ZONE TECH

www.zonetech.in

ZONE TECH

Best Institute For Assistant & Junior Engineer

CIVIL ENGINEERING

E-Book

for

(Degree Students)

CE

B.Tech

Fluid Mechanics & Open Channel Flow

SSC-JE
STAFF SELECTION COMMISSION-JE

RRB-JE
RAILWAY RECRUITMENT BOARD-JE

State-AE/JE
STATE LEVEL EXAMINATIONS

PSUs
PUBLIC SECTOR EXAMINATIONS

CONTACT US: ZONE TECH
H.O.: M-28, Madhuban Colony, Near Tonk Bridge, Tonk Phatak, Jaipur
B.O.: F-26/A, Behind Pinkcity Honda Showroom, Gatta Stand, Tonk Road, Jaipur
+91-9828747676
+91-9462447676
www.zonetech.in
www.onlinezonetech.in
info@zonetech.in

CHAPTER - 1 (PROPERTIES OF FLUID)

Fluid Statics:-

It is the branch of fluid Mechanics in which we study the fluid at rest condition.

Ex:- Water in glass.

Fluid Kinematics:- It is branch of fluid mechanics in which study of motion of fluid without considering cause of motion i.e. force. Here we consider velocity, acceleration etc for study.

Velocity (v), $v = \frac{ds}{dt}$

Acceleration (a), $a = \frac{dv}{dt}$

Angular velocity (ω), $\omega = \frac{d\theta}{dt}$

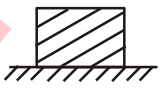
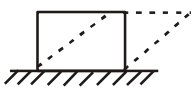
Fluid Dynamics:- It is branch of fluid mechanics in which study of motion of fluid with considering cause of motion i.e. force.

Fluids (Liquid and gas):- The substance which continuously deform under the application of a tangential or shear stress. No matter how small is the value of tangential force.

Note:-

In solid, stress is proportional to strain, but in fluid, stress is proportional to strain rate. When a constant shear stress is applied, a solid eventually stops deforming at some fixed strain angle, where as fluid never stops and approaches a constant rate of strain.

Difference between solid as well as fluid (Liquid or gases):-

Solid	Fluid
	
• Solid deform permanently only when the applied force is more than the elastic limit. • Solid regains their original position when the applied load is remove or it is less than the elastic limit of the material.	• Fluid deform permanently no matter how small is the value of applied tangential force. • Fluids are not regain their original position when applied load is removed.

ZONE TECH

Types of Fluid:-

1. **Ideal Fluid:-**

- An ideal fluid does not have surface tension, viscosity and are incompressible (i.e. bulk modulus $= \infty$)
- No such fluid exist in practical situation.
- However, fluids like air and water have very low value of viscosity and can be treated as ideal fluids for all practical purpose.

2. **Real Fluid:-** A real fluid is one which has viscosity, surface tension and are compressible. Naturally available all fluid are real.

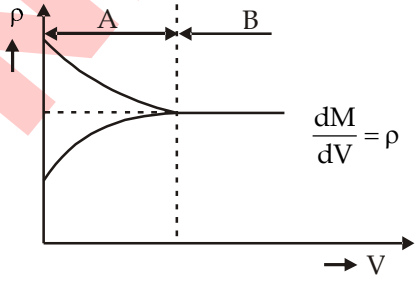
Method of Study:- There are two methods of study.

1. Microscopic Approach
2. Macroscopic Approach

1. **Microscopic Approach:-** In Microscopic approach our attention is focused on each and every molecules so that particle motion information is very very correct because of this huge time and cost consumed.
2. **Macroscopic Approach:-** In macroscopic approach our attention is focused on certain quantity of matter without going the event occuring at molecular level. It is also called as classical way of study.

Concept of continuum or Fluid as a continuous Medium:-

- Concept of continum gives the criteria to apply macroscopic approach.
- It gives the minimum volume which must be considered so the system should be continuous and averaging is meaningful. Here intermolecular distance between molecules is very very less as compare to characteristic dimension of system.
- The continuum concept is not valid above atmospheric level.



