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PART - A

2 Marks Questions

1.

Sol. • **Normal Stress (σ):**
It is the stress developed in a body when the load acts **perpendicular (normal)** to the cross-sectional area.
$$\sigma = P / A$$

It causes either **tensile** or **compressive** deformation along the line of action of load.

• **Shear Stress (τ):**
It is the stress developed when the load acts **tangentially** or **parallel** to the surface.
$$\tau = V / A$$

It causes **sliding** or **angular distortion** in the material.
2.

Sol. **Difference between Normal Stress and Normal Strain**

• **Normal Stress (σ):**
It is the **intensity of internal resisting force** set up per unit area, when the material is subjected to normal loading.
Unit: N/mm^2

• **Normal Strain (ϵ):**
It is the **ratio of change in length to the original length** produced by the action of normal stress.
$$\epsilon = \Delta L / L$$

It is a dimensionless quantity.
3.

Sol. **Difference between Nominal Stress and Actual Stress**

• **Nominal Stress:**
It is based on the **original cross-sectional area** before any deformation occurs.
$$\sigma_{\text{nominal}} = P / A_0$$

It is used for design calculations.

• **Actual Stress:**
It is based on the **instantaneous or deformed area** under the applied load.
$$\sigma_{\text{act}} = P / A_{\text{inst}}$$

After necking in tension test, actual stress is greater than nominal stress.
4.

Sol. **Difference between Yield Stress and Ultimate Stress**

• **Yield Stress:**
It is the **minimum stress** at which the material starts **permanent deformation** (plastic flow) without further increase in load.

• **Ultimate Stress:**
It is the **maximum stress** that the material can withstand before necking or failure occurs.
It represents the **maximum load carrying capacity** of the material.
5.

Sol. **Definitions**

(a) **Elasticity**
Elasticity is the property of a material by virtue of which it **regains its original shape and size** after removal of the applied load.

(b) **Elastic Limit**

It is the **maximum stress** up to which the material obeys Hooke's law and regains its original shape completely on unloading.

(c) Proof Stress

It is the stress corresponding to a specified **permanent (plastic) strain**, generally 0.2% or 0.1% strain, used for materials without a clear yield point.

(d) Factor of Safety (F.O.S.)

It is the ratio of **ultimate (or yield) stress** to the **working (permissible) stress**.

F.O.S. = Ultimate or Yield Stress / Working or permissible stress

6.

Sol. Relation between E, G and K

For an isotropic material:

$$E = 2G(1 + \mu)$$

$$E = 3K(1 - 2\mu)$$

Eliminating Poisson's ratio (μ):

$$E = 9KG / (3K + G)$$

Where,

E = Modulus of Elasticity

G = Modulus of Rigidity

K = Bulk Modulus

μ = Poisson's Ratio

7.

Sol. Poisson's Ratio

Poisson's ratio (μ or $1/m$) is the ratio of **lateral strain to longitudinal strain** under uniaxial loading.

μ = Lateral strain / Longitudinal strain

It indicates how much a material contracts laterally when stretched longitudinally.

For most metals, $\mu = 0.25$ to 0.35 .

8.

Sol. Bulk Modulus (K)

As per Hooke's law under hydrostatic stress condition, Bulk modulus is the ratio of **Normal Stress** to the **volumetric strain** it produces. It measures the material's **resistance to uniform compression**. Higher value of K indicates that the material is **incompressible**.

9.

Sol. Volumetric Strain

Volumetric strain (ϵ_v) is the **change in volume with respect to unit original volume**.

$$\epsilon_v = \Delta V / V$$

For a rectangular element under 3D stress:

$$\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$$

It represents total dimensional change in all directions.

10.

Sol. Let's carefully calculate step by step.

Given data:

- Width $b = 150 \text{ mm} = 0.15 \text{ m}$
- Depth $d = 300 \text{ mm} = 0.30 \text{ m}$
- Bending moment $M = 100 \text{ kNm} = 100 \times 10^6 \text{ kNmm}$

Moment of inertia of rectangular section

$$I = \frac{bd^3}{12}$$



$$I = \frac{150 \times 300^3}{12} = \frac{150 \times 27,000,000}{12} = 337.5 \times 10^6 \text{ mm}^4$$

Distance of extreme fiber from neutral axis

$$y = \frac{d}{2} = \frac{300}{2} = 150 \text{ mm}$$

Bending stress formula

$$\sigma = \frac{My}{I}$$

Substitute values:

$$\sigma = \frac{(100 \times 10^6) \times 150}{337.5 \times 10^6}$$

$$\sigma = \frac{15,000 \times 10^6}{337.5 \times 10^6} = 44.44 \text{ N/mm}^2$$

$$\sigma = 44.44 \text{ MPa}$$

11.

Sol. Pure Bending

When a beam is subjected to a **constant bending moment** and **zero shear force** over a length, it is said to be under **pure bending**.

Under this condition, bending stresses are uniform and no shear stresses exist.

12.

Sol. Neutral Axis and Moment of Resistance

- **Neutral Axis (N.A.):**

It is the axis in the beam's cross-section where **bending stress is zero**. Fibres along the N.A. do not change in length.

- **Moment of Resistance (MR):**

The internal resisting moment developed by the stresses in a cross-section, equal to the applied bending moment.

$$MR = \sigma_{\max} \cdot Z$$

where $Z = I / y_{\max}$ is the section modulus.

13.

Sol. Bending Stress Formula

From simple bending theory:

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Therefore,

$$\sigma = \frac{M \times y}{I}$$

This shows that **bending stress varies linearly** with distance from the neutral axis.

14.

Sol. Relation between Bending Moment and Bending Stress

Bending moment is directly proportional to bending stress:

$$M = \frac{\sigma_{\max} \times I}{y} = \sigma_{\max} \cdot Z$$

Thus,

$$\sigma = \frac{M.y}{I}$$

Greater the bending moment, higher will be the bending stress in the beam.

15.

Sol. Uniform Bending

Uniform bending occurs when **bending stress is constant** along a portion of the beam.

For example, in a simply supported beam with two equal loads symmetrically placed, the central portion bends to form an arc of a circle - called **uniform bending**.

16.

Sol. Shear Stresses in Beams

When two force couple acting on two parallel face of the body in such a way that they are same in magnitude but opposite in nature known as shear force, that is responsible for shear stress.

The shear stress at any layer is:

$$\tau = \frac{V \times Q}{I \times b}$$

Where:

V = Shear force,

Q = First moment of area about N.A.,

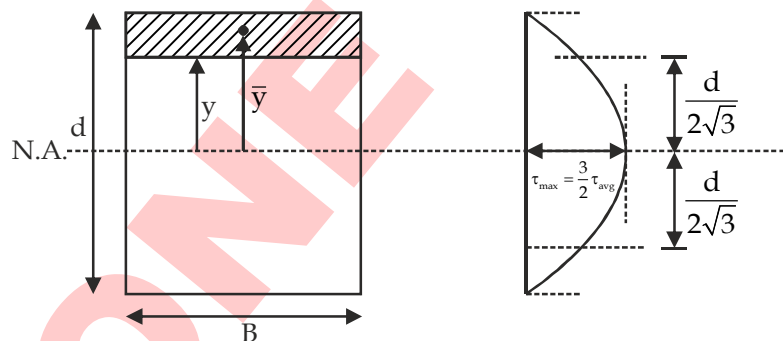
I = Moment of inertia,

b = Width of section.

Shear stress varies parabolically in rectangular or circular beams.

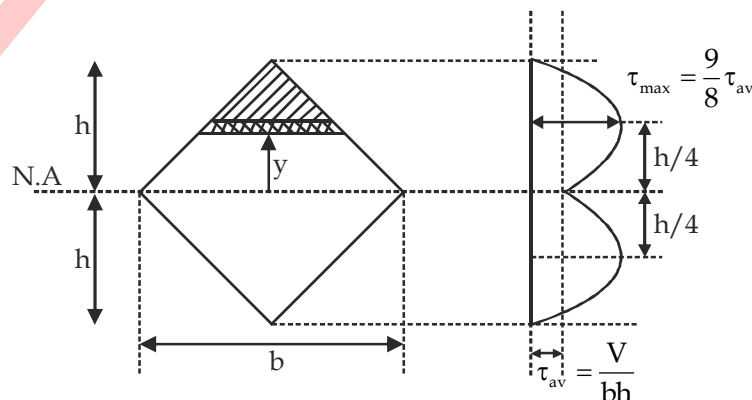
17.

Sol. a. Rectangle Section

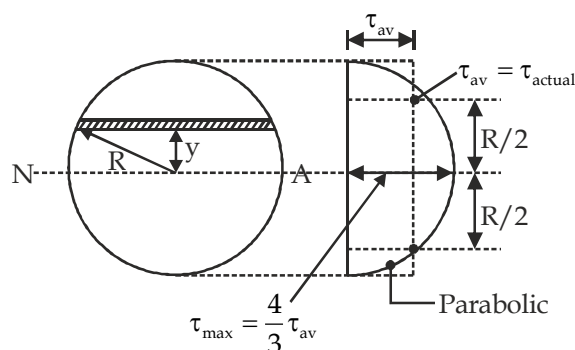


b. Square Section : Same as Rectangle Section

c. Square section with diagonal vertical



d. Circular section



18.

Sol. **Torsion, Torsional Rigidity, and Polar Moment of Inertia**• **Torsion:**

The twisting of a shaft when subjected to a torque about its longitudinal axis.

• **Torsional Rigidity:**

The torque required to produce a unit angle of twist per unit length.

Torsional Rigidity = $TL / \theta = GJ$ • **Polar Moment of Inertia (J):**

The moment of inertia of the cross-sectional area about its polar axis.

For solid circular shaft, $J = \frac{\pi d^4}{32}$

19.

Sol. **Assumptions in Pure Torsion Theory**

1. Material is homogeneous and isotropic.
2. Cross-sections are circular and remain plane and circular after twisting.
3. Radial lines remain straight and radial.
4. Twist is uniform along the length.
5. Stress is within the elastic limit and obeys Hooke's law.
6. The shaft is of uniform diameter.

20.

Sol. **Strength of Shaft**The strength of a shaft is its ability to **transmit torque safely** without failure due to shear stress or excessive twist.

Design is based on:

- Maximum shear stress theory, and
- Angle of twist limit (torsional rigidity).

21.

Sol.

Given :

Diameter of the shaft,

$$D = 150 \text{ mm}$$

Maximum shear stress,

$$\tau = 45 \text{ N/mm}^2$$

Let

T = Maximum torque transmitted by the shaft.

Using equation,

$$T = \frac{\pi}{16} \tau D^3 = \frac{\pi}{16} \times 45 \times 150^3$$

$$= 29820586 \text{ N-mm} = 29820.586 \text{ N-m.}$$

22.

Sol.

Given:

Outer diameter, $D_o = 20 \text{ cm} = 200 \text{ mm}$ Inner diameter, $D_i = 10 \text{ cm} = 100 \text{ mm}$ Maximum shear stress, $\tau = 40 \text{ N/mm}^2$ Let T = Maximum torque transmitted by the shaft.

Using equation,

$$\begin{aligned}
 T &= \frac{\pi}{16} \tau \left[\frac{D_o^4 - D_i^4}{D_o} \right] = \frac{\pi}{16} \times 40 \left[\frac{200^4 - 100^4}{200} \right] \\
 &= \frac{\pi}{16} \times 40 \left[\frac{16 \times 10^8 - 1 \times 10^8}{200} \right] = 58904860 \text{ N-mm} \\
 &= 58904.86 \text{ N-m.}
 \end{aligned}$$

23.

Sol.

Given data:

- Type: Cantilever beam
- Length $L = 2 \text{ m}$
- Load: Triangularly distributed load (UDL varying from 0 at free end to 25 kN/m at fixed end)
- Maximum intensity at fixed end $w_{\max} = 25 \text{ kN/m}$

Step 1: Type of loadSince the load varies linearly from 0 to $w_{\max}$, it forms a triangular load.Hence, the total load (W) on the beam is given by the area of the triangle:

$$W = \frac{1}{2} \times \text{base} \times \text{height}$$

$$W = \frac{1}{2} \times L \times w_{\max}$$

$$W = \frac{1}{2} \times 2 \times 25 = 25 \text{ kN}$$

Step 2: Shear force at the fixed end

For a cantilever with triangular load (zero at free end, maximum at fixed end):

Shear Force at fixed end = Total Load = W

Because the shear at the fixed end equals the total resultant load acting downward along the span.

$$V_{\text{fixed}} = 25 \text{ kN}$$

24.

Sol.

Definition:The point of contraflexure in a bending-moment diagram is the point where the **bending moment changes its sign**, i.e., from positive to negative or vice-versa.At this point, the bending moment is **zero**, and the curvature of the beam changes direction.**Example:**In overhanging or continuous beams, the bending-moment curve passes through zero between a region of sagging and one of hogging. That zero-moment point is called the **point of contraflexure**.

25.

Sol.

Definition:A structure is said to be **statically indeterminate** when the number of unknown reactions or internal forces exceeds the number of available equilibrium equations.For a **plane truss**,

$$\text{Degree of Indeterminacy} = m + r - 2j$$

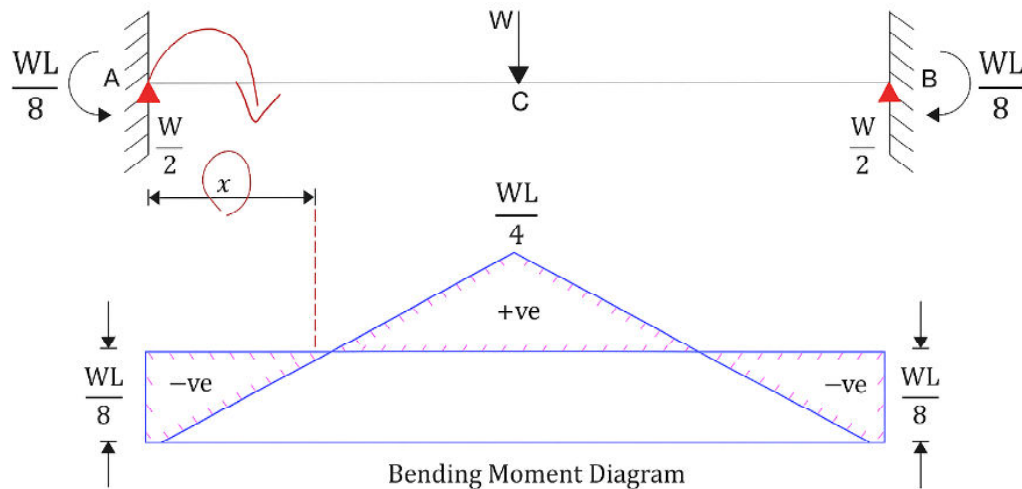
where

 m = no. of members, r = no. of reactions, j = no. of joints.

