C,

$$R_C = \frac{30 \times 11}{2} - \frac{5R_B}{11}$$

$$R_C = 165 - \frac{5R_B}{11} \quad \dots(ii)$$

Moment in span AB at any distance 'x' measured from A

$$M_{AB} = \left(165 - \frac{6R_B}{11} \right) \times -30 \times \frac{x}{2}$$

$$M_{AB} = \left(165 - \frac{6R_B}{11} \right) \times -15x^2 \quad \dots(iii)$$

$$\Rightarrow \frac{\partial M_{AB}}{\partial R_B} = \frac{-6x}{11} \quad \dots(iv)$$

Similarly, moment in span CB at any distance 'x' measured from C

$$M_{CB} = \left(165 - \frac{5R_B}{11} \right) \times -30 \times \frac{x}{2}$$

$$M_{CB} = \left(165 - \frac{5R_B}{11} \right) \times -15x^2 \quad \dots(v)$$

$$\Rightarrow \frac{\partial M_{CB}}{\partial R_B} = \frac{-5x}{11} \quad \dots(vi)$$

∴ Total strain energy (v) in beam,

$$U = \int_0^5 \frac{M_{AB}^2 dx}{2EI} + \int_0^6 \frac{M_{CB}^2 dx}{2E(2I)}$$

∴ Deflection at, B (δ_B),

$$\delta_B = \frac{\partial u}{\partial R_B} = \int_0^5 \frac{2M_{AB} \frac{\partial M_{AB}}{\partial R_B}}{2EI} dx + \int_0^6 \frac{2M_{CB} \frac{\partial M_{CB}}{\partial R_B}}{4EI} dx$$

Putting value from equation (iii), (iv), (v) and (vi), we get

$$\delta_B = \int_0^5 \frac{\left[\left(165 - \frac{6R_B}{11} \right) x - 15x^2 \right] \left(-\frac{6x}{11} \right) dx}{EI} + \int_0^6 \frac{\left[\left(165 - \frac{5R_B}{11} \right) x - 15x^2 \right] \left(-\frac{5x}{11} \right) dx}{EI}$$

Now, as there is a support at B, therefore deflection at B must be zero

i.e., $\delta_B = 0$

$$\Rightarrow \int_0^5 \frac{\left[\left(165 - \frac{6R_B}{11} \right) x - 15x^2 \right] \left(-\frac{6x}{11} \right) dx}{EI} + \int_0^6 \frac{\left[\left(165 - \frac{5R_B}{11} \right) x - 15x^2 \right] \left(-\frac{5x}{11} \right) dx}{2EI} = 0$$

After solving this equation, we get

$$R_B = 205.025 \text{ kN}$$

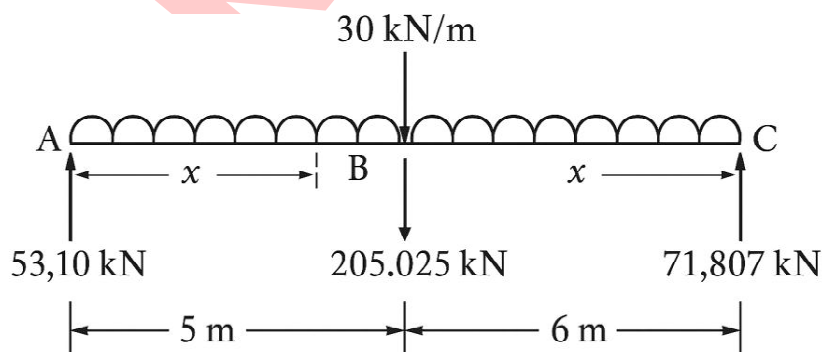
From equation (i),

$$R_A = 165 - \frac{6R_B}{11} = 165 - \frac{6 \times 205.025}{11} = 53.17 \text{ kN}$$

From equation (ii),

$$R_B = 165 - \frac{5R_B}{11} = 165 - \frac{5 \times 205.025}{11} = 71.807 \text{ kN}$$

Shear force diagram



Portion AB ('x' measured from A)

$$\text{S.F.} = 53.17 - 30x$$

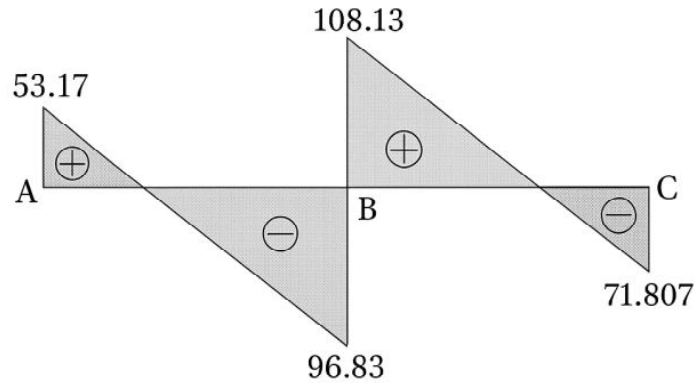
∴ S.F. (just left of B) = $53.17 - 30 \times 5 = -96.83 \text{ kN}$

Portion CB ('x' measured from C)

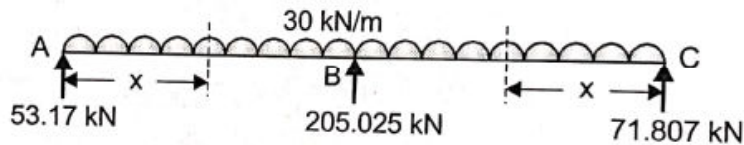
$$\text{S.F.} = -71.807 + 30x$$

∴ S.F. (just right of B) = $-71.807 + 30 \times 6 = 108.13 \text{ kN}$

S.F.D. will be



Bending moment diagram



Portion AB ('x' measured from A)

$$M = 53.17x - 30 \cdot x \cdot \frac{x}{2} = 53.17x - 15x^2$$

∴ Bending moment at B

$$M_B = 53.17 \times 5 - 15 \times 5^2 = -109.15 \text{ kN-m}$$

For maximum M,

$$\frac{dM}{dx} = 0 \Rightarrow x = 1.77 \text{ m}, M_{\max} = 47.12 \text{ kN-m}$$

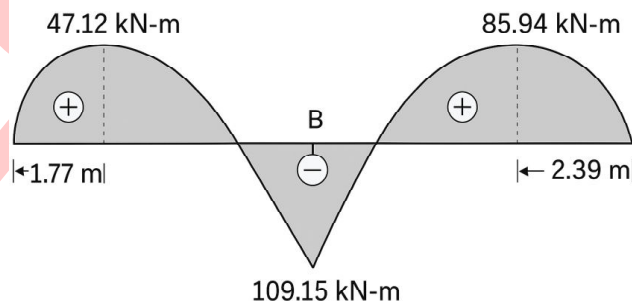
Portion CB ('x' measured from C)

$$M = 71.807x - 15x^2$$

For maximum M,

$$\frac{dM}{dx} = 0 \Rightarrow x = 2.39 \text{ m}, M_{\max} = 85.94 \text{ kN-m}$$

BMD will be

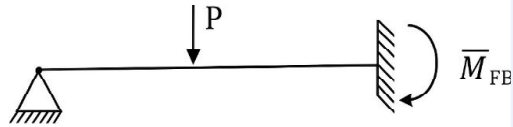


Q.34

Sol. Fixed end moments

$$\bar{M}_{FBA} = +\frac{3PL}{16} = \frac{+3 \times 24 \times 3}{16} = +\frac{27}{2} = +13.5 \text{ kN-m}$$

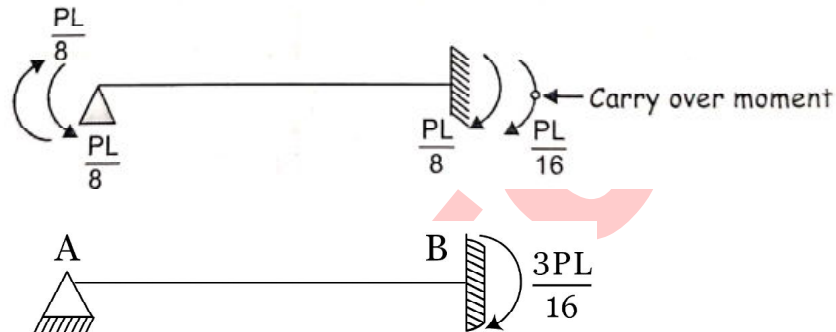
Conceptual Background:



We know that



Hence



Hence,

$$M_{FBA} = +\frac{3PL}{16}$$

$$M_{FBC} = +\frac{M_0}{4} = +\frac{40}{4} = 10$$

$$M_{FBC} = 10 \text{ kNm}$$

Hence,

$$\bar{M}_{FBA} = +13.5 \text{ kNm}$$

$$M_{FBC} = +10 \text{ kNm}, M_{FCB} = +10 \text{ kNm},$$

Slope-deflection equation :

$$M_{BA} = \bar{M}_{FBA} + \frac{3EI}{L} \left(\theta_B - \frac{\delta}{l} \right) = 13.5 + \frac{3E(1.5l)}{3} (\theta_B)$$

$$\Rightarrow M_{BA} = 13.5 + 1.5EI\theta_B \quad \dots(1)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L} \left\{ 2\theta_B + \theta_C - \frac{3\delta}{l} \right\} = 10 + \frac{2EI}{3} \{ 2\theta_B \}$$

$$\Rightarrow M_{BC} = 10 + \frac{4EI\theta_B}{3} \quad \dots(2)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L} \left\{ 2\theta_C + \theta_B - \frac{3\delta}{l} \right\} = 10 + \frac{2EI}{3} \{ \theta_B \}$$

$$\Rightarrow M_{CB} = 10 + \frac{2EI\theta_B}{3} \quad \dots(3)$$

Joint equilibrium equation

$$M_{BA} + M_{BC} = 0$$

$$13.5 + 1.5EI\theta_B + 10 + \frac{4EI\theta_B}{3} = 0$$

$$\Rightarrow -23.5 = \left(1.5 + \frac{4}{3}\right)EI\theta_B$$

$$\Rightarrow EI\theta_B = -8.294$$

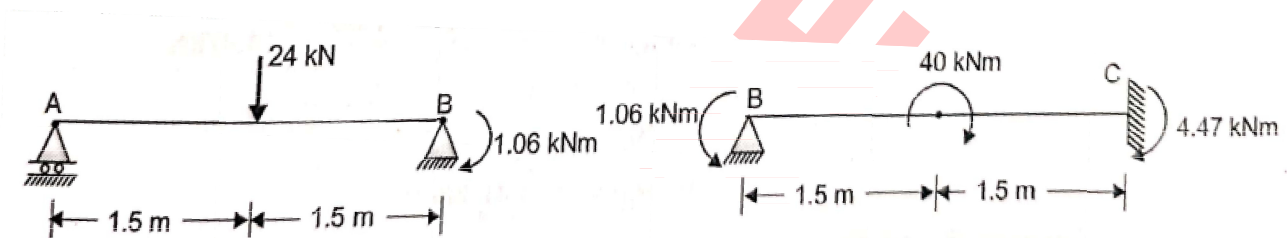
By putting this value in Eqn. (1), Eqn. (2) and Eqn. (3)

$$M_{BA} = 1.06 \text{ kNm}$$

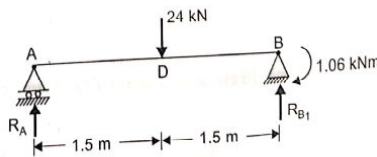
$$M_{BC} = -1.06 \text{ kNm}$$

$$M_{CB} = +4.47 \text{ kNm}$$

F.B.D



Bending moment diagram and SFD :
For Part AB



$$R_A + R_{B1} = 24$$

$$\sum M_B = 0$$

$$R_A \times 3 = 24 \times 1.5 - 1.06$$

$$\Rightarrow R_A = 11.646 \text{ kN}$$

$$\text{and } R_{B1} = 12.354 \text{ kN}$$

From A to D

$$M = R_A \times x = 11.646x \text{ kNm}$$

$$M_{x=1.5m} = (11.646 \times 1.5) = 17.469 \text{ kNm}$$

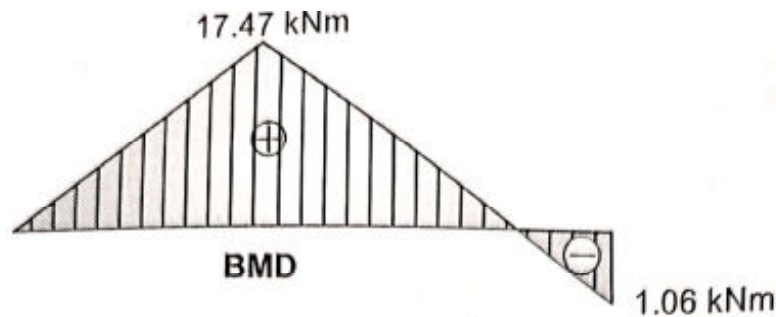
From B to D (x from right side)

$$M = R_{B1} \times x - 1.06$$

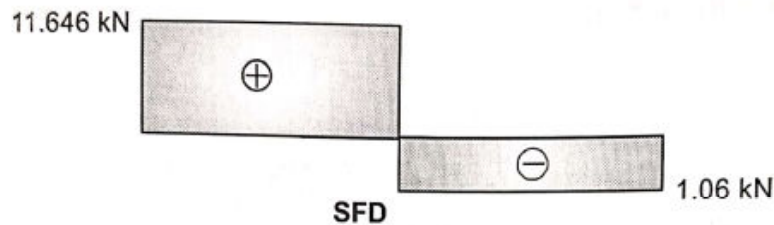
$$M_{x=1.5m} = 12.354 \times 1.5 - 1.06$$

$$= 18.531 - 1.06 = 17.471 \text{ kNm}$$

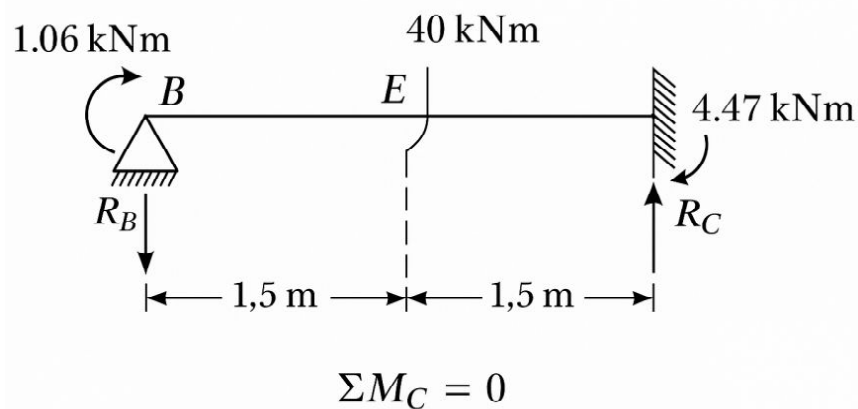
$$M_{x=0} = -1.06 \text{ kNm, Hence the BMD will be}$$



Shear force diagram will be as shown below



For Part BC



$$\Sigma M_C = 0$$

$$R_{B_2} \times 3 - 1.06 + 40 + 4.47 = 0$$

$\Rightarrow$

$$R_{B_2} = \frac{-40 - 4.47 + 1.06}{3} = -14.47 \text{ kN}$$

Also,

$$R_{B_2} + R_C = 0$$

$\therefore$

$$R_C = +14.47 \text{ kN}$$

BMD (Between B and E)

$$M = R_{B_2} x - 1.06$$

$$M|_{x=0} = -1.06$$

$$M|_{x=1.5\text{m}} = -14.47 \times 1.5 - 1.06 = -22.765$$

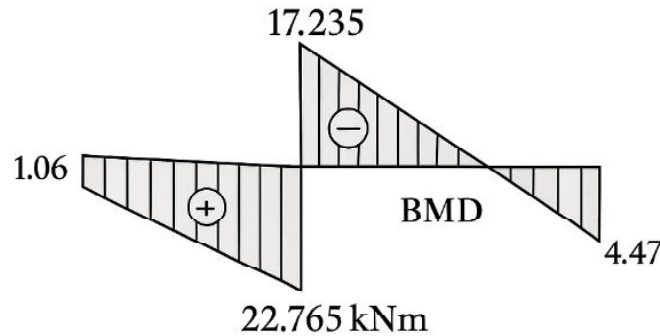
BMD (Between C and E) (x from right side)

$$M = R_C x - 4.47$$

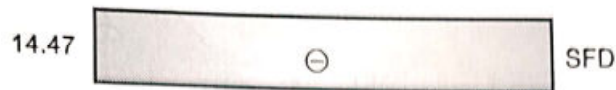
$$M_{x=0} = -4.47 \text{ kNm}$$

$$M_{x=1.5\text{m}} = +14.47 \times 1.5 - 4.47 = 17.235 \text{ kNm}$$

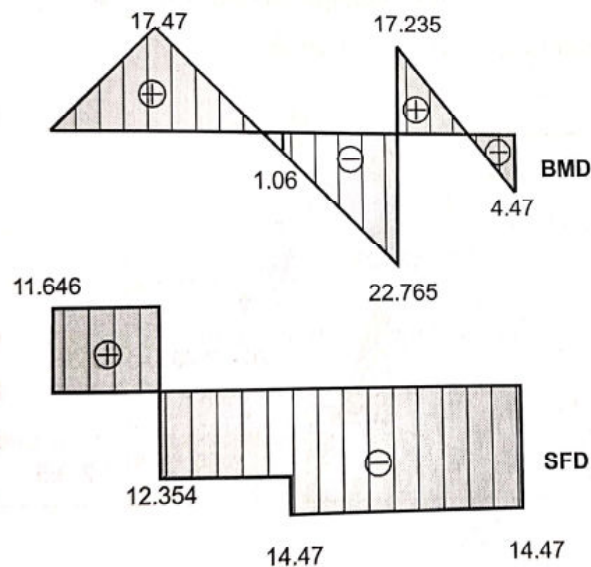
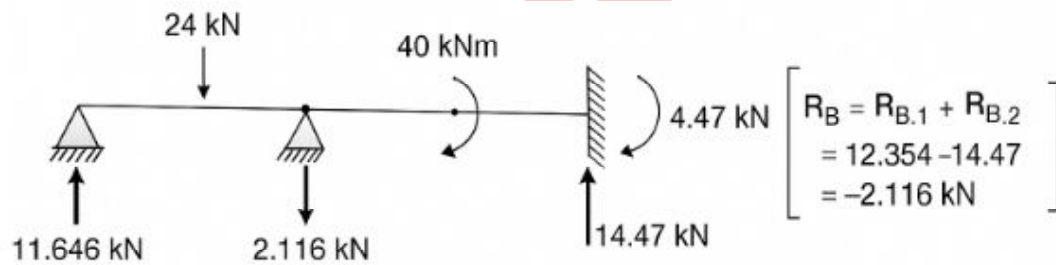
Hence the BMD for part BC can be known as follows