A = domain of molecular effect

B = domain of continuum

Knudsen Number $(K_n) = \frac{\lambda}{L}$

λ = Mean free path

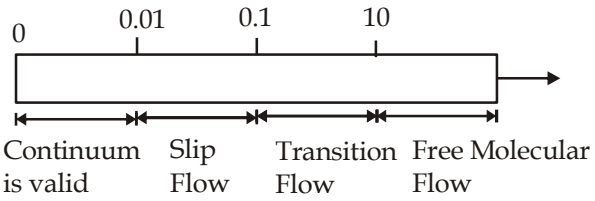
L = Charateristics dimension of the problem

If $K_n < 0.01$ (Continuum is valid)

$0.01 < K_n < 0.1$ (Slip flow)

$0.1 < K_n < 10$ (Transition flow)

$K_n > 10$ (free molecular flow)



Some properties of fluid:-

(1) **Density (ρ):-** It is the ratio of the mass of fluid substance to its volume.

$$\rho = \frac{\text{Mass (M)}}{\text{Volume (V)}}$$

- **Unit**

$$\text{In MKS} = kg/m^3$$
$$\text{In CGS} = gram/cm^3$$

$$1\frac{gram}{cm^3} = \frac{10^{-3}kg}{10^{-6}m^3} = 10^3\ kg/m^3$$

- **Dimension:-**

$M^1L^{-3}T^0$

Note:-

$$\rho_{water} = 1\frac{gram}{cm^3} = 1000\frac{kg}{m^3},\ \rho_{Hg} = 13.6 \times 10^3\ kg/m^3$$

Effect of density:-

1. Density of liquid is not a function of pressure and temperature.
2. For gases if pressure increases then density also increases. If temperature increases, density decreases.

(2) **Specific Volume (V):-** Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume.

$$\text{Specific Volume} = \frac{\text{Volume of fluid (V)}}{\text{Mass of fluid (M)}}$$

$$V = \frac{1}{\frac{M}{V}} = \frac{1}{\rho}$$

Unit:- In MKS $\rightarrow m^3/kg$

CGS $\rightarrow cm^3/gram$

Dimension:- $M^{-1}L^3T^0$

(3) **Specific Gravity (s):-** It is another way of representation of relative density.

It is the ratio of the density of the substance (fluid) with respect to the density of a standard fluid.

$$S = \frac{\text{Density of Fluid}}{\text{Density of Standard Fluid}}$$

ZONE TECH

For liquid:- Standard fluid $\Rightarrow$ water, $\rho_w = 1000\ kg/m^3$

For gases:- Standard fluid $\Rightarrow$ Air $= \rho_{air} = 1.21\ kg/m^3$

- **Dimension:-** $M^{\circ}L^{\circ}T^{\circ}$
- Unitless
- **For Hg**:- In MKS $\rightarrow$ $\rho_{Hg} = 13.6 \times 10^3\ kg/m^3$

$$S = \frac{13.6 \times 10^3}{10^3} = 13.6$$

(4) **Relative Density (R.D.):-**

$$R.D. = \frac{\text{Density of fluid 1}}{\text{Density of fluid 2}}$$

- Dimension- $M^{\circ}L^{\circ}T^{\circ}$
- Unitless

(5) **Specific weight or weight density (w):-** This is the ratio between weight of fluid to its volume.

$$w = \frac{mg}{\forall}$$

$$w = \rho g$$

Unit:- $\frac{Kg}{m^3} \times \frac{m}{sec^2}$ or $\frac{N}{m^3}$

$$= \frac{Kg}{m^2 \times sec^2},$$

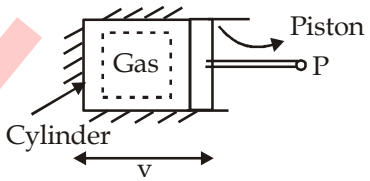
Dimension $= M^1L^{-2}T^{-2}$

$w_{water} = 10^3 \times 9.81\ N/m^3 = 9810\ N/m^3$

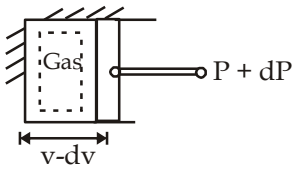
$w_{Hg} = 13.6 \times 10^3 \times 9.81\ N/m^3$

(6) **Compressibility (β):-**

It is the reciprocal of bulk modulus of elasticity (K):- Bulk modulus is defined as ratio of compressive stress to volumetric strain.