If this value is greater than zero, the truss is **statically indeterminate**.

26.

Sol.



$$M_x = -\frac{WL}{8} + \frac{W}{2} \times x = 0$$

$$\frac{W}{2} \times x = \frac{WL}{8}$$

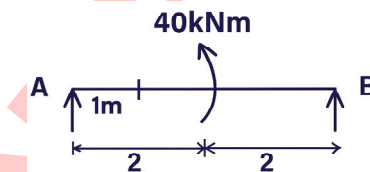
$$x = \frac{WL}{8} \times \frac{2}{W}$$

$$x = \frac{L}{4}$$

RHS
 ↺ +VE
 ↻ -VE

27.

Sol.



$$R_A + R_B = 0$$

$$R_B \times 4 + 40 = 0 \quad [R_B = -10]$$

$$R_A = +10$$

So moment at 1m from left will be

$$M_{at 1m} = 10 \times 1 = +10 \text{ kN-m}$$

28.

Sol.

Shear Force:

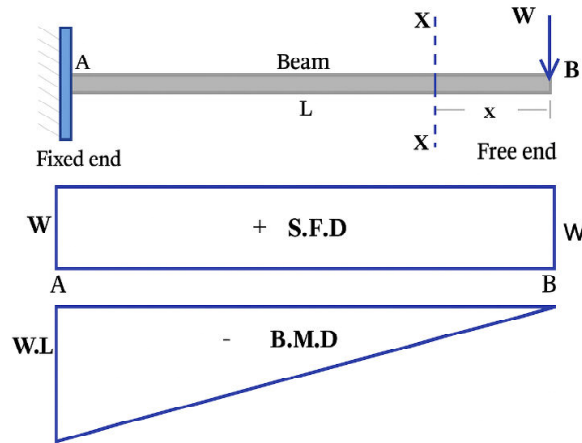
- **Positive (upward)** on the **left** side of a section.
- **Negative (downward)** on the left side (or upward on right side).

Bending Moment:

- **Positive (sagging)** → beam bends concave upward (tension at bottom).
- **Negative (hogging)** → beam bends concave downward (tension at top).

29.

Sol.



30.

Sol. In an overhanging beam, the bending moment changes sign between the overhanging portion (hogging) and the main span (sagging).

Hence, there is **only one point of contraflexure**.

31.

Sol. A **thrust diagram** is a graphical representation of the **axial force** or **horizontal thrust** in an arch or structure along its span.

It shows the variation of compressive or tensile thrust at different points of the structure.

32.

Sol.

Aspect	Point Load	Uniformly Distributed Load (UDL)
Nature of load	Acts at a single point	Acts continuously over the span
Units	kN	kN/m
S.F. Diagram	S.F.D. is rectangular	S.F.D. is linear
B.M. Diagram	Triangular	Parabolic

33.

Sol. According to maxwell theorem

$$60 \times 10 + 20 \times 4 = P \times 5$$

$$600 + 80 = 5P$$

$$P = \frac{680}{5} = 136 \text{ kN.}$$

34.

Sol.

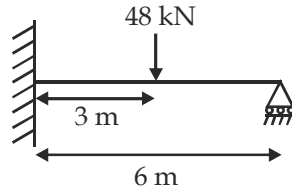
1. Double-integration method
2. Moment-area method
3. Macaulay's method
4. Conjugate-beam method
5. Strain-energy (Castigliano's theorem)

Each gives the same results but varies in computational simplicity depending on loading type.



35.

Sol.



Given :

Length,

Point load,

Let

Using equation, we get

$$L = 6 \text{ m}$$

$$W = 48 \text{ kN}$$

$$P = \text{Reaction at the rigid prop}$$

$$\frac{P \times (6)^3}{3EI} = \frac{48 \times (3)^3}{3EI} + \frac{48 \times (3)^2 \times 3}{2EI}$$

On solving we get

$$P = \frac{5}{16} \times 48 = 15 \text{ kN}$$

36.

Sol. Conjugate Beam:

A fictitious beam having the **same length and loading as the real beam**, but its support conditions are **interchanged** (fixed $\leftrightarrow$ free, hinge $\leftrightarrow$ roller).

The **bending moment diagram of the real beam divided by EI** is treated as loading on the conjugate beam to find slope and deflection.

Flexural Rigidity (EI):

It is the product of the modulus of elasticity (E) and moment of inertia (I) of the beam section.

It indicates the **resistance offered to bending**.

Propped Beam:

A beam that is fixed at one end and **supported (propped)** at the other.

It is **statically indeterminate** and offers less deflection than a cantilever.

37.

Sol. A **propped cantilever** is a beam **fixed at one end** and **simply supported (propped)** at the other end. It combines the stiffness of a fixed beam with the simplicity of a simply supported beam. It has **one degree of indeterminacy**.

38.

Sol. Propping reduces the **deflection and bending moment** in beams.

It provides **additional support** to prevent excessive sagging, especially during construction or in long-span structures.

It enhances **stiffness** and decreases the effective span.

39.

Sol. (i) **Both ends fixed**

$$P_{cr} = \frac{4\pi^2 EI}{L^2}$$

(ii) **One end fixed and the other free**

$$P_{cr} = \frac{\pi^2 EI}{4L^2}$$

Additional:

(iii) **Both ends hinged**



$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

(iv) One end fixed and the other hinged

$$P_{cr} = \frac{2\pi^2 EI}{L^2}$$

40.

Sol. **Column:**

column is a vertical compression member which experience axial compressive load only. For column generally length is **greater than 3 times its least lateral dimension**.

Strut:

A compression member in **any position** (not necessarily vertical) used in frameworks or trusses.

Crippling (Buckling) Load:

The **axial compressive load at which a long column fails suddenly** by lateral buckling.

For elastic columns, it is given by **Euler's formula**.

41.

Sol. **Failure of Short Column:**

A **short column** fails by **crushing** of the material due to direct compressive stress.

The entire cross-section is uniformly stressed and no buckling occurs.

The load at failure is determined by:

$$P = \sigma_c \cdot A$$

where σ_c = crushing stress and A = cross-sectional area.

Failure of Long Column:

A **long column** fails by **buckling (elastic instability)** at a stress much lower than crushing stress.

Under axial compression, it bends laterally and losses equilibrium.

The **Euler's critical (buckling) load** is:

$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

where, E = Young's modulus, I = Least moment of inertia, L_e = Effective length.

42.

Sol. In a **thin-walled cylindrical pressure vessel**, two principal tensile stresses are developed:

1. Hoop (circumferential) stress

Acts tangentially and tends to split the shell longitudinally.

$$\sigma_h = Pd / 2t$$

2. Longitudinal stress

Acts along the length and tends to split the shell circumferentially.

$$\sigma_L = Pd / 4t$$

Since $\sigma_h = 2\sigma_L$, the **hoop stress** is the **maximum** and therefore governs the **design thickness** of the cylinder wall.

43.

Sol. • **Principal Planes:**

Planes in a stressed body on which **shear stress is zero** and only **normal stresses** act are called principal planes.

• **Principal Stresses:**

The **normal stresses** acting on principal planes are called principal stresses.

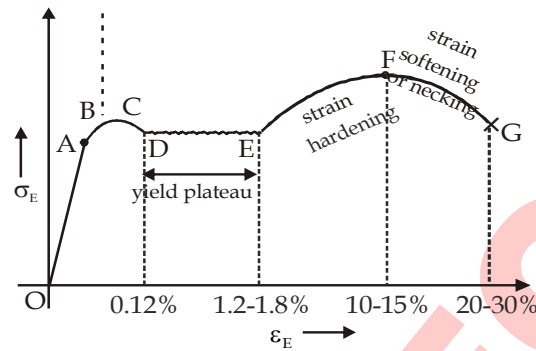
They represent the **maximum and minimum normal stresses** at a point.



PART - B

5 Marks Questions

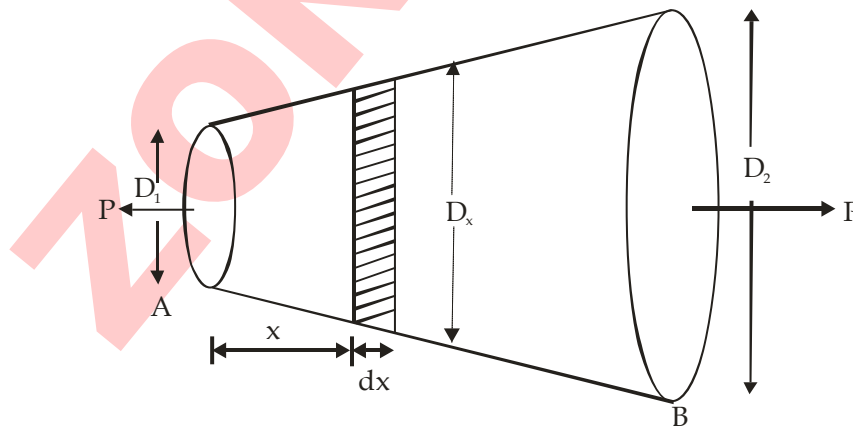
44.
Sol.



OA = Linear curve
A = Proportional limit
B = Elastic limit
C = Upper yield point
D = Lower yield point
DE = Plastic region
EF = Strain hardening region
FG = Necking region
F = Ultimate stress point
G = Fracture point

45.

Sol. Elongation of a uniform tapered bar having circular C/S due to external load 'P'.



Consider a strip at distance x from 'A'

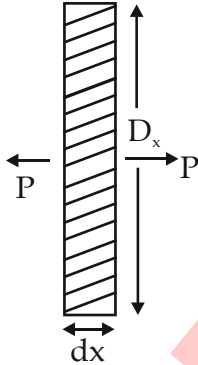
$$\text{Dia of strip} = D_x = D_1 + \left(\frac{D_2 - D_1}{L} \right) x$$

$$D_x = D_1 + kx \quad \left\{ \text{where } k = \frac{D_2 - D_1}{L} \right\}$$

Length of strip = dx

Force acting on strip = $P_x = P$

Then elongation of strip (Assume to be prismatic)



$$d\delta = \frac{Pdx}{A_x E} = \frac{Pdx}{\frac{\pi}{4} (D_x)^2 \times E}$$

$$d\delta = \frac{4Pdx}{\pi E (D_1 + Kx)^2}$$

$$\text{Then total elongation of } \Delta = \int_0^L d\delta = \int_0^L \frac{4Pdx}{\pi E (D_1 + kx)^2} dx$$

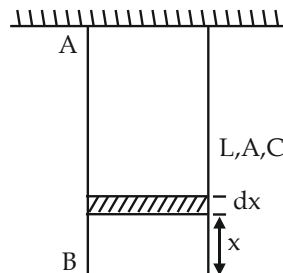
So after integration of this equation between limit of 0 to L we will get

$$\Delta = \frac{4PL}{\pi E D_1 D_2}$$

46.

Sol. Elongation of a prismatic bar due to its self weight

If unit weight of bar is γ



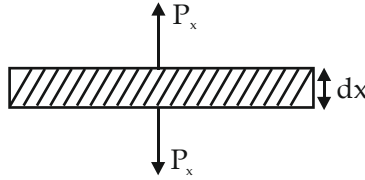
For strip at distance ' x ' from lower end B,

C/S area of strip = A

Length of strip = dx

Total Force acting on strip = $\gamma \cdot A \cdot x$ = Weight of body below strip

$$P_x = (\gamma \times A \times x)$$



then elongation of strip at x

$$d\delta = \frac{P_x \cdot dx}{A \cdot E}$$

$$d\delta = \frac{\gamma A x dx}{A E} = \frac{\gamma x}{E} dx$$

$$\text{Total elongation of bar, } \Delta L = \int_0^L dx = \int_0^L d\delta = \int_0^L \frac{\gamma x}{E} dx$$

$$\Delta L = \frac{\gamma}{E} \left[\frac{x^2}{2} \right]_0^L$$

$$\Delta L = \frac{\gamma \cdot L \cdot L}{2E} = \frac{\gamma \cdot L^2}{2E} \quad \text{Or}$$

$$\Delta L = \frac{\gamma \cdot L \cdot L}{2E} \times \frac{A}{A} = \frac{(\gamma \cdot A \cdot L) \times L}{2AE} = \frac{WL}{2AE}$$

$$\Delta L = \frac{WL}{2AE}$$

Where W = Total weight of bar = $(\gamma \times A \times L)$

47.

Sol. Stresses :

- Normal Stress (σ):** Force per unit area acting **perpendicular** to the surface.
 - Tensile $\rightarrow$ pulls apart.
 - Compressive $\rightarrow$ pushes together.
- Shear Stress (τ):** Force per unit area acting **tangentially**, tending to slide layers.
- Volumetric (Hydrostatic) Stress:** Equal pressure acting on all sides.

Strains:

- Linear Strain:** Change in dimension/original dimension in the direction of applied load.
- Lateral Strain:** Change in lateral dimension/original dimension.
- Volumetric Strain:** Change in volume/original volume.
- Shear Strain:** Angular distortion between layers.

48.

Sol. Stress

$$\sigma = \frac{P}{A} = \frac{17 \times 10^3}{\frac{\pi (12)^2}{4}} = 150.31 \text{ N/mm}^2$$

Strain

$$\epsilon = \frac{\Delta L}{L} = \frac{0.22}{300} = 7.333 \times 10^{-4}$$

$\therefore$

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{150.31}{7.333 \times 10^{-4}} = 2.05 \times 10^5 \text{ N/mm}^2 = 210 \text{ kN/mm}^2$$

49.