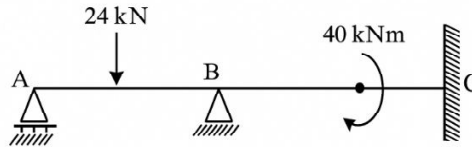
SFD for part BC is shown as below



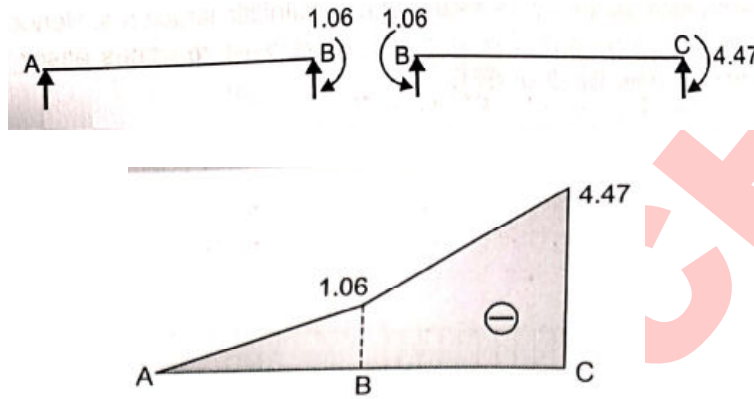
By combining both part,



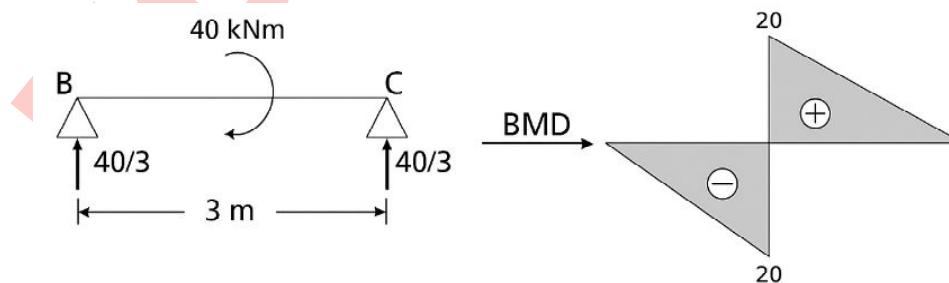
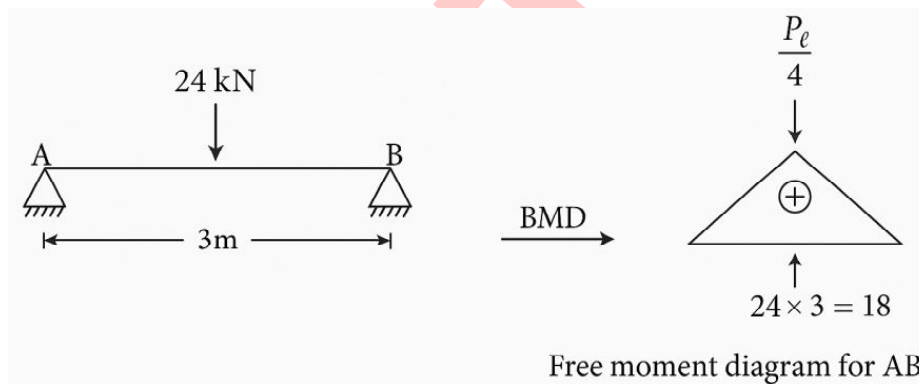
Alternative method for drawing BMD:



First draw the end moment diagram

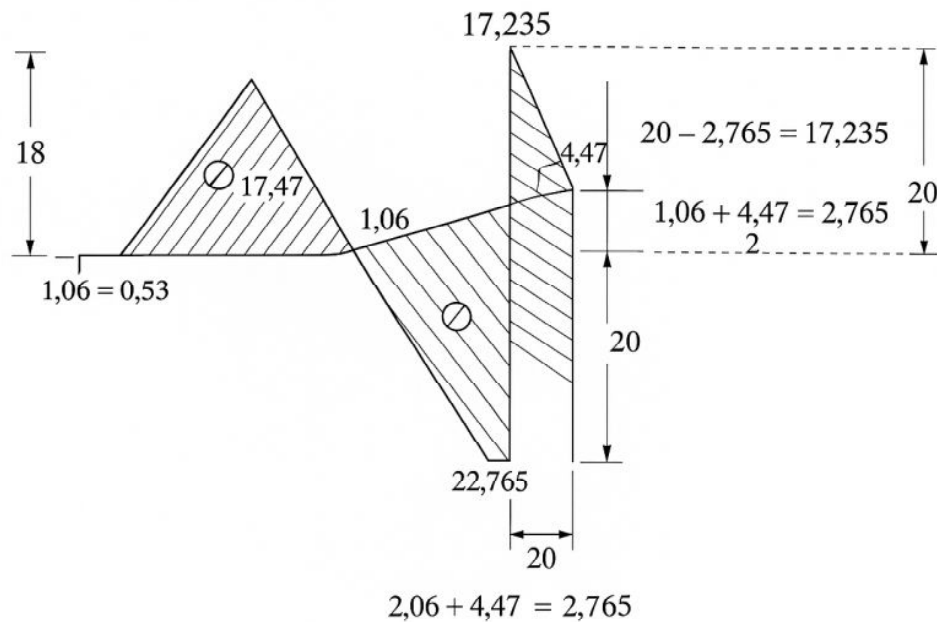


After plotting end moments, Free moment diagram (assuming simply supported condition with loading) is superimposed.



After superimposition, the BMD will look like as shown below

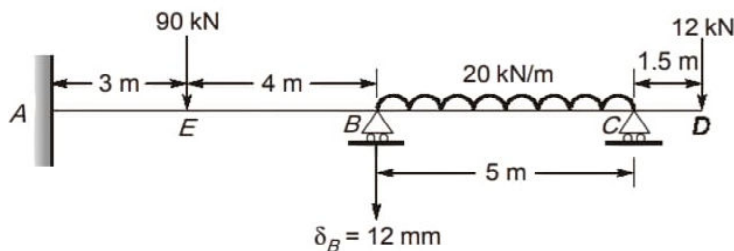


**Note:**

- The Bending moment diagram from both the method is same only the way of representation is different.
- In solid mechanics generally we deal with deterministic structures. Hence we can directly calculate the unknown reactions and if someone can calculate reactions easily then the 1st method will become easy to draw BMD or SFD.
- In structure analysis part, we generally deal with indeterminate structures where we can't find the reactions directly by using equilibrium equations. For that we have to estimate the end moments first. Hence in these cases the fixed end moment method i.e. 2nd one will be easy.
- For shear force, we have to determine reactions as in previous method, and after that we can draw S.F.D. which will be some as in previous method.

Q.35

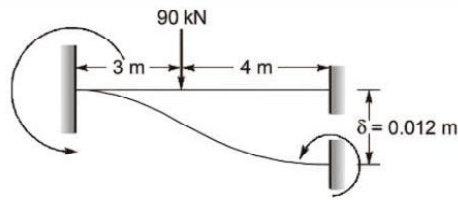
Sol.



$$E = 210 \text{ GPa}, I = 5131.6 \times 10^4 \text{ mm}^4 = \frac{5131.6 \times 10^4}{10^{12}}$$

$$EI = 210 \times 10^6 \times \frac{5131.6 \times 10^4}{10^{12}} = 10776.36 \text{ kN} \cdot \text{m}^2$$

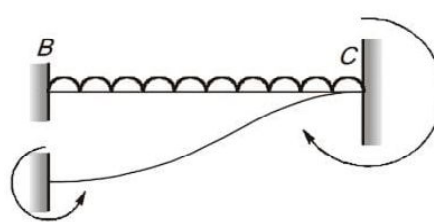
Step- 1: FEMS due to applied loads and sinking of supports.



$$M_{FAB} = \frac{90 \times 3 \times 4^2}{7^2} - \frac{6 \times 10776.36 \times 0.012}{7^2} = -104 \text{ kNm}$$