Initial condition



Final condition

Change in pressure = Final pressure - Initial pressure

$= P + dP - P = dP$

Change in volume $= (V - dv) - V = -dV$

$$\text{Volumetric Strain} = \frac{\text{change in volume}}{\text{Volume}} = \frac{-dV}{V}$$

Negative sign shows decrease in volume

$$K = \frac{dP}{\left(\frac{-dV}{V}\right)} \quad \text{....(1)}$$

We know that $\beta = \frac{1}{k}$, $\beta = -\frac{1}{V}\frac{dV}{dP}$ ....(2)

In general,

$m = \rho v$

differentiate it

$0 = \rho dv + v d\rho \qquad \because dm = 0$

$$-v d\rho = \rho dv \Rightarrow \frac{d\rho}{\rho} = \frac{-dV}{V}$$

Put in equation (1)

$$K = \frac{dP}{\frac{d\rho}{\rho}} \Rightarrow K = \rho\frac{dP}{d\rho} \quad \text{....(3)}$$

So $\beta = \frac{1}{\rho}\frac{d\rho}{dP}$ ....(4)

For liquid

$d\rho = 0$

$K = \infty$ ....(5)

$\beta = 0$ ....(6)

For gases

$d\rho \neq 0$

$K \neq \infty$ ....(7)

$\beta \neq 0$ ....(8)

(a) Isothermal compressibility of a gas:-

For isothermal process

$PV = \text{constant}$

differentiate it

$$vdp + pdv = 0 \Rightarrow pdv = -vdp$$

$$-\frac{dV}{V} = \frac{dP}{P} \Rightarrow P = \frac{dP}{\left(\frac{-dV}{V}\right)}$$

dP = Bulk modulus of elasticity (K)

$K_{iso} = P \qquad \because P = \rho RT$

or $K_{iso} = \rho RT$ and $\beta_{iso} = \frac{1}{P} = \frac{1}{\rho RT}$

Unit of K is Pascal

ZONE TECH

(b) Adiabatic compressibility of a gas:-

For adiabatic process

$$PV^{\gamma} = \text{constant}$$

$$V^{\gamma}dp + \left[\gamma V^{\gamma-1}dV\right]P = 0$$

$$V^{\gamma-1}\left[VdP + \gamma PdV\right] = 0$$

$$VdP = -\gamma\ \rho\, dV$$

$$\gamma P = \frac{dP}{\left(\frac{-dV}{V}\right)}$$

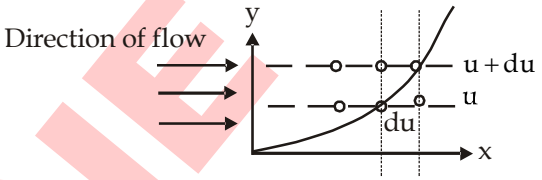
$$K = \gamma P$$

$$K_{adiabatic} = \gamma P$$

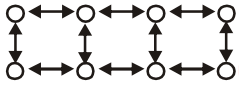
Adiabatic bulk modulus $= \gamma P =$ Adaiabtic Index × Pressure

$$\beta_{adiabatic} = \frac{1}{\gamma P}$$

Viscosity (μ):- Viscosity is measurement of "Internal Resistance of flow".



Cause of viscosity :-

 cause of viscosity is Attraction force between molecules which is Cohesive force

Definition:-

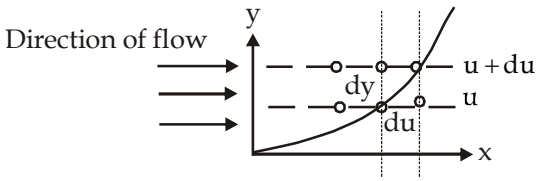
When the two adjacent layer of a fluid are in relative motion then they resist the motion of each other and this fundamental property of fluid is known as viscosity.