Sol. Given: Young's modulus, $E = 1.2 \times 10^8 \text{ N/mm}^2$

Poisson's ratio, $\mu = \frac{1}{4}$

Let

K = Bulk modulus

Using equation,

$$K = \frac{E}{3(1-2\mu)} = \frac{1.2 \times 10^8}{3\left(1-\frac{2}{4}\right)} = \frac{1.2 \times 10^8}{3 \times \frac{1}{2}}$$

$$= \frac{2 \times 1.2 \times 10^8}{3} = 0.8 \times 10^8 \text{ N/mm}^2$$

50.

Sol. RELATION BETWEEN E AND N

We know that linear strain of a diagonal is given by

$$e = \frac{1}{2} \phi = \frac{q}{2N} \quad \dots(i)$$

The state of simple shear produces tensile and compressive stresses along the diagonal planes, and the value of each such direct stress is numerically equal to q . Thus, along diagonal AC, there is a tensile stress $p (= q)$ and along diagonal BD, there is compressive stress $p (= q)$.

Hence, the linear strain e of the diagonal AC, due to two mutually perpendicular direct stresses is given by

$$e = \frac{p}{E} - \left(-\frac{p}{mE} \right) = \frac{p}{E} \left(1 + \frac{1}{m} \right) = \frac{q}{E} \left(1 + \frac{1}{m} \right) \quad \dots(ii)$$

Equating (i) and (ii), we get

$$\frac{q}{2N} = \frac{q}{E} \left(1 + \frac{1}{m} \right)$$

or

$$E = 2N \left(1 + \frac{1}{m} \right)$$

Also,

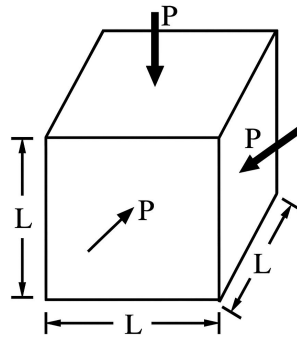
$$N = \frac{mE}{2(m+1)}$$

51.

Sol. BULK MODULUS

When a body is subjected to three mutually perpendicular like stresses of equal intensity (p), the ratio of direct stress (p) to the corresponding volumetric strain (e_v) is defined as the bulk modulus K for the material of the body.





Thus
$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{p}{e_v}$$

RELATION BETWEEN E AND K

Let a cube of side L be subjected to three mutually perpendicular like compressive stresses of equal intensity p .

Total linear strain of each side $= e = \frac{p}{E} - \frac{p}{mE} - \frac{p}{mE}$

Hence
$$\frac{\delta L}{L} = e = \frac{p}{E} \left(1 - \frac{2}{m} \right) \quad \dots(i)$$

Now $V = L^3$

or $\delta V = 3L^2 \delta L$

$\therefore \frac{\delta V}{V} = e_v = \frac{3\delta L}{L} = 3e = \frac{3p}{E} \left(1 - \frac{2}{m} \right) \quad \dots(ii)$

Also, by definition of bulk modulus, $\frac{p}{e_v} = K$

or
$$e_v = \frac{\delta V}{V} = \frac{p}{K} \quad \dots(iii)$$

Equating (ii) and (iii)

$$\frac{p}{K} = \frac{3p}{E} \left(1 - \frac{2}{m} \right)$$

or
$$E = 3K \left(1 - \frac{2}{m} \right)$$

or
$$K = \frac{mE}{3(m-2)}$$

This is the desired relation between E and K in terms of m .

52.

Sol. The following are the important assumptions:

1. The material of the beam is homogeneous and isotropic.
2. The value of Young's modulus of elasticity is the same in tension and compression.
also.
3. The transverse sections which were plane before bending, remain plane after bending
4. The beam is initially straight and all longitudinal filaments bend into circular arcs with a common

centre of curvature.

5. The radius of curvature is large compared with the dimensions of the cross-section.
6. Each layer of the beam is free to expand or contract, independently of the layer, above or below it.

53.

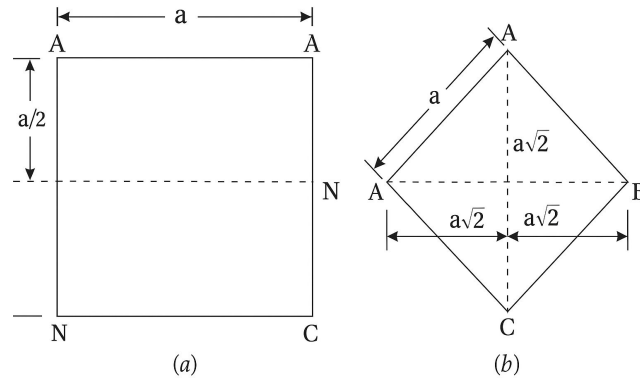
Sol.

Given :

Bending stress = σ

1st Case

Fig. (a) shows the square beam section when two sides are horizontal.



Let M_1 = Moment of resistance of the square beam when two sides are horizontal. Moment of resistance is given by.

$$M = \sigma \times Z$$

$\therefore$

$$M_1 = \sigma \times Z_1$$

where Z_1 = Section modulus

$$Z_1 = \frac{I}{y_{\max}} = \frac{a \times a^3}{\frac{12}{a/2}} = \frac{a^4}{12} \times \frac{2}{a} = \frac{a^3}{6}$$

$\therefore$

$$M_1 = \sigma \times \frac{a^3}{6}$$

2nd Case

Fig.(b) shows the square beam section when one diagonal is vertical.

Let M_2 = Moment of resistance of the beam in this position

$$\therefore M_2 = \sigma \times Z_2$$

where Z_2 = Section modulus for the section shown in Fig. (b)

$$Z_2 = \frac{I_2}{y_{\max}} = \frac{2 \times \frac{bh^3}{12}}{\left(\frac{a}{\sqrt{2}}\right)}$$

($\therefore$ M.O.I. of a triangle about its base = $\frac{bh^3}{12}$. There are two triangles triangles)



$$Z_2 = \frac{\frac{2}{12} \times a\sqrt{2} \left(\frac{a}{\sqrt{2}}\right)^3}{\left(\frac{a}{\sqrt{2}}\right)} \quad (\because \text{Here base} = b = a\sqrt{2} \text{ and } h = \frac{a}{\sqrt{2}})$$

$$Z_2 = \frac{a^3}{6 \times \sqrt{2}}$$

$$\therefore M_2 = \sigma \times \frac{a^3}{6 \times \sqrt{2}}$$

Ratio of moment of resistance of the section in two positions

$$= \frac{M_1}{M_2} = \frac{\left(\frac{\sigma \times a^3}{6}\right)}{\left(\sigma \times \frac{a^3}{6 \times \sqrt{2}}\right)} = \sqrt{2} = 1.414$$

54.

Sol. Moment of inertia of the section about the neutral axis

$$I = \frac{12 \times 2^3}{12} \text{ cm}^4 = 8 \text{ cm}^4$$

As we know that

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

$$\therefore f = \frac{E}{R} \cdot y$$

$$\therefore f_{\max} = \frac{E}{R} y_{\max}$$

$$= \frac{2 \times 10^6}{10 \times 100} \times \left(\frac{2}{2}\right) \text{ kg./cm}^2$$

$$= 2000 \text{ kg./cm}^2$$

$$M = \frac{E}{R} \cdot I$$

$$= \frac{2 \times 10^6}{10 \times 100} \times 8 \text{ kg.cm.}$$

$$= 16000 \text{ kg.cm.}$$

55.

Sol. Given :

Diameter,

$$D = 100 \text{ mm}$$

$\therefore$ Radius,

$$R = \frac{100}{2} = 50 \text{ mm}$$

Shear force,

$$F = 5 \text{ kN} = 5000 \text{ N.}$$

(i) Average shear stress is given by,

$$\tau_{\text{avg}} = \frac{\text{Shear force}}{\text{Area of circular section}}$$

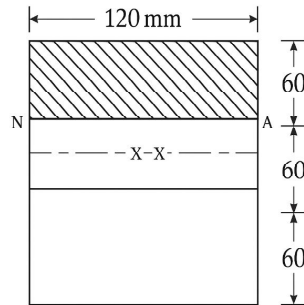
$$= \frac{5000}{\pi(50)^2} = 0.6366 \text{ N/mm}^2$$

(ii) Maximum shear stress for a circular section is given by equation.

$$\begin{aligned} \therefore \tau_{\max} &= \frac{4}{3} \times \tau_{\text{avg}} \\ &= \frac{4}{3} \times 0.6366 = 0.8488 \text{ N/mm}^2. \end{aligned}$$

56.

Sol.



$$I_{xx} = \frac{1}{12} \times 120(180)^3 = 5832 \times 10^4 \text{ mm}^4$$

$$F = \frac{wL}{2} = \frac{w \times 25}{2} = 12.5w \text{ kN}$$

(where w is the safe U.D.L. in N/m)
At the glued joint ab.

$$\begin{aligned} q &= \frac{F}{I_z} (A\bar{y}) = \frac{12.5w}{5832 \times 10^4 \times 120} [120 \times 60 \times 60] \\ &= 7.716 \times 10^{-4} w \text{ N/mm}^2 \end{aligned}$$

But permissible value of shear stress is 0.5 N/mm^2

$$\begin{aligned} \therefore 7.716 \times 10^{-4} w &= 0.5 \\ \text{From which } w &= 648 \text{ N/m} \end{aligned}$$

57.

Sol.

Given :

Shear force,

Base width,

Height,

Maximum shear stress is given by equation,

$$F = 50 \text{ kN} = 50000 \text{ N}$$

$$b = 250 \text{ mm}$$

$$h = 200 \text{ mm}$$

$$\therefore \tau_{\max} = \frac{3F}{bh} = \frac{3 \times 50000}{250 \times 200} = 3 \text{ N/mm}^2$$

Shear stress at N.A. is given by equation,

$$\therefore \tau = \frac{8F}{3bh} = \frac{8 \times 50000}{3 \times 250 \times 200} = 2.67 \text{ N/mm}^2.$$

58.

Sol.

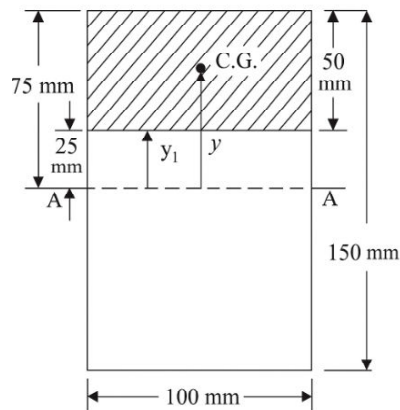
Given :

Width,

$$b = 100 \text{ mm}$$

Depth,
Shear force,

$d = 150 \text{ mm}$
 $F = 4500 \text{ N}$



Let τ = Shear stress at a distance of 25 mm above the neutral axis.

Using equation, we get

$$\tau = F \cdot \frac{A\bar{y}}{I \cdot b} \quad \dots(i)$$

where A = Area of the beam above y_1

$$A = 100 \times 50 = 5000 \text{ mm}^2$$

(Shaded area of Fig.)

$\bar{y}$ = Distance of the C.G. of the area A from neutral axis

$$\bar{y} = 25 + \frac{50}{2} = 50 \text{ mm}$$

I = M.O.I. of the total section

$$I = \frac{bd^3}{12}$$

$$I = \frac{100 \times 150^3}{12} = 28125000 \text{ mm}^4$$

b = Actual width of section at a distance y_1 from N.A. = 100 mm Substituting these values in the above equation (i), we get

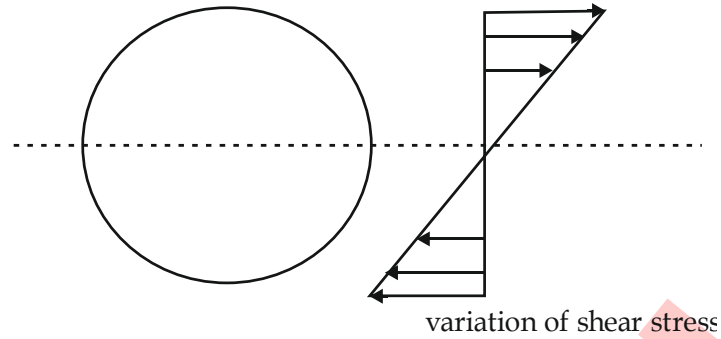
$$\tau = \frac{4500 \times 5000 \times 50}{28125000 \times 100} = 0.4 \text{ N / mm}^2$$

59.

Sol. We know that the equation of torsion is

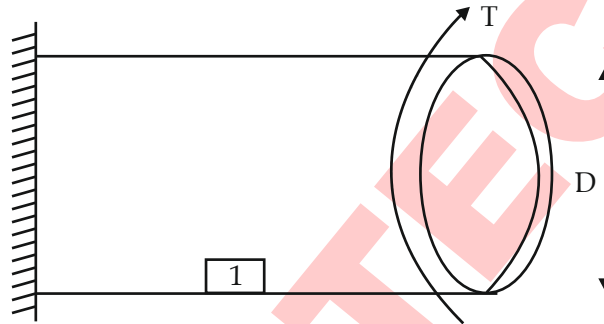
$$\tau_{\max} = \frac{T r_{\max}}{J} = \frac{TR}{J}$$

If the value of torque and polar moment of inertia is constant then $\tau \propto R$ is linear



60.

Sol. As we know that under Pure torsion :-



Maximum shear stress at outer fibre, $\tau = \frac{T r_{\max}}{J}$

where,

$$\text{Polar MOI} = J = \frac{\pi D^4}{32}$$

$$r_{\max} = \frac{D}{2}$$

So, maximum shear stress will be $\tau = \frac{T \times (D/2)}{\frac{\pi D^4}{32}}$

$$\tau = \frac{16T}{\pi D^3}$$

61.

Sol.

Given:

Diameter = 150 mm

Shear stress = 110 N/mm²

Calculate = Maximum Torque

We know that $\tau = \frac{16T}{\pi D^3}$

$$\tau = \frac{16 \times T}{\pi (150)^3} = 110$$

$$\Rightarrow T = 72.89 \text{ kN-m}$$



62.