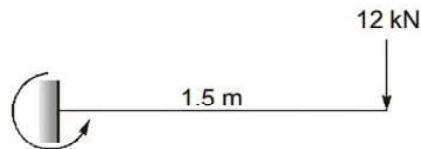
$$M_{FBA} = +\frac{90 \times 3^2 \times 4}{7^2} - \frac{6 \times 10776.36 \times 0.012}{7^2} = 50.29 \text{ kNm}$$

$$M_{FBC} = -\frac{20 \times 5^2}{12} + \frac{6 \times 10776.36 \times 0.012}{5^2} = -10.63 \text{ kNm}$$



$$M_{FCB} = +\frac{20 \times 5^2}{12} + \frac{6 \times 10776.36 \times 0.012}{5^2} = 72.7 \text{ kNm}$$

$$M_{FCD} = -18 \text{ kNm}$$



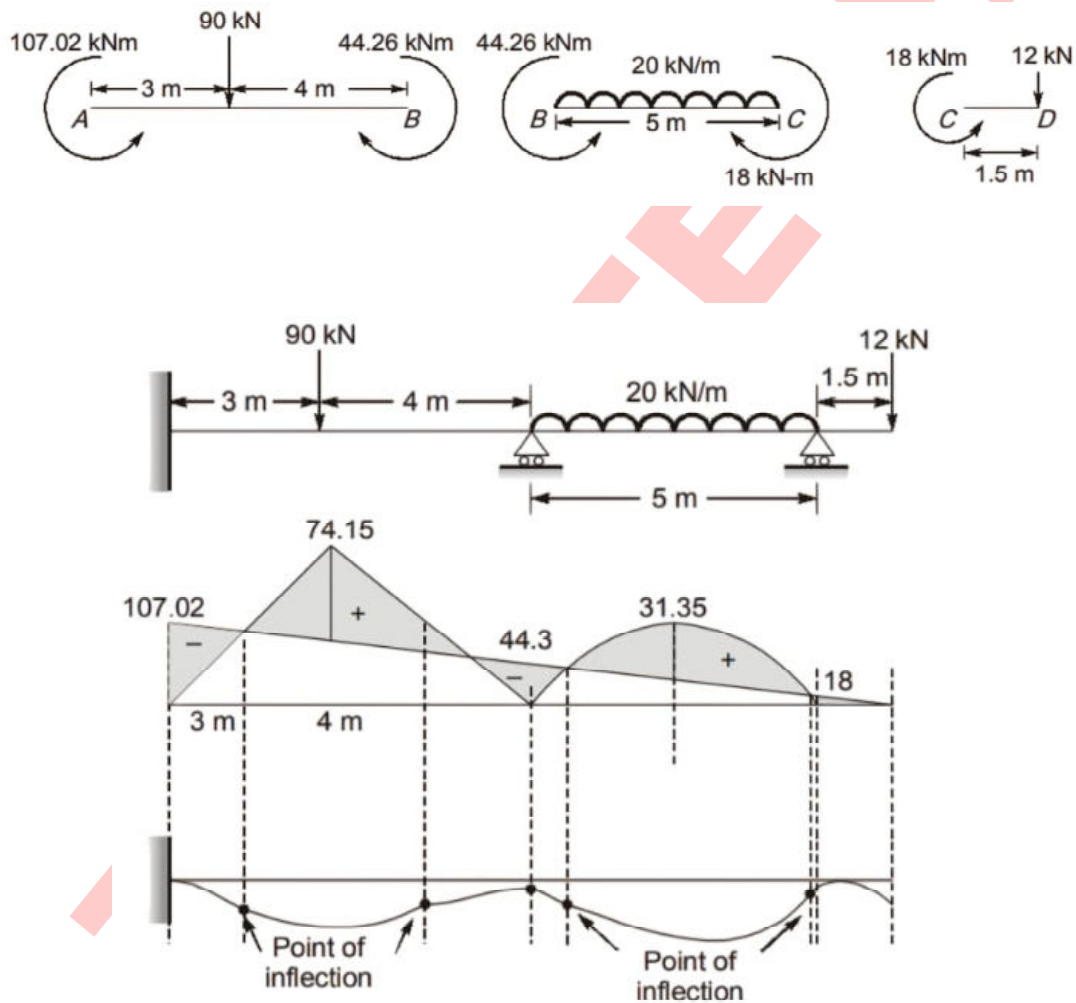
Step-2: Distribution factors

Joint	Member	k	Σk	$DF = \frac{k}{\Sigma k}$
B	BA	$I/7$	$0.293I$	0.49
	BC	$\frac{3}{4} \times \frac{I}{5} = \frac{3I}{20}$		0.51

Step-3: End Moment distribution

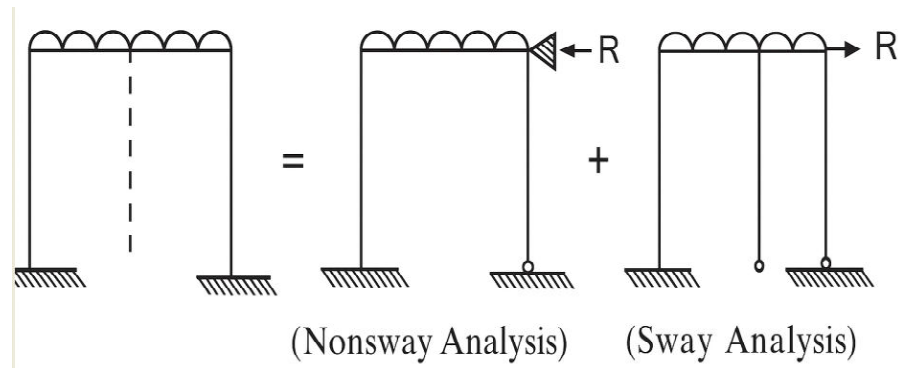
Joint	A	B		C	D
DF	0	0.49	0.51	1	0
FEMS	-104	50.29	-10.63	72.7	-18
Release C COM			-27.35	-54.7	0
Net FEMS	-104	50.29	-37.98	+18.0	-18
Bal B		-6.03	-6.28		
COM	-3.02			0	
Final and moments	-107.02	44.26	-44.26	+18	-18

Step-4: BMD



Q.36

Sol. In this frame Side sway occurs if there is a lack of symmetry either with respect to loading or with respect to geometry.



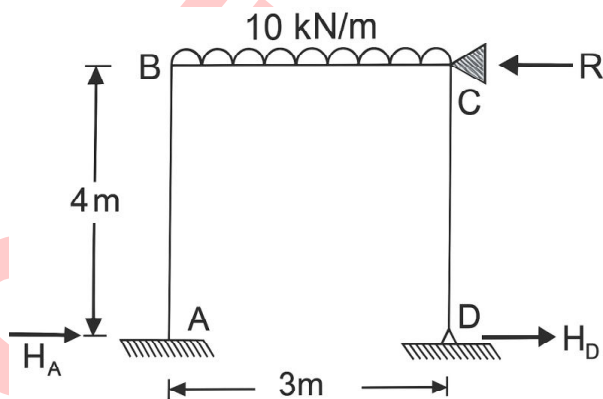
Distribution factors

Joint	Member	Relative Stiffness	Total Relative Stiffness	Distribution Factor
B	BA	$I/4$	$7I/12$	$3/7$
	BC	$I/3$		$4/7$
C	CB	$I/3$	$17I/24$	$8/17$
	CD	$(3/4) \times (2I/4) = 3I/8$		$9/17$

Nonsway Analysis

Objective: To find restraining force R.

Fixed end moment



$$M_{FAB} = M_{FBA} = 0$$

$$M_{FBC} = \frac{-10 \times 3^2}{12} = -7.50 \text{ kNm}$$

$$M_{FCB} = 7.50 \text{ kNm}$$

$$M_{FCD} = M_{FDC} = 0$$

A		B		C		D		
		$\frac{3}{7}$	$\frac{4}{7}$		$\frac{8}{17}$	$\frac{9}{17}$		
0	0		-7.50	7.50		0	0	FEM
	+3.21		+4.29	-3.53		-3.9		Bal
+1.605			-1.765	+2.145				C/O
	+0.756		+1.009	-1.009		-1.136		Bal
+0.378			-0.505	-0.505				C/O
	+0.016		+0.289	-0.238		-0.077		Bal
+0.016			-0.019	+0.145				C/O
	+0.034		+0.034	-0.068		-0.077		Bal
+0.051			-0.019	-0.016				C/O
	+0.015		+0.019	-0.018		-0.018		Bal
2.117	4.248		-4.248	5.468		-5.468	0	

Hence, $M_{AB} = 2.117$, $M_{BA} = 4.248$, $M_{BC} = -4.248$, $M_{CB} = 5.468$, $M_{CD} = -5.468$, $M_{DC} = 0$

Note that we can avoid calculation of moments on legs AB and CD while doing moment distribution. Moments of these legs can finally be obtained using equilibrium equation once M_{BC} and M_{CB} have been calculated. For example, the analysis can be done in the central part follows:

A		B		C		D		
		$\frac{4}{7}$		$\frac{8}{17}$				
0	0		-7.5	7.5		0	0	FEM
			$7.5 \times \frac{4}{7}$	$-7.5 \times \frac{8}{17}$				Bal
			$\frac{-7.5 \times 4}{17}$	$7.5 \times \frac{2}{7}$				C/O
			$\frac{7.5 \times 4}{17} \times \frac{4}{7}$	$-7.5 \times \frac{2}{7} \times \frac{8}{17}$				Bal
								C/O
			-4.248	+5.468				