- Viscosity is property of fluid not a property of flow.
- It is the internal resistance of flow.
- For liquids cohesive force is very very high as compare to gases so viscosity of liquid is very high as compare to gases.
- Viscosity in gases because of random motion of molecules.
- Dynamic visocity of water is approximately 50 times that of air.

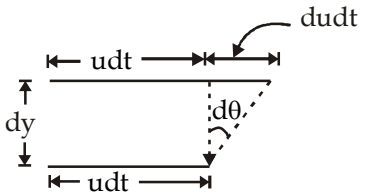
$$\mu_{H_2O} >>>>> \mu_{air}$$

ZONE TECH

Shear strain:-



In time interval = dt



Shear strain

$$\tan d\theta = \frac{dudt}{dy}$$

If θ is very small $\tan d\theta \approx d\theta$

$$d\theta = \frac{dudt}{dy}$$

where $d\theta$ = Shear strain or angular deformation

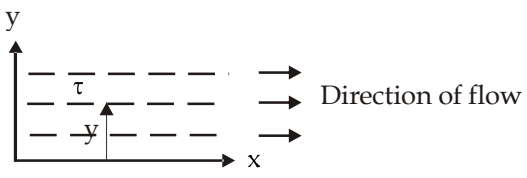
$$\frac{d\theta}{dt} = \frac{du}{dy}$$

where $\frac{d\theta}{dt}$ = Rate of shear strain or rate of angular deformation in direction of flow

$\frac{du}{dy}$ = Velocity gradient in the transverse direction of flow.

Newton's law of viscosity:-

It has been experimentally verified that the shear stress on the layer at a distance 'y' from the surface is directly proportional to the rate of shear strain or the rate of angular deformation.



$$\tau \propto \frac{d\theta}{dt}$$

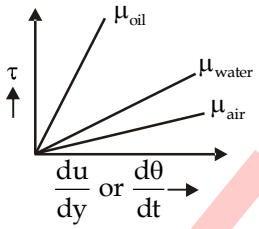
ZONE TECH

By defination $\frac{d\theta}{dt} = \frac{du}{dy}$

$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}$$

μ = constant (not a universal constant) or Coefficient of viscosity or Absolute viscosity or Dynamic viscosity



The slope of curve $\tau = \mu \frac{du}{dy}$ is μ. Which gives viscosity but μ remains same with flow i.e. if more will be slope, more will be viscosity thats why $\mu_{oil} > \mu_{water} > \mu_{air}$

Unit of viscosity (μ):-

Dynamic Viscosity (μ)

In SI

$$\frac{N}{m^2} \times \text{sec or Pa} \times \text{sec}$$

In MKS

$$kg \frac{m}{sec^2} \times \frac{1}{m^2} \times \text{sec}$$

$$\frac{kg}{m \times sec}$$

In CGS

$$1 \text{ CGS} = \frac{gram}{cm \times sec}$$

$$1 \text{ Poise} = \frac{gram}{cm \times sec} = \frac{1}{10} \frac{kg}{m \times sec} = \frac{1}{10} \frac{N}{m^2} \times \text{sec} = 0.1 \text{ pa} - \text{sec}$$

Dimension :- $M^1L^{-1}T^{-1}$

Kinematic viscosity (ν) :- It is the ratio between dynamic viscosity and density of fluid.

$$\nu = \frac{\mu}{\rho}$$

ZONE TECH

Unit of kinematic viscosity :-

In MKS

$$\frac{kg/m-sec}{kg/m^3}=\frac{m^2}{sec}$$

In CGS

$$1\ CGS=\frac{cm^2}{sec}$$

$$1\ stoke=\frac{cm^2}{sec}=\frac{1}{10^4}\frac{m^2}{sec}$$

Dimension :- $M^0L^2T^{-1}$

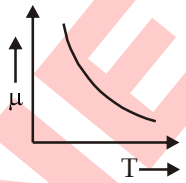
Effect of different conditions on viscosity of fluid:-

1. For Dynamic Viscosity or Absolute viscosity :-

(A) Effect of temperature

- **Liquids :-** In liquids cause of viscosity is cohesive force so as we increase temperature bonds between molecules breaks so force of attraction decreases hence internal resistance decreases because of this dynamic viscosity decreases.