Sol. The polar modulus is a measure of a shaft's torsional strength, defined as the ratio of its polar moment of inertia to the radius of its outermost fibers. Also called torsional section modulus, it quantifies how effectively a cross-section can resist twisting. It is denoted as Z_p and is calculated using the formula $Z_p = J/r$

Soild shaft

• **Formula :** $Z_p = \frac{\pi d^3}{16}$

• **Where:** d is the diameter of the shaft.

Hollow shaft

• **Formula:** $Z_p = \frac{\pi(D^4 - d^4)}{16D}$

• **Where:**

- D is the outer diameter of the shaft.

- d is the inner diameter of the shaft.

63.

Sol.

Given :

External dia.,

Power,

Speed,

Max. shear stress,

Let

Using equation,

$$D_0 = 120 \text{ mm}$$

$$P = 300 \text{ kW} = 300,000 \text{ W}$$

$$N = 200 \text{ r.p.m.}$$

$$\tau = 60 \text{ N/mm}^2$$

$$D_i = \text{Internal dia. of shaft}$$

$$P = \frac{2\pi NT}{60}$$

$$300,000 = \frac{2\pi \times 200 \times T}{60}$$

$$\therefore T = \frac{300,000 \times 60}{2\pi \times 200} = 14323.9 \text{ N-m}$$

$$= 14323.9 \times 1000 \text{ Nmm} = 14323900 \text{ N-mm}$$

Now using equation,

$$T = \frac{\pi}{16} \times \tau \times \frac{(D_0^4 - D_i^4)}{D_0}$$

$$\text{or } 14323900 = \frac{\pi}{16} \times 60 \times \frac{(120^4 - D_i^4)}{120}$$

$$\text{or } \frac{14323900 \times 16 \times 120}{\pi \times 60} = 120^4 - D_i^4$$

$$145902000 = 207360000 - D_i^4$$

$$\text{or } D_i^4 = 207360000 - 145902000 = 61458000$$

$$\therefore D_i = (61458000)^{1/4} = 88.5 \text{ mm}$$

64.

Sol.

Given :

Dia. of shaft,

$$D = 10 \text{ cm}$$

∴ Area of shaft,

$$A = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2 = 7854 \text{ mm}^2$$

Length of shaft,

$$l = 1.25 \text{ m} = 125 \text{ cm}$$

∴ Volume of shaft,

$$V = A \times l = 78.54 \times 125 = 9817.5 \text{ cm}^3 = 9817.5 \times 10^3 \text{ mm}^3$$

Maximum allowable shear stress,

$$\tau = 50 \text{ N/mm}^2$$

(shear stress is maximum on the surface of the shaft)

Modulus of rigidity,

$$C = 8 \times 10^4 \text{ N/mm}^2$$

Let U = Shear strain energy stored in the shaft.

Using equation, we get

$$U = \frac{\tau^2}{4C} \times V = \frac{50^2}{4 \times 8 \times 10^4} \times 9817.5 \times 10^3$$

$$= 76699 \text{ N-mm.}$$

65.

Sol. Shear force (S.F.)

At any cross-section of a beam, the **shear force** is the algebraic sum of all the transverse forces (loads and reactions) acting on either side of the section.

It represents the tendency of one part of the beam to slide over the other along the section.

Bending moment (B.M.)

At any cross-section of a beam, the **bending moment** is the algebraic sum of the moments of all the forces, taken about that section, on either side of it.

It represents the rotational effect causing the beam to bend, and is usually taken positive when it produces sagging.

66.

Sol. Shear force diagram (S.F.D.)

A shear-force diagram is the **graphical representation of the variation of shear force along the span of a beam**.

On the horizontal axis the distance along the beam is plotted, while ordinates represent the shear force at the corresponding sections, with sign according to the chosen convention.

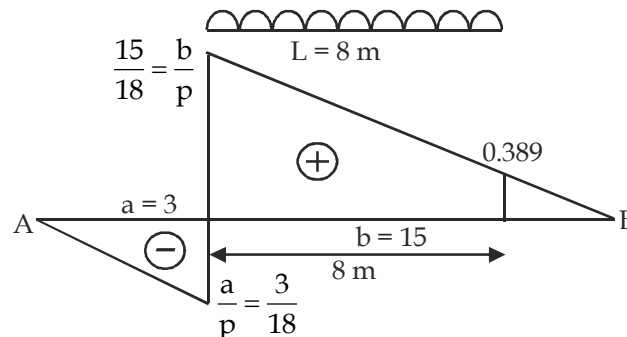
Bending moment diagram (B.M.D.)

A bending-moment diagram is the **graph between bending moment and the distance along the beam**.

It shows how the bending moment changes from one section to another, and is obtained by calculating the bending moment at various key points and joining them by straight or curved lines, depending upon the loading.

67.

Sol.



For maximum negative shear force ordinate $= \frac{a}{l} = \frac{3}{18} = 0.167$

Max negative shear force $\Rightarrow$

keep UDL over negative area of ILD = Load intensity $\times$ Area under ILD

$$= 10 \times \frac{1}{2} \times 3 \times \frac{3}{10}$$

$$= 2.5 \text{ kN}$$

for positive shear force ordinate $= \frac{b}{l}$

$$= \frac{15}{18} = 0.833$$

Max positive shear force $\Rightarrow$ keep UDL over positive area of ILD over 8 m length from max ordinate = Load intensity $\times$ Area under UDL

$$= 10 \times \left[\left(\frac{0.833 + 0.389}{2} \right) \times 8 \right]$$

$$= 48.88 \text{ kN}$$

68.

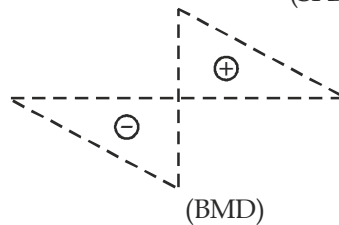
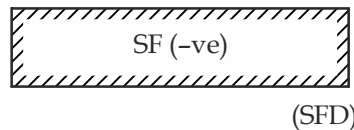
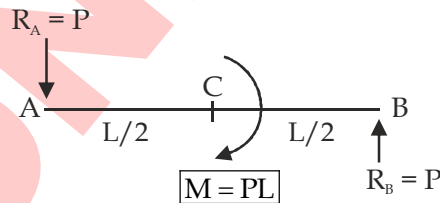
Sol. Step 1:

$$R_A + R_B = 0$$

$$M_A = 0 \Rightarrow R_B \times L = PL$$

$$R_B = P \quad R_A = P$$

$$R_A = P \text{ Sign change}$$



Step 2: SF Calculation

$$(SF)_B = -P$$

$$(SF)_A = -P$$

Step 3: BM Calculation

$$(BM)_A = 0$$

$$(BM)_B = 0$$

$$(BM)_C \text{ without Couple} \Rightarrow P \times \frac{L}{2}$$

$$(BM)_C \text{ with Couple} \Rightarrow P \times \frac{L}{2} - PL = \frac{-PL}{2}$$

69.

Sol. Point of contraflexure

A point of contraflexure on a beam is a section where the bending moment **changes sign**, i.e. where the bending moment is zero and the curve of the bending-moment diagram crosses the base line. At this point the beam curvature changes from sagging to hogging or vice versa.

When the elastic curve of the beam is drawn, the corresponding point on the deflected shape is called the **point of inflexion**. For ordinary beams these two practically refer to the same physical location, so in strength-of-materials problems they are treated as identical.

70.

Sol. For a beam subjected to a distributed load of intensity $w(x)$ kN/m along its length, the rate of change of shear force with respect to the distance x is equal to the **load intensity**.

$$\frac{dV}{dx} = -w(x)$$

i.e. the algebraic difference in shear force between two nearby sections equals the total load acting between those sections.

71.

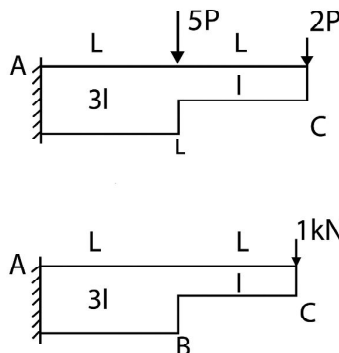
Sol. The rate of change of bending moment along the beam equals the **shear force** at that section:

$$\frac{dM}{dx} = V$$

Thus, if the shear force at a point is zero, the bending moment is either maximum or minimum at that point.

72.

Sol.



Portion	CB	BA
Origion	Ⓒ	Ⓑ
Limit	0 to L	0 to L
M	$(-2P.x)$	$(-5P.x) - 2P(x+2)$
m	$(-1.x)$	$-1(x+1)$
I	I	3I

Deflection at c, y_c will be-

$$y_c = \int_0^L \frac{M.m}{EI} + \int_0^L \frac{M.m}{3EI}$$

$$y_c = \int_0^L \frac{(-2P.x)(-x).dx}{EI} + \int_0^L \frac{[5Px + 2P(x+L)] \cdot [x+L]}{3EI}$$

$$y_c = \frac{2P.L^3}{3EI} + \frac{\frac{7}{3}PL^3 + \frac{9}{2}PL^3 + 2PL^3}{3EI}$$

$$y_c = \frac{65}{18EI}$$

73.

Sol.

Given :

Length, $L = 4 \text{ m} = 4000 \text{ mm}$

Point load at centre = w

Slope at the ends,

$$\theta_A = \theta_B = 1^\circ = \frac{1 \times \pi}{180} = 0.01745 \text{ radians}$$

Let

y_c = Deflection at the centre

Using equation, for the slope at the supports, we get

$$\theta_A = \frac{WL^2}{16EI} \quad \text{(Numerically)}$$

or

$$0.01745 = \frac{WL^2}{16EI} \quad \dots(i)$$

Now using equation, we get

$$y_c = \frac{WL^3}{48EI}$$

$$= \frac{WL^2}{16EI} \times \frac{L}{3} \quad \left(\because \frac{WL^3}{48EI} = \frac{WL^2}{16EI} \times \frac{L}{3} \right)$$

$$= 0.01745 \times \frac{4000}{3} \quad \therefore \frac{WL^2}{16EI} = 0.01745 \text{ from equation (i)}$$

$$= 23.26 \text{ mm.}$$

74.

Sol.

Given:

Length = L

U.d.l. = w



Sinking of prop = δ

The downward deflection (y_1) of the free end of cantilever due to uniformly distributed

load is equal to $\frac{wL^4}{8EI}$

The upward deflection (y_2) of the free end due to prop reaction P will be equal to $\frac{PL^3}{3EI}$.

Now using equation, we get

$$y_1 = y_2 + \delta$$

$$\frac{wL^4}{8EI} = \frac{PL^3}{3EI} + \delta$$

$$\text{or } \frac{PL^3}{3EI} = \frac{wL^4}{8EI} - \delta$$

$$\text{or } P = \frac{3EI}{L^3} \left(\frac{wL^4}{8EI} - \delta \right)$$

75.

Sol. The relations between an actual beam and the corresponding conjugate beam for different end conditions are given in Table-

S. No.	Actual Beam	Conjugate Beam
1.	Simply supported or roller supported end (Deflection = 0 but slope exists)	Simply supported end B.M. = 0 but S.F. exists
2.	Free end (slope and deflection exist)	Fixed end (S.F. and B.M. exist)
3.	Fixed end (slope and deflection are zero)	Free end (S.F. and B.M. are zero)
4.	Slope at any section	S.F. at the corresponding section
5.	Deflection at any section	B.M. at the corresponding section
6.	Given system of loading	The loading diagram is M/EI diagram
7.	B.M. diagram positive (sagging)	M/EI load diagram is positive (i.e., loading is downward)
8.	B.M. diagram negative (hogging)	M/EI load diagram is negative (i.e., loading is upward)

76.

Sol. ADVANTAGES OF FIXED BEAMS

The following are the advantages of a fixed beam over a simply supported beam:

- For the same loading, the maximum deflection of a fixed beam is less than that of a simply supported beam.
- For the same loading, the fixed beam is subjected to a lesser maximum bending moment.
- The slope at both ends of a fixed beam is zero.
- The beam is more stable and stronger.

DISADVANTAGES OF FIXED BEAMS

The following are the disadvantages of a fixed beam over a simply supported beam:

- Any settlement of the support induces significant stress, and temperature variations can cause large stresses.
- A fixed beam is more complex to design and construct properly compared to a simple beam.

77.

Sol. Assumptions in Euler's theory

1. The column is **perfectly straight**, homogeneous and prismatic before loading.
2. The load is **exactly axial**, applied through the centroid of every cross-section.
3. The material is **linearly elastic** and obeys Hooke's law; stresses remain within the elastic limit.
4. The **self-weight** of the column is neglected.
5. Deflections are **small** compared with the length, so that the slope of the elastic curve is small and d^2y/dx^2 may be used.
6. The column fails **only by elastic buckling** and not by crushing or by inelastic instability.
7. End conditions (hinged, fixed, etc.) are **perfectly ideal**, giving the assumed restraints against rotation and translation.

Validity in practice

Real columns are never perfectly straight, loads are seldom exactly central, and end fixity is neither perfectly hinged nor perfectly fixed. There are also residual stresses and local imperfections. Hence Euler's formula gives reliable results only for **long, slender columns** in the elastic range. For short and intermediate columns, or where end conditions are not ideal, Euler's theory by itself is not applicable and empirical formulas such as Rankine's must be used.

78.

Sol. End conditions of a column

End condition denotes the **degree of restraint offered at the ends** of a column against rotation and lateral displacement. It governs the effective length and therefore the buckling load.

Important ideal end conditions are:

1. Both ends hinged (pinned-pinned)

Ends are free to rotate but cannot translate. The column has an effective length equal to its actual length, $L_e = L$

2. Both ends fixed (fixed-fixed)

Ends are restrained against both rotation and lateral displacement. Buckling takes place with points of contraflexure near the ends. Effective length $L_e = L/2$

3. One end fixed and the other hinged (fixed-pinned)

One end is built-in, the other is free to rotate but not to translate. Effective length $L_e = 0.7L$.