Now,

$$M_{BA} + M_{BC} = 0 \Rightarrow M_{BA} = -M_{BC} = 4.248$$

$$M_{CB} + M_{CD} = 0 \Rightarrow M_{CD} = -M_{CB} = -5.468$$

Also, from (i) and (ii) ;

$$M_{AB} = M_{FAB} + \frac{M_{BA} - M_{FBA}}{2}$$

$$\left[\begin{array}{l} M_{AB} = M_{FAB} + \frac{2EI}{l}(\theta_B) \quad \dots(i) \\ M_{AB} = M_{FBA} + \frac{2EI}{l}(2\theta_B) \quad \dots(ii) \end{array} \right]$$

Since $\delta = 0$ and end A is fixed

$$M_{AB} = 0 + \frac{4.248 - 0}{2}$$

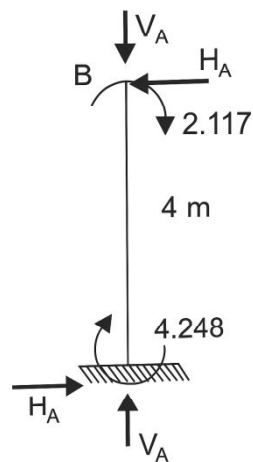
$$M_{AB} = 2.124 \approx 2.117$$

$$M_{DC} = 0 \quad [\because \text{hinge}]$$

Free body diagram

For part AB

Our aim is to calculate H_A



$$\Sigma M_B = 0$$

$\Rightarrow$

$$H_A \times 4 = 2.117 + 4.248$$

$\Rightarrow$

$$H_A = 1.59125 \text{ kN}$$

For part CD:

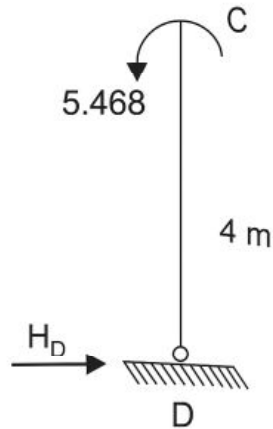
$$\Sigma M_C = 0$$

$\Rightarrow$

$$H_D \times 4 = -5.468$$

$\Rightarrow$

$$H_D = -1.367 \text{ kN}$$



Restraining force,

$$R = H_A + H_D \text{ [from } \Sigma H = 0]$$

$$R = H_A + H_D$$

$\Rightarrow$

$$R = 1.59125 - 1.367$$

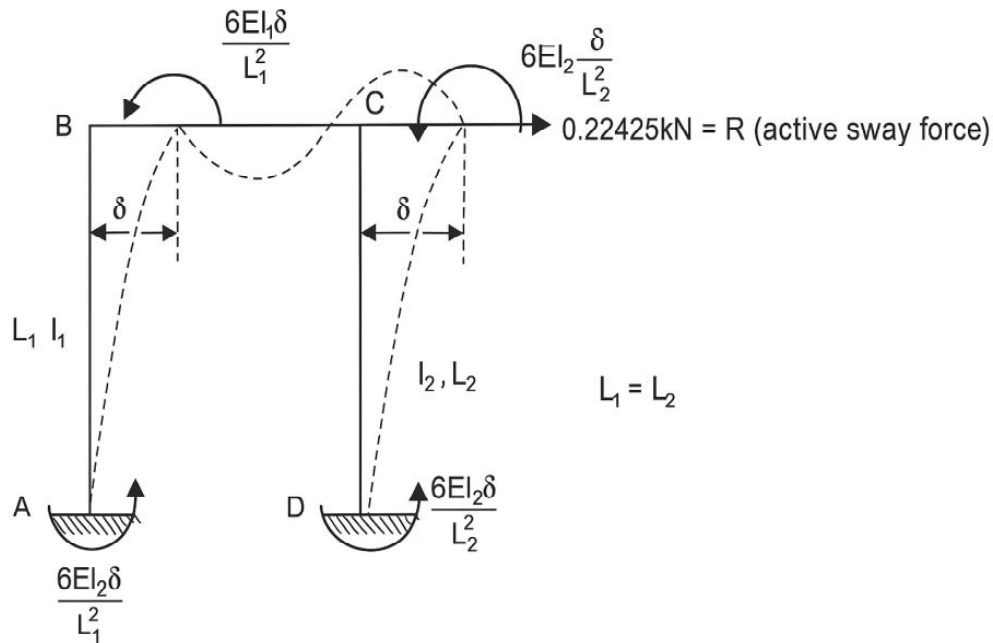
$\Rightarrow$

$$R = 0.22425 \text{ kN}$$

Sway Analysis

- Remove all external loading
- Apply arbitrary sway force (S) opposite to the restraining force i.e., rightward in this case.
- In sway analysis, fixed end moment are not known in advance hence arbitrary values are assumed for fixed end moments, like 100, 10 etc.
- Joint moment determined
- Arbitrary sway force determined
- Correction factor determination; $\text{correction factor} = \frac{\text{Actual sway force (R)}}{\text{Arbitrary sway force (S)}}$
- Actual sway moment calculation = end moments $\times$ (correction factor)
- Total end moment = (Actual sway moment) + (Nonsway moment)

Sway analysis



if we suppose, $\frac{6EI_1\delta}{L_1^2} = 10$ [Here only magnitude considered]

then, $M_{FAB} = M_{FBA} = -10$ [(-ve) sign taken because of anticlockwise moment]

Hence, $\frac{6E(2I)\delta}{L^2} = 20$

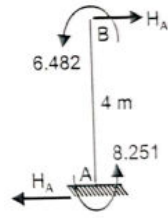
$M_{FCD} = M_{FDC} = -20$ [$\because I_2 = 2I$]

So,

A		B		C		D
		3/7	4/7	8/17	9/17	
-10	-10	0		0	-20	-20
					+10	+20
-10	-10	0			-10	0
2.143	4.286	2.353	5.714	2.857	4.706	5.294
-0.504	-1.008	-0.672	-1.345	-0.673	-1.344	-1.513
0.144	0.288	0.159	0.384	0.192	0.317	0.356
-0.034	-0.068	-0.046	-0.091	-0.046	-0.091	-0.101
		0.02	0.026	0.022	0.024	0.024

FEM → considering all joints to be fixed
In actual case, joint D is hinged hence BM at D is permanently released
New FEM
Bal
C/O
Bal
C/O
Bal
C/O
Bal
C/O
Bal

Calculation of H_A :



$$M_{AB} = -8.251$$

$$M_{BA} = -6.482$$

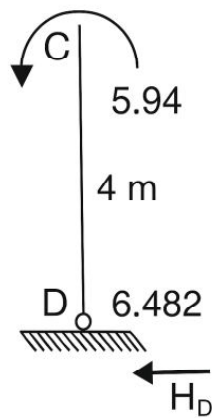
$$\sum M_B = 0$$

$$H_A \times 4 = 8.251 + 6.482$$

$$H_A = 3.68325 \text{ kN}$$

⇒

Calculation of H_D



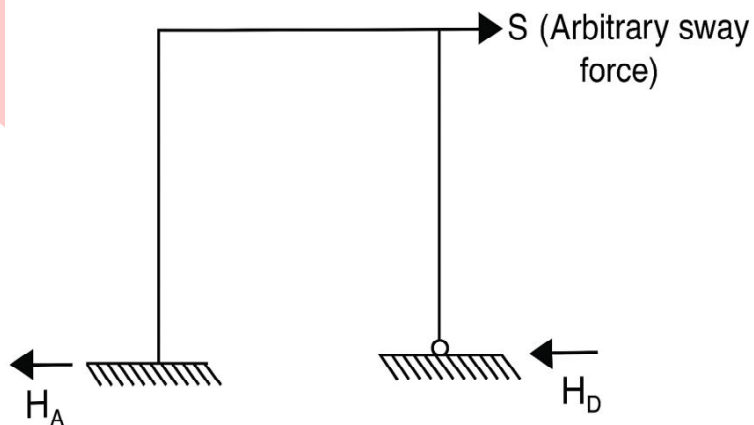
$$M_{CD} = -5.94$$

$$\sum M_C = 0$$

$$H_D \times 4 = 5.94$$

$$H_D = 1.485 \text{ kN}$$

⇒



Hence, Arbitrary sway force = (S) = 3.68325 + 1.485 = 5.16825 kN

$$\text{Correction factor} = \frac{R}{S} = \frac{0.22425}{5.16825} = 0.04338$$

∴ Actual end moments (for sway) are:

$$M_{AB} = -8.251 \times 0.04338 = -0.358 \text{ kNm}$$

$$M_{BA} = -6.482 \times 0.04338 = -0.281 \text{ kNm}$$

$$M_{BC} = 6.482 \times 0.04338 = +0.281 \text{ kNm}$$

$$M_{CB} = 5.94 \times 0.04338 = +0.2576 \text{ kNm}$$

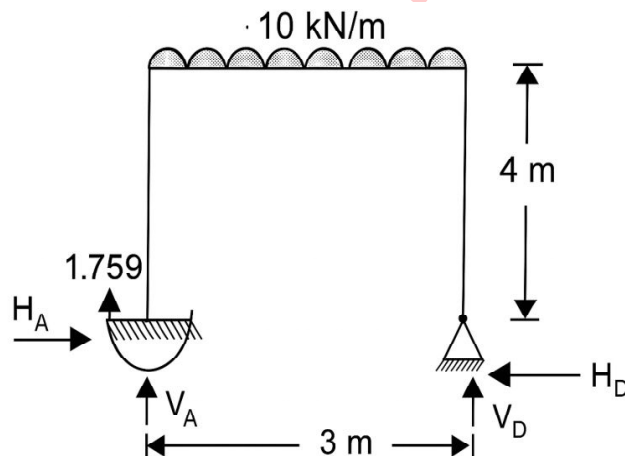
$$M_{CD} = -5.94 \times 0.04338 = -0.2576 \text{ kNm}$$

$$M_{DC} = 0 \times 0.4338 = 0$$

⇒ Total end moments :