$T\uparrow \Rightarrow \mu\downarrow$



- **Gases :-** In gases cohesive force is negligible but cause of viscosity is random motion of molecules as we know that $V_{rms}\Rightarrow\sqrt{\frac{3RT}{M}}$. As temperature is increases, V_{rms} increases. Because of this resistance to flow increases hence dynamic viscosity increases.

$T\uparrow \Rightarrow \mu\uparrow$

(B) Effect of pressure

- **Liquids :-** In liquids due to change in pressure cohesive forces not affected so dynamic viscosity remain same or no change in viscosity. ($P\uparrow \Rightarrow \mu$ Remain same)

- **Gases :-** In gases according to boyle's law $P\propto\frac{1}{V}$ so as pressure increases, volume decreases so PV remain constant. As we know that $v_{rms}=\sqrt{\frac{3RT}{M}}=\sqrt{\frac{3PV}{M}}$ so V_{rms} remain constant hence by increasing pressure there is no effect on viscosity. ($P\uparrow \Rightarrow \mu$ Remain same)

Note :- Dynamic viscosity is a function of temperature only.

ZONE TECH

2. For Kinematic Viscosity (ν)

(A) Effect of temperature

- As we increase temperature, there is no change in density but dynamic viscosity decreases for liquids. As we know that $\nu = \frac{\mu}{\rho}$ so by increasing temperature kinematic viscosity decreases. $(T\uparrow \Rightarrow \nu\downarrow)$
- As we increase temperature density decreases $(\rho \propto \frac{1}{T})$ and dynamic viscosity increases so by increasing temperature kinematic viscosity increases $(T\uparrow = \nu\uparrow\uparrow)$

(B) Effect of pressure

- As pressure increases there is no change in density and dynamic viscosity of liquids. As we know that $\nu = \frac{\mu}{\rho}$ so by increasing pressure kinematic viscosity remains same ($P\uparrow \Rightarrow \mu$ and ν remain same)
- As pressure increases density increases $(P \propto \rho)$ and no change in dynamic viscosity of gases. As we know that $\nu = \frac{\mu}{\rho}$ so by increasing pressure kinematic viscosity decreases $(P\uparrow \Rightarrow \nu\downarrow)$

Note :- Kinematic viscosity of air is 15.2 times that of water at 20°C. $(\upsilon_{air} \approx 15\ \upsilon_{water})$

Linearisation of Newton's law of viscosity:-

According to Newton's law of viscosity

$$\tau = \mu\frac{du}{dy}$$

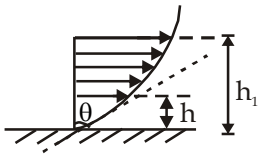
Shear stress at the wall is τ_w or τ_0

$$\tau_w = \mu\left.\frac{du}{dy}\right|_{Y=0}$$

u = velocity profile (variation of velocity)



External flow

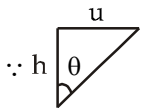


$$\tau_w = \mu\left.\frac{du}{dy}\right|_{Y=0}$$

ZONE TECH

where $\left.\frac{du}{dy}\right|_{Y=0}$ = Slope of tangent at $Y = 0$

So $\tau_w = \mu\theta$

$\because$ 

$\tan\theta = \frac{u}{h}$

If θ is very small $\tan\theta \approx \theta$

So $\theta = \frac{u}{h}$

$\therefore$ $\tau_w = \mu\frac{u}{h}$

where u = Relative Velocity (u - 0)

h = Very small gap

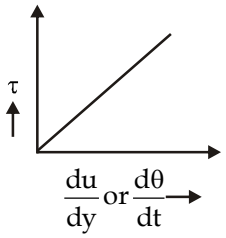
Note:-

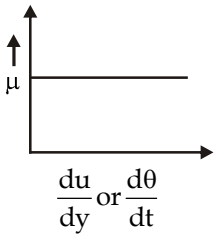
The linearisation of Newton's law of viscosity is only valid for the small gaps (in mm).