4. One end fixed and the other free (fixed-free or cantilever)

One end is built-in and the other is completely free. This is the least stable condition.

Effective length $L_e = 2L$.

These ideal conditions are used while applying Euler and Rankine column formulas.

79.

Sol. Equivalent length (effective length)

The equivalent or effective length L_e of a column is defined as:

The length of an **equivalent pin-ended column** which would have the same critical buckling load as that of the actual column with given end conditions.

For a column of actual length L , $L_e = KL$

where K is the end-condition factor (1 for hinged-hinged, 2 for fixed-free, etc.)

so that any column with arbitrary end conditions can be treated by replacing its actual length by the equivalent length L_e .

80.

Sol. Slenderness ratio

The slenderness ratio of a column is the **ratio of its equivalent length to the least radius of gyration of its cross-section**, i.e.

$$\lambda = L_e / k$$

where L_e is the effective length and k is the least radius of gyration.

Limitations of Euler's formula

1. It is valid only for **long, slender columns** having high slenderness ratio (beyond a certain critical

value).

2. The column must remain **elastic** and stresses must not exceed the proportional limit.
3. It assumes **perfect straightness, central loading and ideal end conditions**, which are not fully realised in practice.
4. It neglects **initial curvature, residual stresses and eccentricity of load**.
5. For **short and intermediate columns** the formula gives unrealistically high and therefore unsafe values of critical load; it cannot predict crushing failure.

Hence Euler's formula is restricted to long columns and must be supplemented by empirical relations for other cases.

81.

Sol. In Rankine-Gordon formula the crippling load of a column is given by

$$\sigma_{cr} = \frac{\sigma_c \times A}{1 + a \left[\frac{L_e}{k} \right]^2}$$

where σ_c is the crushing stress of the material, A is cross-sectional area, L_e/k is the slenderness ratio and a is called **Rankine's constant**.

Thus Rankine's constant is a material constant which empirically combines the effect of end conditions and elastic instability.

Approximate values used in practice are:

- For **cast iron**:
 $A = 1/1600$
- (For comparison, for mild steel, $a = 1/7500$)

82.

Sol. **Explanation:**

For short or long columns Rankine's Formula is used.

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_E}$$

Where P is crippling load by Rankine formula; P_c is crushing load; P_E is crippling load by Euler's formula.

For Short column: P_E is very large so $\frac{1}{P_E}$ will be very small and negligible as compared to $\frac{1}{P_c}$. so

$$\frac{1}{P} = \frac{1}{P_c} \rightarrow P = P_c$$

For Long column: P_E is small so $\frac{1}{P_E}$ will be large as compared to $\frac{1}{P_c}$. Hence the value of $\frac{1}{P_c}$ can be neglected. So

$$\frac{1}{P} = \frac{1}{P_E} \rightarrow P = P_E$$

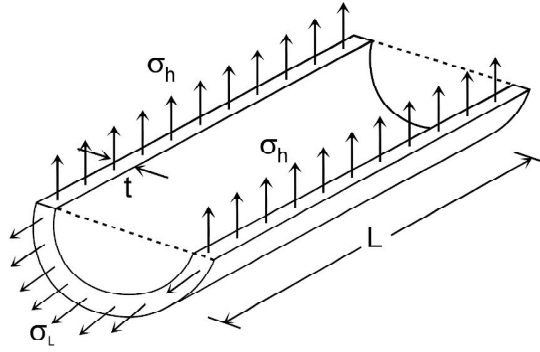
Hence the crippling load by Rankine's formula for a long column is approximately equal to the crippling load by Euler's formula.

83.

Sol. **Concept:**

- Circumferential stress is the stress acting along the circumferential direction, it is generally tensile in nature.
- Longitudinal stress is the stress which acts along the length and it is also tensile in nature whereas radial stress which acts in the direction of the radius is compressive in nature.





Circumferential stress/Hoop stress, $\sigma_h = \frac{PD}{2t}$

Longitudinal stress, $\sigma_L = \frac{PD}{4t}$

Where P = pressure due to flowing fluid, d = diameter, and t = thickness of shell

Calculation:

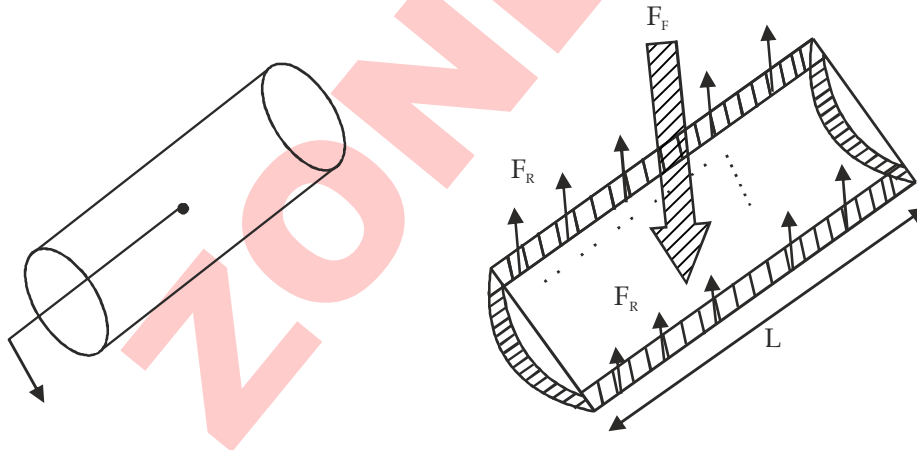
$$\text{Ratio} = \frac{\sigma_h}{\sigma_L} = \frac{PD/2t}{PD/4t} = 2$$

$$\sigma_h = 2\sigma_L$$

In thin cylindrical shell, the value of circumferential stress as compared to the longitudinal stress is Double.

84.

Sol. Hoop stress (σ_h) or circumferential stress:- Stress along the diameter or circumference of cylinder.



At equilibrium

$$F_f = 2F_R$$

$$P \times A = 2 \times \sigma \times A$$

$$(P \times L \times D) = 2 \times \sigma(L \times t)$$

$$\sigma_{\text{hoop}} = \frac{PD}{2t}$$

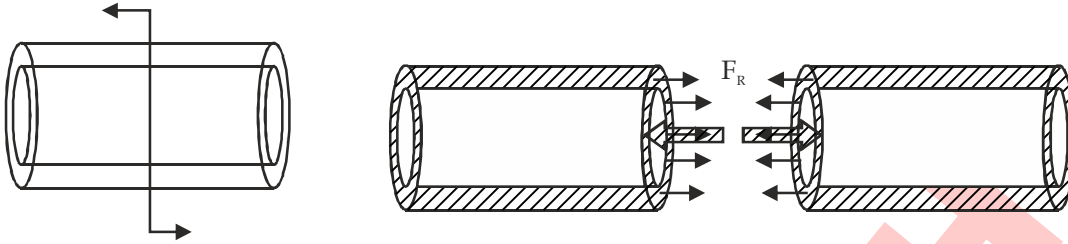
OR



$$\sigma_{\text{hoop}} = \frac{PR}{t}$$

85.

Sol. b. **Longitudinal stress (σ_{long}) or Meridional stress:-** Stress along the length.



In equilibrium condition

$$F_i = F_R$$

$$(P \times A) = (\sigma \times A)$$

$$P \times \frac{\pi D^2}{4} = \sigma \times (\pi D \times t)$$

$$\sigma_{\text{Long}} = \frac{PD}{4t}$$

OR

$$\sigma_{\text{Longi}} = \frac{PR}{2t}$$

86.

Sol. **Explanation:**

Maximum shear stress (τ_{max}) for the biaxial condition is

$$\tau_{\text{max}} = \frac{\sigma_x - \sigma_y}{2}$$

Where τ_{max} = Maximum shear stress, σ_x = Normal stress in the x-direction, σ_y = Normal stress in the y-direction

Derivation:

The shear stress at any angle θ is given by

$$\tau_{xy} = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau \cos 2\theta \quad \dots(i)$$

For maximum shear stress $\frac{d\tau_{xy}}{d\theta} = 0$

So, Differentiating equation (i) we get

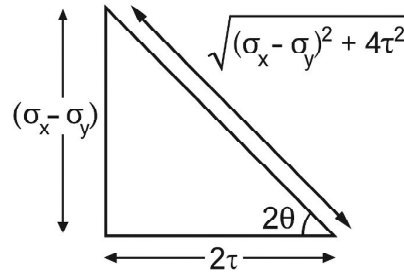
$$\frac{\sigma_x - \sigma_y}{2} (-\cos 2\theta) \times 2 + \tau (\sin 2\theta) \times 2 = 0$$

$$\frac{\sigma_x - \sigma_y}{2} (\cos 2\theta) = \tau (\sin 2\theta)$$

$$\tan 2\theta = \frac{\sigma_x - \sigma_y}{2\tau}$$

This can be assumed as a triangle as shown below





From the above triangle we have

$$\sin 2\theta = \frac{\sigma_x - \sigma_y}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$$

$$\cos 2\theta = \frac{2\tau}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$$

Putting these values in equation (i), we get

$$\tau_{\max} = \frac{\sigma_x - \sigma_y}{2} \times \frac{(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}} + \frac{2\tau^2}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$$

$$\tau_{\max} = \frac{(\sigma_x - \sigma_y)^2 + 4\tau^2}{2\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$$

$$\tau_{\max} = \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}{2}$$

But for biaxial condition $\tau = 0$, So above formula reduces to

$$\tau_{\max} = \frac{\sigma_x - \sigma_y}{2}$$

87.

Sol. Mohr's circle is a **graphical representation of the state of stress at a point**. It is drawn on a coordinate system with abscissa equal to the normal stress (σ) and ordinate equal to the shear stress (τ) on a plane.

For a given biaxial stress system ($\sigma_x, \sigma_y, \tau_{xy}$) the points representing the two mutually perpendicular planes are plotted and a circle is drawn with centre. The intersections of the circle with the x-axis give the **principal stresses** σ_1 and σ_2 , while the maximum ordinate gives the **maximum shear stress** $\tau_{\max}$. Mohr's circle thus provides a convenient graphical method for finding principal stresses, principal planes and maximum shear stress in a complex state of stress.



PART - C

20 Marks Questions

88.

Sol.

$$\text{Area of the rod, } A = \frac{\pi}{4} \times 40^2 = 1256.63 \text{ mm}^2$$

$$\begin{aligned} \text{Tensile stress, } \sigma &= \frac{P}{A} = \frac{80000}{1256.63} \\ \sigma &= 63.66 \text{ N/mm}^2 \end{aligned}$$

$$\text{Strain along the length } (\epsilon_l) = \frac{\sigma}{E} = 9.09 \times 10^{-4}$$

$$\begin{aligned} \therefore \text{Increase in length } (\Delta l) &= \epsilon_l \times L = 9.09 \times 10^{-4} \times 3000 \\ \Delta l &= 2.728 \text{ mm.} \end{aligned}$$

$$\text{Strain along diameter } (\epsilon_d) = -\mu \times \epsilon_l$$

$$\epsilon_d = -\frac{1}{3} \times 9.09 \times 10^{-4}$$

$$\epsilon_d = -3.03 \times 10^{-4}$$

$$\begin{aligned} \therefore \text{Decrease in diameter } (\Delta D) &= \epsilon_d \times D = 3.03 \times 10^{-4} \times 40 \\ \Delta D &= -121.2 \times 10^{-4} \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Now Volumetric strain } (\epsilon_v) &= \epsilon_l + 2\epsilon_d \\ &= 9.09 \times 10^{-4} - 2 \times 3.03 \times 10^{-4} \\ &= 3.03 \times 10^{-4} \end{aligned}$$

$$\text{Hence increase in volume } (\Delta V) = \epsilon_v \times V = 3.03 \times 10^{-4} \times \left(\frac{\pi}{4} \times D^2 \times l \right)$$

$$\Delta V = 3.03 \times 10^{-4} \times \left(\frac{\pi}{4} \times 40^2 \times 3000 \right)$$

$$\Delta V = 1142 \text{ mm}^3$$

89.

Sol.

Given

$$d = 2 \text{ cm ; } l = 100 \text{ cm.}$$

$$\text{Longitudinal strain} = 4 \text{ times lateral strain}$$

$$E = 1 \times 10^6 \text{ kg./cm.}^2 \quad p = 1000 \text{ kg./cm.}^2$$

To find

$$\text{Rigidity modulus } C = ? \quad \text{Bulk modulus } K = ?$$

$$\delta V = ?$$

$$\text{Now, Poisson's ratio} = \frac{1}{m} = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{1}{4} = 0.25$$

$$E = 2C \left(1 + \frac{1}{m} \right)$$

$$C = \frac{E}{2 \left(1 + \frac{1}{m} \right)}$$

$$= 4 \times 10^5 \text{ kg/cm}^2$$

$$E = 3k \left(1 - \frac{2}{m} \right)$$

$$k = \frac{1 \times 10^6}{3(1 - 2 \times 0.25)}$$

$$= \frac{2}{3} \times 10^6 \text{ kg/cm}^2$$

Volumetric strain

$$= \epsilon_v = \frac{p}{K} = \frac{1000}{\frac{2}{3} \times 10^6} = 1.5 \times 10^{-3}$$

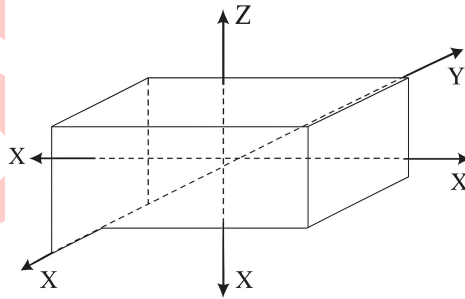
$\therefore$ Decrease in volume $\delta_v = \epsilon_v \times v$

$$= 1.5 \times 10^{-3} \times \frac{\pi}{4} (2)^2 \times 100 \text{ cm}^3$$

$$= 0.471 \text{ cm}^3$$

90.

Sol. Volumetric Strain of a Rectangular Bar Subjected to Three Forces which are Mutually Perpendicular. Consider a rectangular block of dimensions x , y and z subjected to three direct tensile stresses along three mutually perpendicular axis as shown in Fig.