Member	(Non sway end moments)	(Actual sway end Moment)	Total end Moment
AB	2.117	-0.358	1.759
BA	4.248	-0.281	3.967
BC	-4.248	+0.281	-3.967
CB	5.468	+0.2576	5.7256
CD	-5.468	-0.2576	-5.7256
DC	0	0	0

Determining the actual reactions:



$$\sum F_x = 0$$

$$V_A + V_D = 30$$

and

$$\sum M_D = 0$$

⇒

$$1.759 + V_A \times 3 = 10 \times 3 \times 3 / 2$$

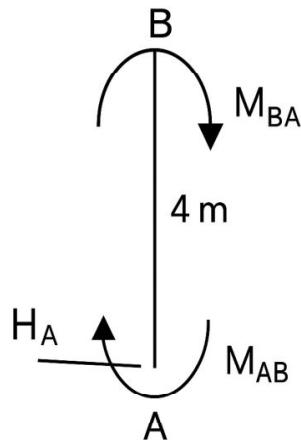
⇒

$$V_A = \frac{45 - 1.759}{3} = 14.4137 \text{ kN} \approx 14.42 \text{ kN}$$

⇒

$$V_D = 15.58 \text{ kN}$$

Part AB



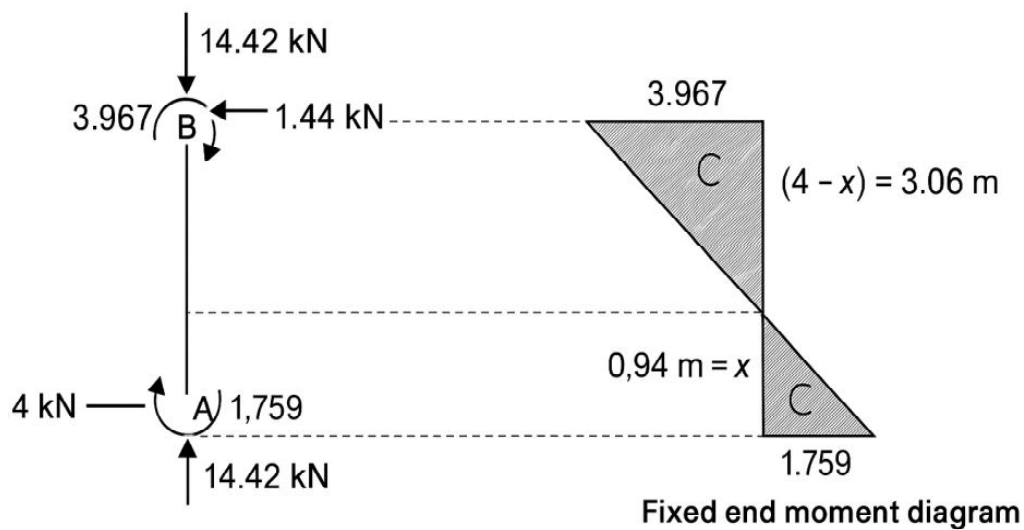
and,

$$H_A = \frac{M_{AB} + M_{BA}}{4} = \frac{1.759 + 3.967}{4} = 1.44 \text{ kN}$$

As

$$H_A - H_D = 0; \Rightarrow H_D = 1.44 \text{ kN}$$

Drawing Bending Moment Diagrams
For part AB,



Finding x ,

$$\frac{1.759}{x} = \frac{3.957}{4-x} \Rightarrow \frac{4-x}{x} = \frac{3.967}{1.759}$$

$\Rightarrow$

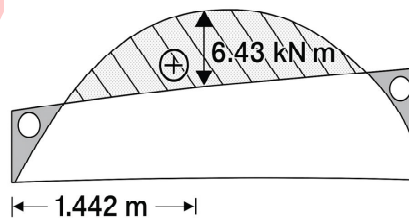
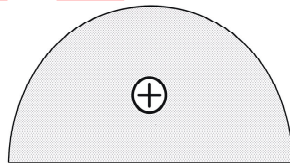
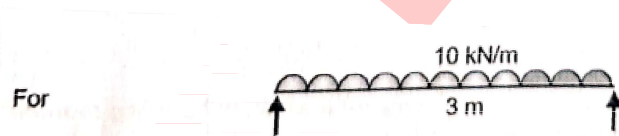
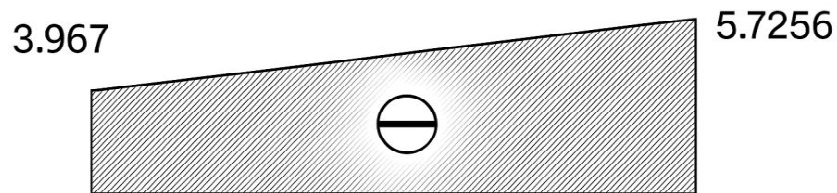
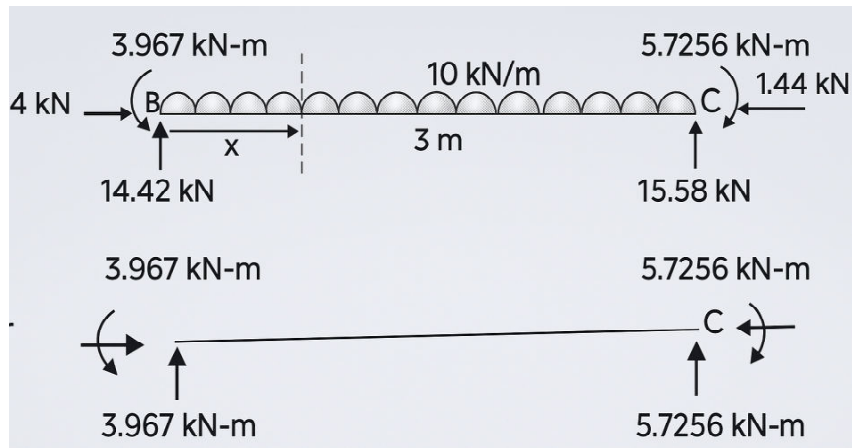
$$\frac{4}{x} = 1 + \frac{3.967}{1.759}$$

$\Rightarrow$

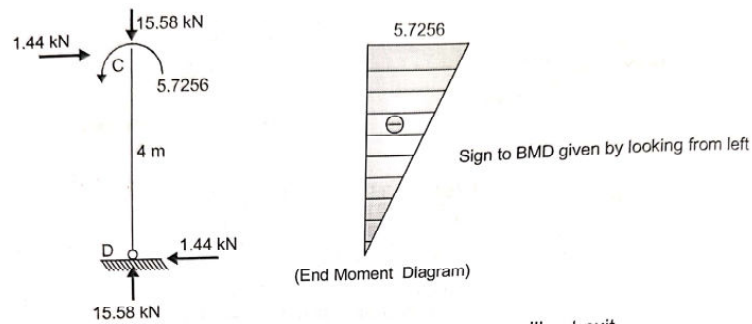
$$x = 0.94 \text{ and } 4 - x = 3.06 \text{ m}$$

Since there is no loading in span AB, hence free moment diagram will not exist

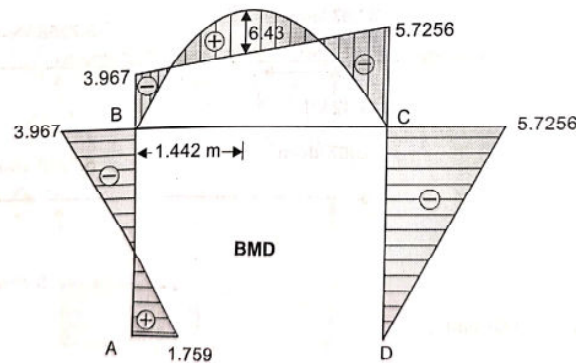
For part BC,



For part CD,

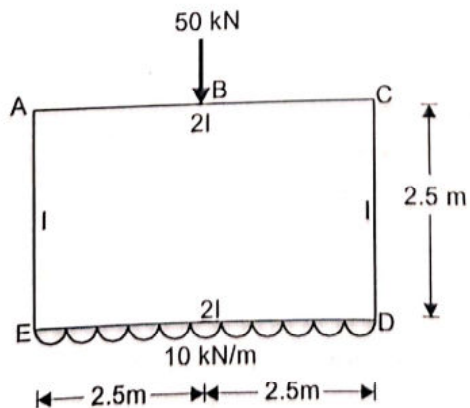


Since there is no loading on CD, hence free moment diagram will not exit
By combining all the parts together, we get

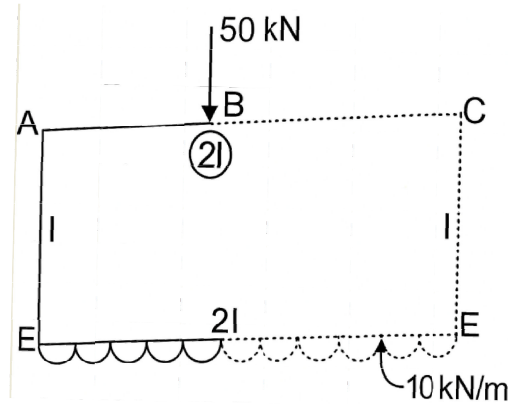


Q.37

Sol.