Newtonion Fluid:-

This fluids Obey Newton's law of viscosity

$\tau = \mu\frac{du}{dy}$





Ex:- Water, air, oils. For such fluids μ does not change with rate of deformation.

Rheology :- It is the branch of science in which we study different type of non-newtonion fluids on the basis of flow behaviour.

Non-Newtonion Fluid:-

These fluids don't obey newton's law of viscosity

Power law $\tau = A\left(\frac{du}{dy}\right)^n$

A = consistency Index

n = Flow Behaviour Index

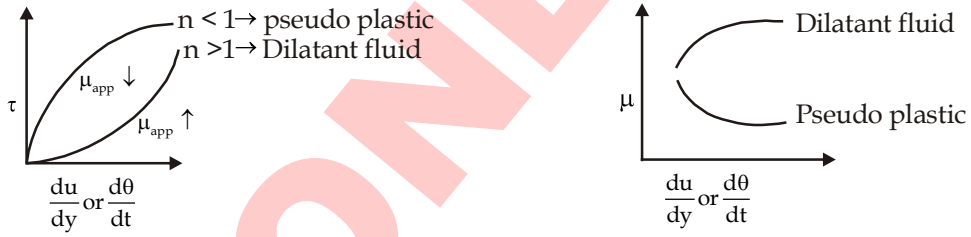
$$\tau = \underbrace{\left[A\left(\frac{du}{dy}\right)^{n-1}\right]}_{\mu_{apparent}}\frac{du}{dy}$$

Slope $= \mu_{apparent}$

Classification of non-newtonian fluids :- Non-newtonian are of two types (a) Purely viscous (b) Visco elastic fluids

Purely viscous fluids further classified as time dependent and time independent fluids

Time independent fluids or deformation dependent



- Pseudo plastic fluid are also known as shear thining fluid.
  Ex:- Milk, blood, paper pulp, molasses.
- Dilatant fluids are also called shear thickening fluids.
  Ex:- Honey, saturated solution of sugar, rice, starch solution.

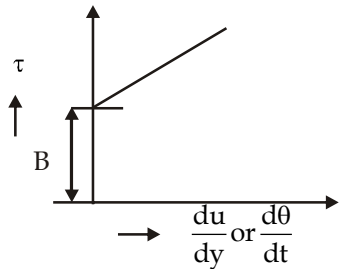
Bingham Plastic or Ideal Plastic Fluid:-

Bingham Plastic (ideal plastic) fluids require a certain minimum shear stress τ_y (yield stress) before they start flowing. Ex:- Tooth paste, Gel, creams.

$$\tau = \mu\frac{du}{dy} + B$$

B = + ve

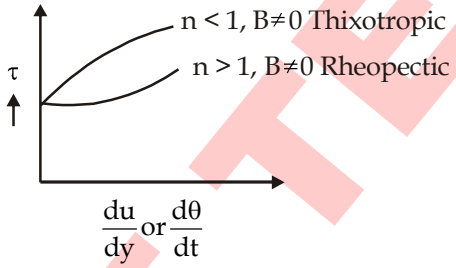
ZONE TECH



where $B =$ Initial yield stress $(B \neq 0)$

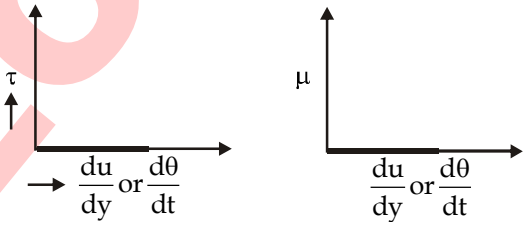
$$\tau = \left[A\left(\frac{du}{dy}\right)^{n-1} \right] \frac{du}{dy} + B$$

Time dependent fluids :-



- Thixotropic fluids shows decrease in apparent viscosity with time under a constant applied shear stress.
  Ex:- Printer's ink, paints, ink, crude oil.
- Rheopectic fluid shows increase in viscosity with time under a constant applied shear stress.
  Ex:- Gypsum paste.