Then volume of block, $V = xyz$. Taking logarithm to both sides, we have

$$\log V = \log x + \log y + \log z.$$

Differentiating the above equation, we get

$$\frac{1}{V} dV = \frac{1}{x} dx + \frac{1}{y} dy + \frac{1}{z} dz$$

or

$$\frac{dV}{V} = \frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}$$

But

$$\frac{dV}{V} = \frac{\text{Change of volume}}{\text{Original volume}} = \text{Volumetric strain}$$

$$\begin{aligned} \frac{dx}{x} &= \frac{\text{Change of dimension } x}{\text{Original dimension } x} \\ &= \text{Strain in the } x\text{-direction} = e_x \end{aligned}$$

Similarly,

$$\frac{dy}{y} = \text{Strain in } y\text{-direction} = e_y$$

and

$$\frac{dz}{z} = \text{Strain in } z\text{-direction} = e_z$$

Substituting these values in equation, we get

$$\frac{dV}{V} = e_x + e_y + e_z$$

Now, Let σ_x = Tensile stress in x-x direction,

σ_y = Tensile stress in y-y direction, and

σ_z = Tensile stress in z-z direction.

E = Young's modulus

μ = Poisson's ratio.

Now σ_x will produce a tensile strain equal to $\frac{\sigma_x}{E}$ in the direction of x, and a compressive strain equal

to $\frac{\mu \times \sigma_x}{E}$ in the direction of y and z. Similarly, σ_y will produce a tensile strain equal to $\frac{\sigma_y}{E}$ in the

direction of y and a compressive strain equal to $\frac{\mu \times \sigma_y}{E}$ in the direction of x and z. Similarly σ_z will

produce a tensile strain equal to $\frac{\sigma_z}{E}$ in the direction of z and a compressive strain equal to $\frac{\mu \times \sigma_z}{E}$ in

the direction of x and y. Hence σ_y and σ_z will produces compressive strains equal to $\frac{\mu \times \sigma_y}{E}$ and

$\frac{\mu \times \sigma_z}{E}$ in the direction of x.

$\therefore$ Net tensile strain along x-direction is given by

$$e_x = \frac{\sigma_x}{E} - \frac{\mu \times \sigma_y}{E} - \frac{\mu \times \sigma_z}{E} = \frac{\sigma_x}{E} - \mu \left(\frac{\sigma_y + \sigma_z}{E} \right)$$

Similarly,

$$e_y = \frac{\sigma_y}{E} - \mu \left(\frac{\sigma_x + \sigma_z}{E} \right)$$

and

$$e_z = \frac{\sigma_z}{E} - \mu \left(\frac{\sigma_x + \sigma_y}{E} \right)$$

Adding all the strains, we get

$$e_x + e_y + e_z = \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z) - \frac{2\mu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$= \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z)(1 - 2\mu)$$

But

$$e_x + e_y + e_z = \text{Volumetric strain} = \frac{dV}{V}$$

$\therefore$

$$\frac{dV}{V} = \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z)(1 - 2\mu)$$

91.

Sol.

Given :

Dimensions of bar

$\therefore$

$\therefore$ Volume,

Load in the direction of x

Load in the direction of y

Load in the direction of z

Value of E

Poisson's ratio,

$$= 300 \text{ mm} \times 100 \text{ mm} \times 40 \text{ mm}$$

$$x = 300 \text{ mm}, y = 100 \text{ mm} \text{ and } z = 40 \text{ mm}$$

$$V = x \times y \times z = 300 \times 100 \times 40$$

$$= 1200000 \text{ mm}^3$$

$$= 5 \text{ kN} = 5000 \text{ N}$$

$$= 6 \text{ kN} = 6000 \text{ N}$$

$$= 4 \text{ kN} = 4000 \text{ N}$$

$$= 2 \times 10^5 \text{ N/mm}^2$$

$$\mu = 0.25$$

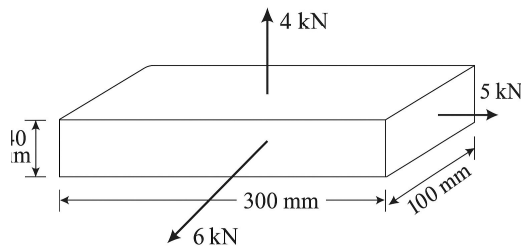


Fig. 2.4

$\therefore$ Stress in the x-direction,

$$\sigma_x = \frac{\text{Load in x-direction}}{y \times z}$$

$$= \frac{5000}{100 \times 40} = 1.25 \text{ N/mm}^2$$

Similarly the stress in y-direction is given by,

$$\sigma_y = \frac{\text{Load in y-direction}}{x \times z}$$

$$= \frac{6000}{300 \times 40} = 0.5 \text{ N/mm}^2$$

And stress in z-direction

$$= \frac{\text{Load in z-direction}}{x \times y}$$

or

$$\sigma_z = \frac{4000}{300 \times 100}$$

$$= 0.133 \text{ N/mm}^2$$

Using equation, we get

$$\frac{dV}{V} = \frac{1}{E} (\sigma_x + \sigma_y + \sigma_z) (1 - 2\mu)$$

$$\frac{dV}{V} = \frac{1}{2 \times 10^5} (1.25 + 0.5 + 0.113) (1 - 2 \times 0.25)$$

$$\frac{dV}{V} = \frac{1.863}{2 \times 10^5 \times 2}$$

$$dV = \frac{1.863}{4 \times 10^5} \times V$$

$$dV = \frac{1.863}{4 \times 10^5} \times 1200000$$

$$dV = 5.589 \text{ mm}^2$$

92.

Sol. The layers above N-N (or N'-N') have been shortened and those below, have been elongated. Due to the decrease in lengths of the layers above N-N, these layers will be subjected to compressive stresses. Due to the increase in the lengths of layers below N-N, these layers will be subjected to tensile stresses.

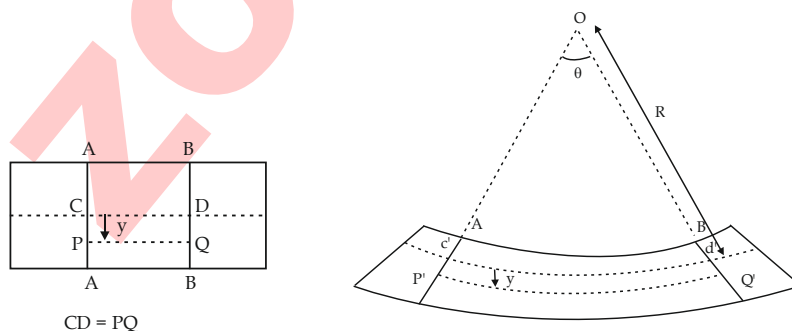
We also see that the top layer has been shortened maximum. As we proceed towards the layer N-N, the decrease in length of the layers decreases. At the layer N-N, there is no change in length. This means the compressive stress will be maximum at the top layer. Similarly the increase in length will be maximum at the bottom layer. As we proceed from bottom layer towards the layer N-N, the increase in length of layers decreases. Hence the amount by which a layer increases or decreases in length, depends upon the position of the layer with respect to N-N. This theory of bending is known as theory of simple bending.

EXPRESSION FOR BENDING STRESS

Fig. shows a small length δx of a beam subjected to a simple bending. Due to the action of bending, the part of length δx will be deformed as shown in Fig. Let A'B' and C'D' meet at O.

Let R = Radius of neutral layer N'N'

θ = Angle subtended at O by A'B' and C'D' produced.



$$CD = PQ$$

$$C'D' = CD \text{ (at NA - No strain, No stress)}$$

$$C'D' = R\theta$$

$$P'Q' = (R + y)\theta$$

$$PQ = C'D' = R\theta$$

Strain in fibre at distance 'y' from NA

$$\epsilon = \frac{P'Q' - PQ}{PQ}$$

$$\epsilon = \frac{(R+y)\theta - R\theta}{R\theta}$$

$$\epsilon = \frac{y}{R}$$

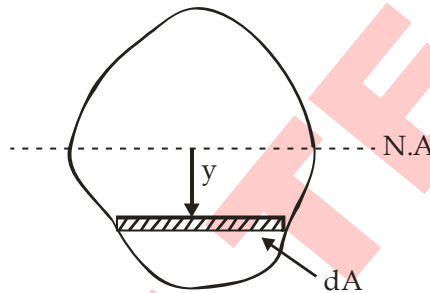
$$\epsilon \propto y$$

where $R \rightarrow$ Radius of curvature

$$\frac{1}{R} \rightarrow \text{Curvature}$$

$$\epsilon = \frac{\sigma}{E} = \frac{y}{R} \dots (1)$$

Stress in fibre at distance 'y' from N.A



$$\sigma = \frac{y}{R} E$$

Force resisted by strip $dF = y \times dF = y \times \sigma \times dA$

$$dM = y \times \frac{yE}{R} dA$$

Total moment carrying capacity of $M = \int dM \text{ section}$

$$= \int \frac{E}{R} y^2 dA$$

$$M = \frac{E}{R} \int y^2 dA$$

$$M = \frac{E}{R} I$$

$$\frac{M}{I} = \frac{E}{R} \dots (ii)$$

From equation (1)

$$\frac{\sigma}{E} = \frac{y}{R}$$

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$$

Bending equation

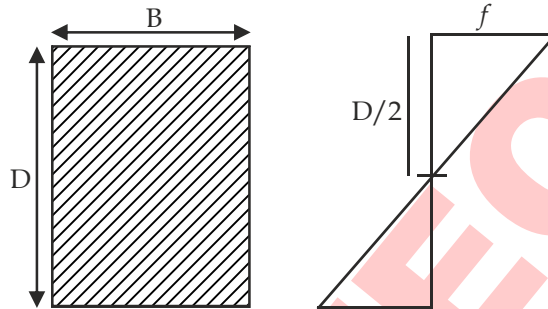
93.

Sol. Section modulus is defined as the ratio of moment of inertia of a section about the neutral axis to the distance of the outermost layer from the neutral axis. It is denoted by the symbol Z . Hence mathematically section modulus is given by - I/Y

Section modulus for various shapes of beam sections

(i) Rectangular section.

Fig. shows a rectangular section of width b and depth d . Let the horizontal centroidal axis be the neutral axis



Section modulus = $Z = \frac{\text{Moment of inertia about the neutral axis}}{\text{Distance of the most distant point of the section from the neutral axis}}$

$$Z = \frac{I}{y_{\max}}$$

But

$$I = \frac{bd^3}{12} \text{ and } y_{\max} = \frac{d}{2}$$

$\therefore$

$$Z = \frac{bd^3}{12} \times \frac{2}{d} = \frac{bd^2}{6}$$

Let f be the maximum stress offered by the beam section.

$\therefore$ Moment of resistance = $M = fZ$

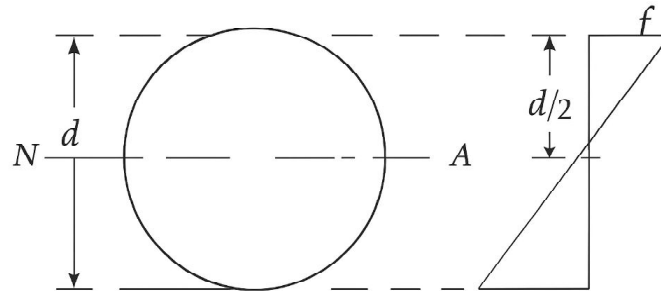
$$= M = f \frac{bd^2}{6}$$

or

$$M = \frac{1}{6} fbd^2$$

(ii) Circular section

Figure shows a circular section of diameter d



Let the diameter of the section be d .

Moment of inertia of the section about the neutral axis = I

$$= \frac{\pi d^4}{64}$$

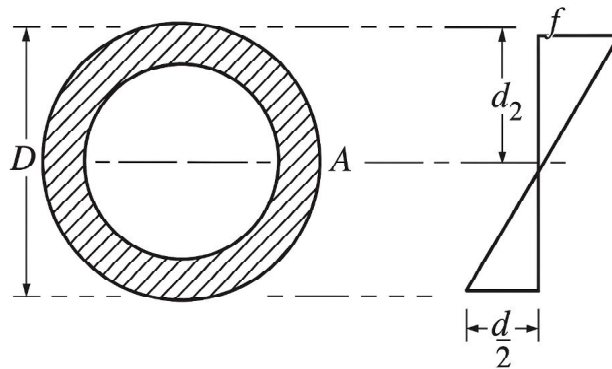
$$y_{\max} = \frac{d}{2}$$

$$\therefore \text{Section modulus} = Z = \frac{I}{y_{\max}} = \frac{\pi d^3}{32}$$

If f be the maximum stress offered by the section, the moment of resistance = $M = fZ$

$$= f \frac{\pi d^3}{32}$$

(iii) Hollow circular section. Fig. shows a hollow circular section of external diameter D and internal diameter d .



Moment of inertia about the neutral axis

$$= I = \frac{\pi}{64} [D^4 - d^4]$$

$$y_{\max} = \frac{D}{2}$$

$$\therefore \text{Section modulus} = Z = \frac{I}{y} = \frac{\pi (D^4 - d^4)}{32D}$$

If f be the maximum stress offered by the section, the moment of resistance = $M = fZ$



94.

Sol.

Given:

External dia.,

$$D = 40 \text{ mm}$$

Internally dia.,

$$d = 20 \text{ mm}$$

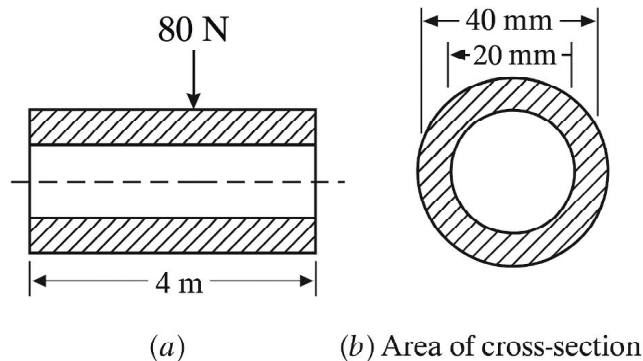
Length,

$$L = 4\text{m} = 4 \times 1000 = 4000 \text{ mm}$$

Point load,

$$W = 80 \text{ N}$$

In case of simply supported beam carrying a point load at the centre, the maximum bending moment is at the centre of the beam.



$$\text{And maximum B.M.} = \frac{W \times L}{4} = 8 \times 10^4 \text{ Nmm}$$

$$\therefore \text{Maximum B.M.} = \frac{80 \times 4000}{4} = 8 \times 10^4 \text{ Nmm}$$

$$\therefore M = 8 \times 10^4 \text{ Nmm}$$

Fig. (b) above shows the cross-section of the pipe.