The structure is symmetrical, hence symmetrical beam approach will be used. We will consider only half of the structure i.e.,



Joint	Member	Stiffness	D.F.
A	AC	$2E \times \frac{2I}{5}$	0.33
	AE	$4E \times \frac{I}{2.5}$	0.67
E	EA	$\frac{4EI}{2.5}$	0.67
	ED	$2E \times \frac{2I}{5}$	0.33

Fixed end moments

$$M_{FAC} = \frac{-PL}{8} = -\frac{50 \times 5}{8} = -31.25$$

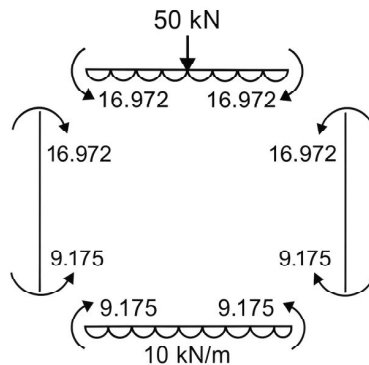
$$M_{FAE} = 0$$

$$M_{FEA} = 0$$

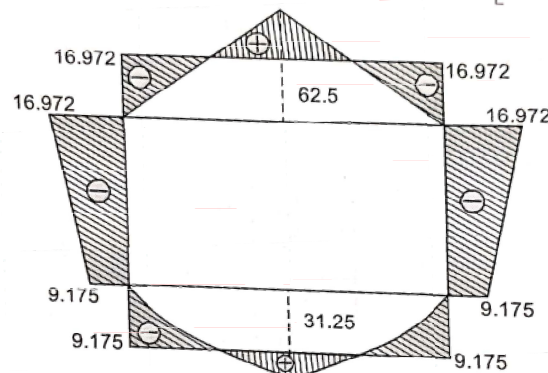
$$M_{FED} = \frac{WL^2}{12} = \frac{10 \times 5^2}{12} = 20.83$$

Joint Member	A		E	
	AC	AE	EA	ED
D.F.	0.33	0.67	0.67	0.33
FEM	-31.250	0	0	20.830
Bal	10.420	20.830	-13.890	-6.940
CO	0	-6.945	10.415	0
Bal	2.315	4.630	-6.943	-3.472
CO	0	-3.472	2.315	0
Bal	1.157	2.315	-1.543	-0.772
CO	0	-0.772	1.157	0
Bal	0.257	0.515	-0.772	-0.385
CO	0	-0.386	0.257	0
Bal	0.129	0.257	-0.171	-0.086
Sum	-16.972	16.972	-9.175	9.175

$\Rightarrow M_{AC} = -16.972$
 $\therefore$ From symmetry $M_{CA} = 16.972$
 Similarly other end moment can be determined.
 The free body diagram is as shown



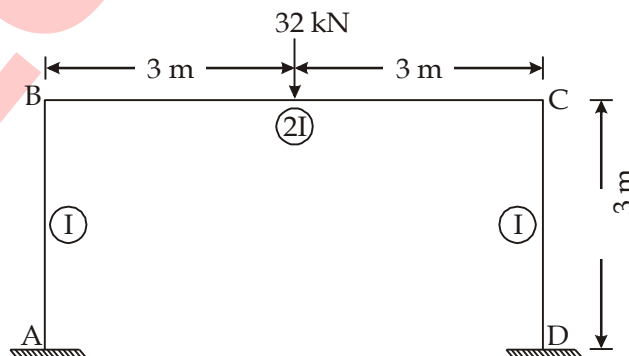
BMD will be (considering sagging as positive and hogging as negative) $\left[M = \frac{PL}{4} = \frac{50 \times 5}{4} = 62.5 \right]$



$$\left[M = \frac{\omega L^2}{8} = \frac{10 \times 25}{8} = 31.25 \right]$$

Q.38

Sol. Due to symmetry in geometry and loading of figure, there will not be any sway in the frame.



Ends A and D are fixed

$$\therefore \theta_A = \theta_D = 0$$

Slope-deflection equations,

$$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$$

$$M_{FBC} = -\frac{32 \times 6}{8} = -24 \text{ kN-m}$$

$$M_{FCB} = +\frac{32 \times 6}{8} = +24 \text{ kN-m}$$

Joint B,

$$M_{BA} = M_{FBA} + \frac{2EI}{3} (2\theta_B + 0) = \frac{4}{3} EI\theta_B$$

$$\begin{aligned} M_{BC} &= M_{FBC} + \frac{2E(2I)}{6} (2\theta_B + \theta_C) \\ &= -24 + \frac{2EI}{3} (2\theta_B + \theta_C) \\ &= \frac{4}{3} EI\theta_B + \frac{2}{3} EI\theta_C - 24 \end{aligned}$$

But, $M_{BA} + M_{BC} = 0$

$$\Rightarrow \frac{4}{3} EI\theta_B + \frac{2}{3} EI\theta_C - 24 + \frac{4}{3} EI\theta_B = 0$$

$$\Rightarrow 4EI\theta_B + EI\theta_C = 36 \quad \dots(i)$$

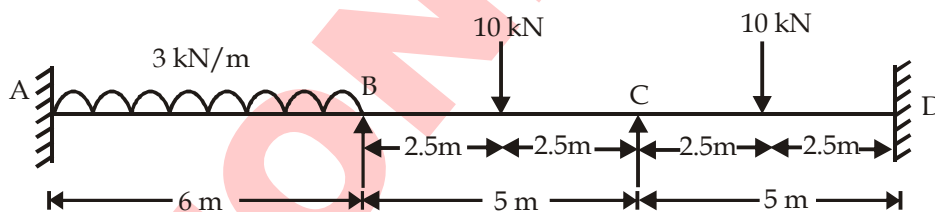
Joint C,

$$\theta_B = \frac{12}{EI} \quad (\text{Clockwise}) (\because \theta_B = -\theta_C)$$

Hence, $\theta_C = -\frac{12}{EI} \quad (\text{Anticlockwise})$

$$\therefore \theta_A = 0, \theta_B = \frac{12}{EI}, \theta_C = (-ve) \frac{12}{EI}, \theta_D = 0$$

Q.39
Sol.



(A) Fixed end moment

$$M_{FAB} = \frac{-wl^2}{12} \Rightarrow \frac{-3 \times 6^2}{12} = -9 \text{ kN-m}$$

$$M_{FBA} = \frac{+wl^2}{12} \Rightarrow \frac{+3 \times 6^2}{12} = 9 \text{ kN-m}$$

$$M_{FBC} = \frac{-wl}{8} \Rightarrow \frac{-10 \times 5}{8} = -6.25 \text{ kN-m}$$

$$M_{FCB} = \frac{+wl}{8} \Rightarrow \frac{10 \times 5}{8} = +6.25 \text{ kN-m}$$

$$M_{FCD} = \frac{-wl}{8} \Rightarrow \frac{-10 \times 5}{8} = -6.25 \text{ kN-m}$$

$$M_{FDC} = \frac{+wl}{8} \Rightarrow \frac{+10 \times 5}{8} = +6.25 \text{ kN-m}$$

(b) Slope deflection equations:

θ_A and θ_D are zero because end A & D are fixed.