Moment of inertia of hollow pipe,

$$\begin{aligned} I &= \frac{\pi}{64} (D^4 - d^4) \\ &= \frac{\pi}{64} (40^4 - 20^4) = \frac{\pi}{64} [2560000 - 160000] \\ &= 117809.7 \text{ mm}^4 \end{aligned}$$

Now using equation,

$$\frac{M}{I} = \frac{\sigma}{y}$$

when y is maximum, stress will be maximum. But y is maximum at the top layer from the N.A.

$$\therefore y_{\max} = \frac{D}{2} = \frac{40}{2} = 20 \text{ mm}$$

"The bending stress will be maximum at the section where B.M. is maximum. This is because

$$\frac{M}{I} = \frac{\sigma}{y} \text{ or } \sigma = \frac{M}{I} \times y.$$

Equation (i) can be written as

$$\frac{M}{I} = \frac{\sigma_{\max}}{y_{\max}}$$

∴

$$\begin{aligned}\sigma_{\max} &= \frac{M}{I} \times y_{\max} \\ &= \frac{8 \times 10^4 \times 20}{117809.7} = 13.58 \text{ N/mm}^2\end{aligned}$$

95.

Sol.

Given:

Depth,

$$d = 300 \text{ mm}$$

Span,

$$L = 4 \text{ m}$$

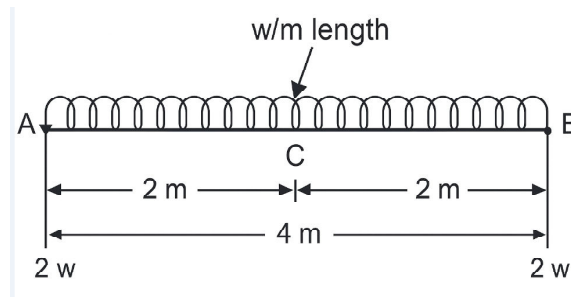
Max. bending stress,

$$\sigma_{\max} = 120 \text{ N/mm}^2$$

Moment of inertia,

$$I = 8 \times 10^6 \text{ mm}^4$$

Let, w = U.D.L. per metre length over the beam in N/m.



The bending stress will be maximum, where bending moment is maximum. For a simply supported beam carrying U.D.L., the bending moment is maximum at the centre of the beam [i.e., at point C of Fig.]

∴

$$\begin{aligned}\text{Max. B.M.} &= 2w \times 2 - 2w \times 1 \\ &= 4w - 2w\end{aligned}$$

$$= 2w \text{ Nm} \quad \left(\text{Also } m = \frac{w \times L^3}{8} = \frac{w \times 4^3}{8} = \frac{16w}{8} = 2w \right)$$

$$= 2w \times 1000 \text{ Nmm}$$

$$M = 2000w \text{ Nmm}$$

or

Now using equation, we get

$$M = \sigma_{\max} \times Z \quad \dots(i)$$

where

$$Z = \frac{I}{y_{\max}} = \frac{8 \times 10^6}{150} \quad \left(\because y_{\max} = \frac{d}{2} = \frac{300}{2} = 150 \text{ mm} \right)$$

Hence above equation (i) becomes as

$$2000w = 120 \times \frac{8 \times 10^6}{150}$$

$$w = \frac{120 \times 8 \times 10^6}{2000 \times 150} = 3200 \text{ N/m}$$

96.

Sol.

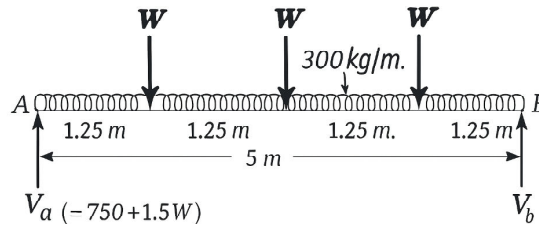


Fig. shows the beam carrying the loading mentioned in the problem.

Each vertical reaction $= \frac{1}{2}$ total load on the span

$$= \frac{300 \times 5 + 3W}{2} = (750 + 1.5W)$$

Max. B.M. will occur at mid span.

Max. B.M.,

$$\begin{aligned} M &= (750 + 1.5W) \times \frac{5}{2} - 300 \times \frac{2.5^2}{2} - W \times 1.25 \\ &= 1875 + 3.75W - 937.50 - 1.25W \\ &= (937.5 + 2.5W) \text{ kg.m.} \\ &= (937.5 + 2.5W) 100 \text{ kg.cm.} \end{aligned}$$

M.R. of the section $= \frac{1}{6} fbd^2$

Stresses in Beams

$$= \frac{1}{6} \times 80 \times 16 \times 30^2 \text{ kg.cm.}$$

Equating the maximum B.M. to the moment of resistance we have,

$$(937.5 + 2.5W) 100 = \frac{1}{6} \times 80 \times 16 \times 30^2$$

$$\therefore 937.50 + 2.5 W = 1920$$

$$\therefore 937.5 + 2.5W = 393 \text{ kg.}$$

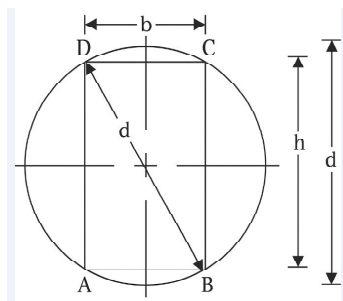
97.

Sol.

Given:

Dia. of log = d

Let ABCD be the strongest rectangular section which can be cut out of the cylindrical log.



Let b = Width of strongest section.

d = Depth of strongest section.



Now section modulus of the rectangular section

$$Z = \frac{I}{y} = \frac{\left(\frac{bh^3}{12}\right)}{\left(\frac{h}{2}\right)} = \frac{bh^2}{6} \quad \dots(i)$$

In the above equation, b and h are variable.

From $\triangle ABCD$,

$$b^2 + h^2 = d^2$$

$$h^2 = d^2 - b^2$$

Substituting the value of h^2 in equation (i), we get

$$Z = \frac{b}{6}[d^2 - b^2] = \frac{1}{6}(bd^2 - b^3) \quad \dots(ii)$$

In the above equation, d is constant and hence only variable is b.

Now for the beam to be strongest, the section modulus should be maximum (or Z should be maximum).

For maximum value of Z,

$$\frac{dZ}{db} = 0$$

$$\text{or } \frac{d}{db} \left[\frac{bd^2 - b^3}{6} \right] = 0 \quad \text{or } \frac{d^2 - 3b^2}{6} = 0$$

$$\text{or } d^2 - 3b^2 = 0 \quad \text{or } d^2 = 3b^2 \quad \dots(iii)$$

But from triangle BCD,

$$d^2 = b^2 + h^2$$

Substituting the value of d^2 in equation (iii), we get

$$b^2 + h^2 = 3b^2 \text{ or } h^2 = 2b^2$$

$$\text{or } h = \sqrt{2} \times b \quad \dots(iv)$$

$$\text{or } \frac{h}{b} = \sqrt{2} = 1.414$$

Numerical Part

Given, $d = 300 \text{ mm}$

But for equation (iii), $d^2 = 3b^2$ or $3b^2 = d^2 = 300^2 = 90000$

$$\text{or } b^2 = \frac{90000}{3} = 30000$$

$$\therefore b = (30000)^{1/2} = 173.2 \text{ mm.}$$

$$\text{From equation (iv) } h = \sqrt{2} b = 1.414 \times 173.2 = 249.95 \text{ mm}$$

98.

Sol.

Given :

Maximum bending stress, $\sigma_{\max} = 12 \text{ N/mm}^2$

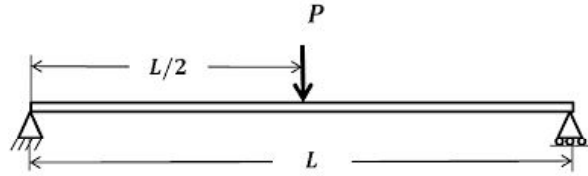
Maximum shear stress, $\tau_{\max} = 1 \text{ N/mm}^2$.

Let b = Width of the beam,

d = Depth of the beam,

L = Span of the beam,

W = Point load at the centre.



Maximum shear force, $F = \frac{W}{2}$

And maximum bending moment = $M = \frac{WL}{4}$

Now average shear stress, $\tau_{avg} = \frac{\text{Shear force}}{\text{Area}} = \frac{\left(\frac{W}{2}\right)}{b \times d} = \frac{W}{2bd}$

Maximum shear stress is given by equation

$$\tau_{max} = 1.5 \times \tau_{avg}$$

or $1 = 1.5 \times \frac{W}{2bd} \quad \left(\because \tau_{max} = 1, \tau_{avg} = \frac{W}{2bd} \right)$

or $\frac{W}{bd} = \frac{2}{1.5} = \frac{4}{3} \quad \dots(i)$

Now using bending equation

$$\frac{M}{I} = \frac{\sigma}{y} \quad \text{or} \quad \sigma = \frac{M}{I} \times y$$

$\therefore$ Maximum bending stress, $\sigma_{max} = \frac{M}{I} \times y_{max}$

$$= \frac{\frac{W \times L}{4} \times \frac{d}{2}}{\frac{bd^3}{12}} \quad \left(\because y_{max} = \frac{d}{2} \right)$$

$$= \frac{12}{8} \times \frac{W.L.d}{bd^3} = 1.5 \frac{W.L}{bd^2}$$

or $12 = 1.5 \frac{W.L}{bd^2} \quad (\because \sigma_{max} = 12)$

$$= 1.5 \frac{W}{bd} \cdot \frac{L}{d}$$

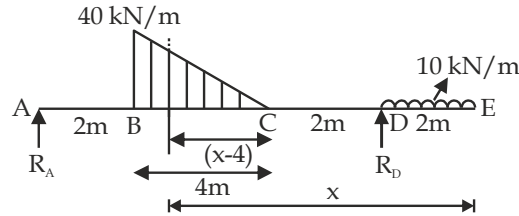
$$= 1.5 \times \frac{4}{3} \times \frac{L}{d} \quad \left[\because \frac{W}{bd} = \frac{4}{3} \text{ from equation (i)} \right]$$

$\therefore 2 \times \frac{L}{d}$

$\therefore \frac{L}{d} = \frac{12}{2} = 6$

99.

Sol.

**Calculation of Reaction R_A and R_D :**

$$R_A + R_D = 20 + \frac{1}{2} \times 40 \times 4$$

$$R_A + R_D = 100 \text{ kN}$$

$$M_A = 0 \Rightarrow (20 \times 9) + \left(\frac{1}{2} \times 40 \times 4 \right) \times \left(2 + \frac{4}{3} \right) = R_D \times 8$$

$$\Rightarrow R_D = 55.83 \text{ kN}$$

$$\Rightarrow R_A = 44.17 \text{ kN}$$

Shear Force Diagram (SFD) Calculation :Section (DE) = $10 \times x$

At point E,

$$\Rightarrow x = 0$$

$$(SF)_E = 0$$

At point D,

$$\Rightarrow x = 2$$

$$(SF)_D = 10 \times 2 = 20 \text{ kN}$$

For Section CD

$$(SF)_D = 20 - 55.83 = -35.83 \text{ kN}$$

$$(SF)_C = -35.83 \text{ kN}$$

For Section BC

$$(SF)_{BC} = 20 + \left[\frac{1}{2} \times \frac{40}{4} \times (x-4) \times (x-4) \right] - 55.83 = 20 + 5(x-4)^2 - 55.83$$

 x is measure from 'E'At point C, $x = 4$

$$(SF)_C = -35.83 \text{ kN}$$

At point B, $x = 8$

$$(SF)_B = 20 + 5(4)^2 - 55.83 = 44.17 \text{ kN}$$

To find out location of zero SF, between BC Section

$$(SF)_{BC} = 0$$

$$20 + 5(x-4)^2 - 55.83 = 0$$

$$\Rightarrow x = 6.677 \text{ m}$$

Bending Moment Diagram (BMD) Calculation :

$$\text{For Section ED, } -10 \times x \times \frac{x}{2} = \frac{-10x^2}{2} = -5x^2$$

At point E, $x = 0$

$$(BM)_E = 0$$

At point D, $x = 2$

$$(BM)_D = -5 \times 4 = -20 \text{ kN-m}$$

For section DC,

$$(BM)_{DC} = -(10 \times 2 \times (x-1)) + (55.83 \times (x-2))$$

At point D, $x = 2$

$$(BM)_D = -20 \text{ kN-m}$$

At point C, $x = 4$

$$(BM)_C = -(10 \times 2 \times 3) + (55.83 \times 2) = 51.66 \text{ kN-m}$$

For section CB,

$$(BM)_{CB} = -20 \times (x-1) + 55.83 \times (x-2) - \frac{1}{2} \times 10(x-4)^2 \times \left[\frac{x-4}{3} \right]$$

At point C, $x = 4$

$$(BM)_C = -20 \times (4-1) + 55.83 \times (4-2) - \frac{1}{2} \times 10(4-4)^2 \times \left[\frac{4-4}{3} \right]$$

$$(BM)_C = 51.66 \text{ kN-m}$$

At point B, $x = 8$

$$(BM)_B = (-20 \times 7) + (55.83 \times 6) - \frac{5}{3} \times 4^3$$

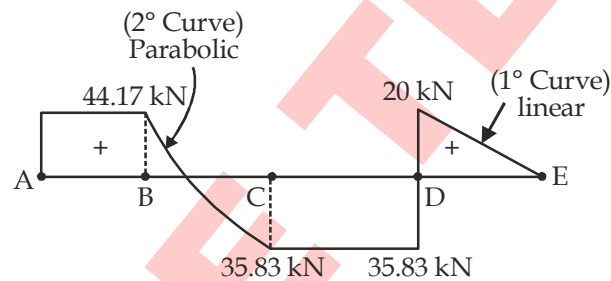
$$= 88.31 \text{ kN-m}$$

As we know that shear force at $x = 6.677 \text{ m}$ equal to 0 hence at this point maximum bending moment occurs