$$M_{AB} = \frac{2EI}{6}[\theta_B] - 9 \quad \dots(i)$$

$$M_{BA} = \frac{2EI}{6}[2\theta_B] + 9 \quad \dots(ii)$$

$$M_{BC} = \frac{2EI}{5}[2\theta_B + \theta_C] - 6.25 \quad \dots(iii)$$

$$M_{CB} = \frac{2EI}{5}[2\theta_C + \theta_B] + 6.25 \quad \dots(iv)$$

$$M_{CD} = \frac{2EI}{5}[2\theta_C] - 6.25 \quad \dots(v)$$

$$M_{DC} = \frac{2EI}{5}[\theta_C] + 6.25 \quad \dots(vi)$$

(c) Equilibrium Equation:-

at joint B, $M_{BA} + M_{BC} = 0$

$$\frac{2EI}{6}[2\theta_B] + 9 + \frac{2EI}{5}[2\theta_B + \theta_C] - 6.25 = 0$$

$$EI \left[\frac{4\theta_B}{6} + \frac{4\theta_B}{5} + \frac{2}{5}\theta_C \right] + 2.75 = 0$$

$$EI \left[\frac{22}{15}\theta_B + \frac{2}{5}\theta_C \right] + 2.75 = 0 \quad \dots(vii)$$

at joint C, $M_{CB} + M_{CD} = 0$

$$\frac{2EI}{5}[2\theta_C + \theta_B] + 6.25 + \frac{2EI}{5}[2\theta_C] - 6.25 = 0$$

$$\frac{3EI}{5}[4\theta_C + \theta_B] = 0$$

$$EI \left[\frac{8}{5}\theta_C + \frac{2\theta_B}{5} \right] = 0 \quad \dots(viii)$$

By equation (7) and (8)

$$\theta_B = \frac{-165}{82EI}$$

$$\theta_C = \frac{165}{328EI}$$

After substitute θ_B & θ_C is slope deflection equations.

$$M_{AB} = -9.67 \text{ kN-m}$$

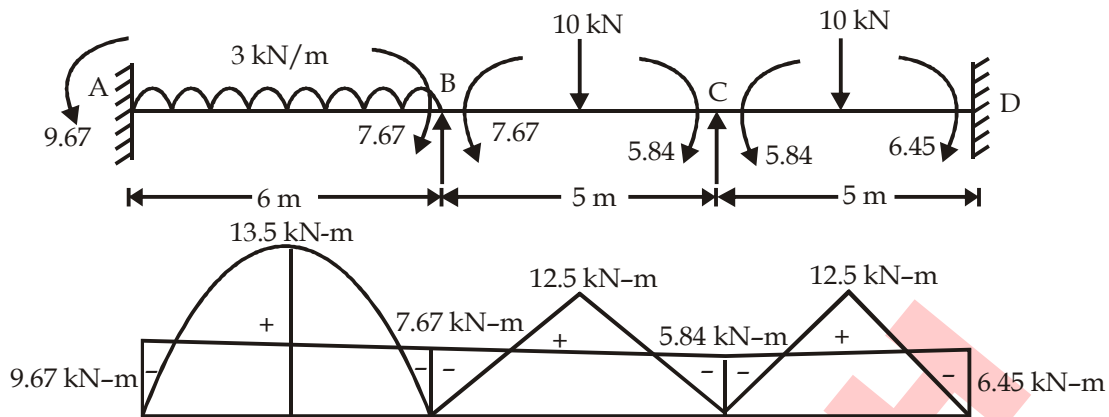
$$M_{BC} = -7.65 \text{ kN-m}$$

$$M_{CD} = -5.84 \text{ kN-m}$$

$$M_{BA} = +7.65 \text{ kN-m}$$

$$M_{CB} = +5.84 \text{ kN-m}$$

$$M_{DC} = +6.45 \text{ kN-m}$$



Q.40

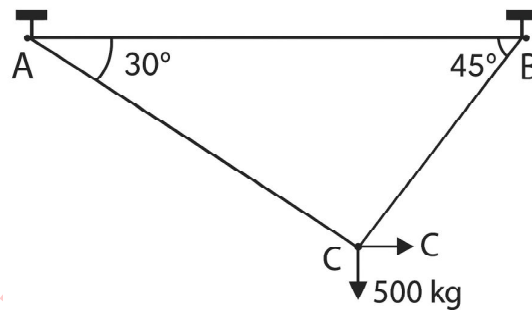
Sol. Given:

Diameter of bars = 8 mm = 0.8 cm

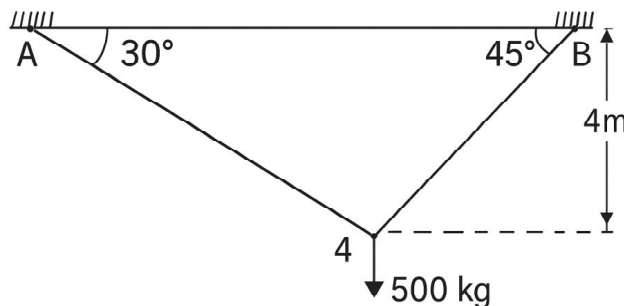
 $E = 2 \times 10^6 \text{ kg/cm}^2$

$$L_{CA} = \frac{4}{\sin 30^\circ} = 8 \text{ m}$$

$$L_{CB} = \frac{4}{\sin 45^\circ} = 4\sqrt{2} \text{ m}$$



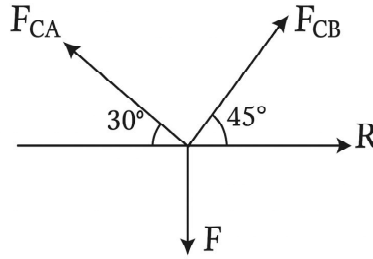
To find out the vertical and horizontal deflection component of deflection of joint C, we will use castigliano theorem.



$$\text{Vertical deflection of joint C} = \frac{\partial U}{\partial F} \bigg|_{\substack{F=500 \text{ kg} \\ R=0}}$$



Horizontal deflection of joint C = $\frac{\partial U}{\partial R} \bigg|_{\substack{F=500 \text{ kg} \\ R=0}}$



U = Strain energy of system

$$U = U_{CA} + U_{CB}$$

$$U_{CA} = \frac{F_{CA}^2 l_{CA}}{2AE}$$

$$U_{CB} = \frac{F_{CB}^2 l_{CB}}{2AE}$$

$$\sum F_V = 0$$

$$\Rightarrow F_{CA} \sin 30^\circ + F_{CB} \sin 45^\circ - F = 0$$

$$\sum F_H = 0$$

...(i)

$$\Rightarrow -F_{CA} \cos 30^\circ + F_{CB} \cos 45^\circ + R = 0$$

...(ii)

From (i) and (ii)

$$F_{CA} (\sin 30^\circ + \cos 30^\circ) - F - R = 0$$

$$F_{CA} = \frac{F + R}{\sin 30^\circ + \cos 30^\circ} = 0.732(F + R)$$

$$F_{CB} = \frac{F - F_{CA} \sin 30^\circ}{\sin 45^\circ} = -0.518(F + R) + 1.414F$$

$$F_{CB} = 0.896F - 0.518R$$

$$\Rightarrow U = \frac{F_{CA}^2 l_{CA}}{2AE} + \frac{F_{CB}^2 l_{CB}}{2AE}$$

$$AE = \frac{\pi}{4} (0.8)^2 \text{ cm}^2 \times 2 \times 10^6 \text{ kg / cm}^2 = 100.48 \times 10^4 \text{ kg}$$

$$\Rightarrow U = \frac{F_{CA}^2 \text{ kg}^2 \times 800 \text{ cm}}{2 \times 100.48 \times 10^4 \text{ kg}} + \frac{F_{CB}^2 \text{ kg}^2 \times 565.685 \text{ cm}}{2 \times 100.48 \times 10^4 \text{ kg}}$$

$$U = \frac{F_{CA}^2}{2 \times 1256} \text{ kgcm} + \frac{F_{CB}^2}{2 \times 1776.253} \text{ kgcm}$$

$$\frac{\partial U}{\partial F} = \left[\frac{2F_{CA} \cdot \frac{\partial F_{CA}}{\partial F}}{2 \times 1256} + \frac{2F_{CB} \cdot \frac{\partial F_{CB}}{\partial F}}{2 \times 1776.253} \right] \text{ cm}$$

$$\frac{\partial U}{\partial R} = \left[\frac{2F_{CA} \cdot \frac{\partial F_{CA}}{\partial R}}{2 \times 1256} + \frac{2F_{CB} \cdot \frac{\partial F_{CB}}{\partial R}}{2 \times 1776.253} \right] \text{cm}$$

$$\Delta V_C = \frac{\partial U}{\partial F} \bigg|_{\substack{F=500 \text{ kg} \\ R=0}} = \frac{2 \times 0.732(500) \times 0.732}{2 \times 1256} \text{cm} + \frac{2 \times (0.896 \times 500) \times (0.896)}{2 \times 1776.253}$$

$$\Delta V_C = \frac{0.427 \text{ cm} \times 0.452 \text{ cm}}{2}$$

$$\Delta V_C = \frac{0.879 \text{ cm}}{2} = \frac{8.79 \text{ mm}}{2} = 4.395 \text{ mm} \downarrow$$

$$\begin{aligned} \Delta_{HC} &= \frac{\partial U}{\partial R} \bigg|_{\substack{F=500 \text{ kg} \\ R=0}} = \frac{2 \times 0.732(500) \times 0.732}{2 \times 1256} + \frac{2 \times 0.896 \times 500 \times (-0.518)}{2 \times 1776.253} \\ &= \frac{0.427 - 0.260}{2} = 0.0835 \text{ cm} = 0.835 \text{ mm} (\rightarrow) \end{aligned}$$

(+)ve value of deflections mean that deflections are in the direction of force F and R i.e., downward and rightward.