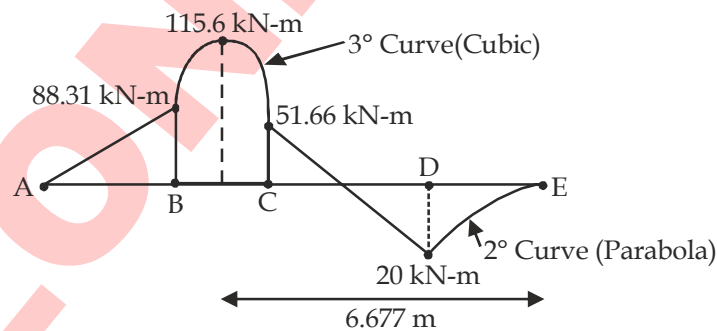
At $x = 6.677 \text{ m}$

$$(BM)_{\max} = -20 \times (6.677 - 1) + 55.83 \times (6.677 - 2) - \frac{1}{2} \times 10(6.677 - 4)^2 \times \left[\frac{6.677 - 4}{3} \right]$$

$$= 115.6 \text{ kN-m}$$



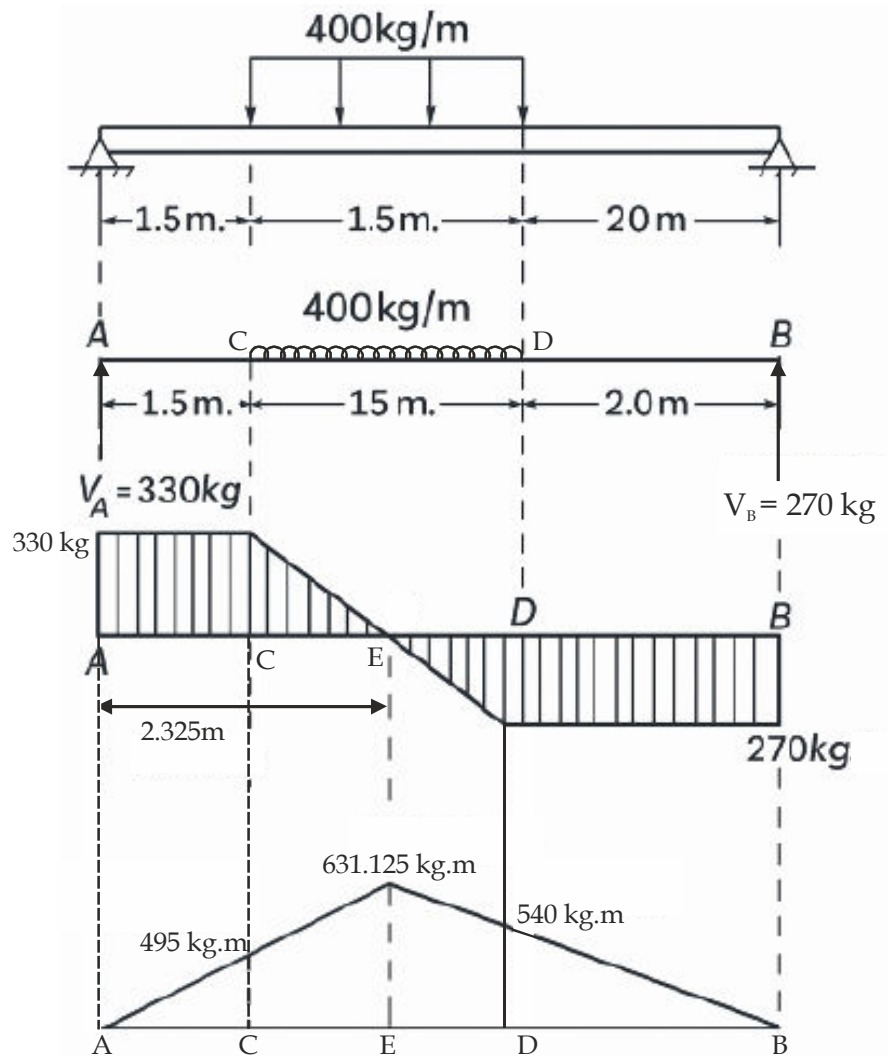
Shear Force Diagram



Bending Moment Diagram

100.

Sol. Let the salient points of the beam be named as shown in the figure,



Let V_a and V_b be the vertical reactions at the left and right supports respectively. Taking moments about the left support, we have

$$V_b \times 5 = 400 \times 1.5 \times (1.5 + 0.75)$$

$$\therefore V_b = 270 \text{ kg}$$

$$\therefore V_a = 400 \times 1.5 - 270$$

$$= 330 \text{ kg}$$

S.F. Between A and C = 330 kg

S.F. Between D and B = -270 kg

Let the S.F. be zero at E,

Let $AE = x$

Equating the S.F. at E to zero we have

$$330 - 400(x - 1.5) = 0$$

$$\therefore x = 2.325 \text{ m}$$



B.M. calculations

$$\text{B.M. at A} = M_a = 0$$

$$\text{B.M. at C} = M_c = 330 \times 1.5 = 495 \text{ kg.m}$$

$$\text{B.M. at D} = M_d = 270 \times 2 = 540 \text{ kg.m}$$

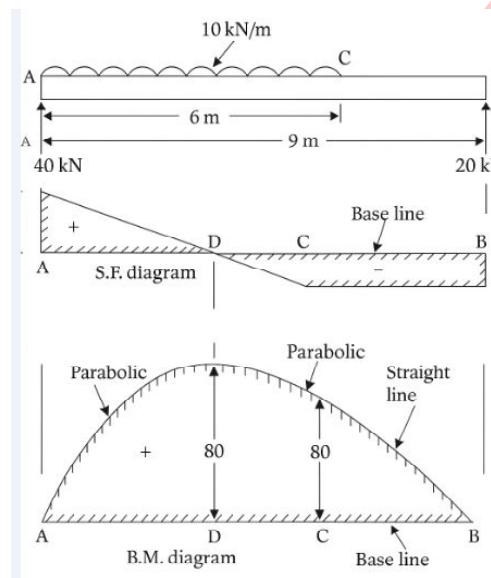
B.M. at 2.325 m from A

$$= M_E = 330 \times 2.325 - \left[400 \times \frac{(2.325 - 1.5)^2}{2} \right] \text{ kg.m}$$

$$= 631.125 \text{ kg.m}$$

101.

Sol. First calculate reactions R_A and R_B



Taking moments of the forces about A, we get

$$R_B \times 9 = 10 \times 6 \times \frac{6}{2} = 180$$

$\therefore$

$$R_B = \frac{180}{9} = 20 \text{ kN}$$

$\therefore$

$$R_A = \text{Total load on beam} - R_B = 10 \times 6 - 20 = 40 \text{ kN.}$$

Shear Force Diagram

Consider any section at a distance x from A between A and C. The shear force at the section is given by,

$$F_x = +R_A - 10x = +40 - 10x \quad \dots(i)$$

Equation (i) shows that shear force varies by a straight line law between A and C.

At A, $x = 0$ hence

$$F_A = +40 - 0 = 40 \text{ kN}$$

At C, $x = 6 \text{ m}$ hence

$$F_C = +40 - 10 \times 6 = -20 \text{ kN}$$

The shear force at A is + 40 kN and at C is - 20 kN. Also shear force between A and C varies by a straight line. This means that somewhere between A and C, the shear force is zero. Let the S.F. is zero at x metre from A. Then substituting the value of S.F. (i.e., F_x) equal to zero in equation (i), we get

$$0 = 40 - 10x$$



$$\therefore x = \frac{40}{10} = 4\text{m}$$

Hence shear force is zero at a distance 4 m from A.

The shear force is constant between C and B. This equal to -20 kN.

Now the shear force diagram is drawn as shown in Fig. In the shear force diagram, distance AD = 4 m. The point D is at a distance 4 m from A.

B.M. Diagram

The B.M. at any section between A and C at a distance x from A is given by,

$$M_x = R_A \times x - 10 \cdot x \cdot \frac{x}{2} = 40x - 5x^2 \quad \dots(ii)$$

Equation (ii) shows that B.M. varies according to parabolic law between A and C.

At A, x = 0 hence

$$M_A = 40 \times 0 - 5 \times 0 = 0$$

At C, x = 6 m hence

$$M_C = 40 \times 6 - 5 \times 6^2 = 240 - 180 = +60 \text{ kNm}$$

At D, x = 4 m hence

$$M_D = 40 \times 4 - 5 \times 4^2 = 160 - 80 = +80 \text{ kNm}$$

The bending moment between C and B varies according to linear law.

B.M. at B is zero whereas at C is 60 kNm.

The bending moment diagram is drawn as shown in Fig.

Maximum Bending Moment

The B.M. is maximum at a point where shear force changes sign. This means that the point where shear force becomes zero from positive value to the negative or vice-versa, the B.M. at that point will be maximum. From the shear force diagram, we know that at point D, the shear force is zero after changing its sign. Hence B.M. is maximum at point D. But the B.M. at D is + 80 kNm.

$$\therefore \text{Max. B.M.} = +80 \text{ kN}$$

102.

Sol.

Given :

Dimension of section,

$$15 \text{ cm} \times 20 \text{ cm}$$

Actual length,

$$l = 6 \text{ m} = 6000 \text{ mm}$$

Young's modulus,

$$E = 17.5 \text{ kN/mm}^2$$

(i) Let P = Crippling load

Using equation, we get

$$P = \frac{\pi^2 EI}{L_{\text{eff}}^2}$$

where

L_e = Effective length

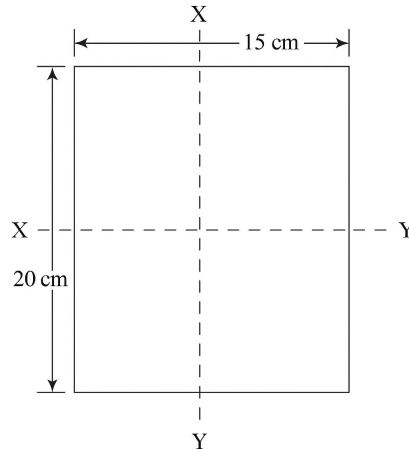
$$= \frac{l}{2}$$

(when both the ends are fixed)

$$= \frac{6000}{2} = 3000 \text{ mm} \quad (\because l = 6000 \text{ mm})$$

I = Least value of moment of inertia

Moment of inertia of the section about X-X axis,



$$I_{xx} = \frac{15 \times 20^3}{12} = 10000 \text{ cm}^4$$

$$= 1000 \times 10^4 \text{ mm}^4$$

And moment of inertia of the section about Y-Y axis,

$$I_{yy} = \frac{20 \times 15^3}{12} = 5625 \text{ cm}^4$$

$$= 5625 \times 10^4 \text{ mm}^4$$

Since I_{yy} is less than I_{xx} , therefore the column will tend to buckle in Y-Y direction. And the value of I will be the least value of the two moment of inertia.

$$\therefore I = 5625 \text{ cm}^4 = 5625 \times 10^4 \text{ mm}^4$$

Substituting the values of $I = 5625 \times 10^4 \text{ mm}^4$ and $L = 3000 \text{ mm}$ in equation (i), we get

$$P = \frac{\pi^2 \times 17.5 \times 5625 \times 10^4}{3000} = 1079.48 \text{ kN}$$

(ii) Safe load for the column
Factor of safety

$$= 3.0 \text{ (given)}$$

$$\therefore \text{ Safe load} = \frac{\text{Crippling load}}{\text{Factor of safety}} = \frac{1079.48}{3} = 359.8 \text{ say } 360 \text{ kN}$$

103.

Sol.

Given:

Actual length of bar, $L = 4 \text{ m} = 4000 \text{ mm}$

Dia. of bar, $d = 5 \text{ cm}$

$$\therefore \text{ Area of bar, } A = \frac{\pi}{4} \times 5^2 = 6.25\pi \text{ cm}^2 = 6.25\pi \times 10^2 \text{ mm}^2 = 625\pi \text{ mm}^2$$

Extension of bar, $\delta L = 4.6 \text{ mm}$

Tensile load, $W = 50 \text{ kN} = 50000 \text{ N}$.

In this problem, the values of Young's modulus (E) is not given. But it can be calculated from the given data.



∴ Young's modulus,

$$E = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{\left(\frac{\text{Tensile load}}{\text{Area}} \right)}{\left(\frac{\text{Extension of bar}}{\text{Length of bar}} \right)}$$

$$\left(\because \text{Stress} = \frac{\text{Load}}{\text{Area}} \text{ and strain} = \frac{\delta L}{L} \right)$$

$$= \frac{\left(\frac{W}{A} \right)}{\frac{\delta L}{L}} = \frac{W}{A} \times \frac{L}{\delta L} = \frac{50000}{625\pi} \times \frac{4000}{4.6} = 2.214 \times 10^4 \text{ N/mm}^2$$

Since the strut is hinged at its both ends,

∴ Effective length, L_e = Actual length = 4000 mm

Let P = Crippling or buckling load.

Using equation, we get

$$P = \frac{\pi^2 EI}{L_e^2}$$

$$P = \frac{\pi^2 \times 2.214 \times 10^4 \times \frac{\pi}{64} \times 5^4 \times 10^4}{4000 \times 4000} \quad \left(\because I = \frac{\pi}{64} \times 5^4 \times 10^4 \text{ mm}^4 \right)$$

$$P = 4189.99 \text{ say } 4190 \text{ N}$$

$$\text{And Safe load} = \frac{\text{Crippling load}}{\text{Factor of safety}} = \frac{4190}{4} = 1047.5 \text{ N.}$$