Ideal fluid:-



$\mu = 0 \rightarrow$ Frictionless and incompressible fluid is called ideal fluid.

$\downarrow$

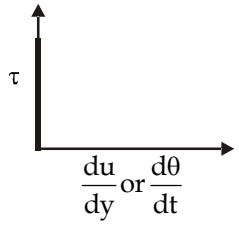
Invisid fluids

Viscous force $= 0$ Cohesive force $= 0$

- An Ideal fluid does not have surface tension and are incompressible (i.e. Bulk Modulus $\approx \infty$)
- No such fluid exist in practical situation.
- However, fluids like air and water have very low value of viscosity and can be treated as ideal fluids for all practical purposes.

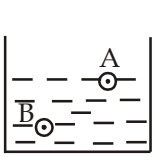
ZONE TECH

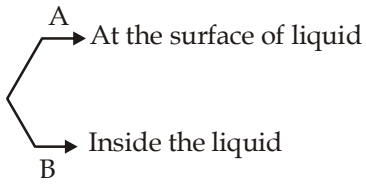
Ideal solids :-

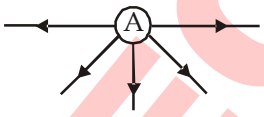


Surface Tension (σ):-

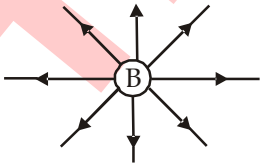
A tensile force is acting on the surface of liquid in contact with a gas or on the surface between two immiscible liquids such that contact surface behave like membrane under tension.



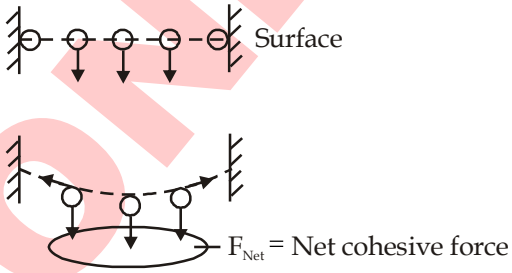




$F_{\text{net cohesive force}} \neq 0$



$F_{\text{net cohesive force}} = 0$



Mathematically:-

$$\sigma = \frac{F}{l}$$

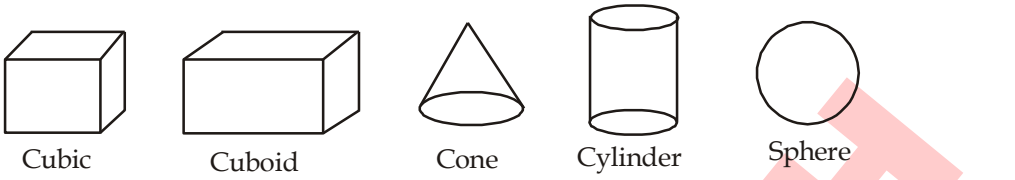
Surface tension is internal force and cause of surface tension is cohesive force

$$\sigma_{H_2O} = 0.073\ \text{N/m} = \frac{1}{15}\text{th of } 1\text{N} \rightarrow \text{Air water interface}$$

$$\sigma_{Hg} = 0.48\ \text{N/m} \quad \rightarrow \text{Air-Mercury Interface}$$

ZONE TECH

- As a result of net downward force, surface will be pulled down causing a curvature to the surface. This in turn develops a tension in the surface which is similar to tension in a stretched member.
- As tempertature increase, surface tension decreases and effect of pressure on surface tension is negligible.



⇓

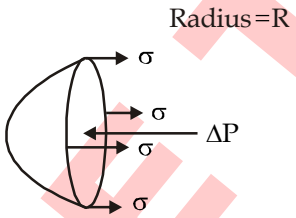
Surface area is minimum

- For constant volume, surface area is minimum for sphere

Definition :- Surface tension is a property of liquid by virtue of which it minimize or tries to minimize its surface area.

(1) Pressure inside a droplet of liquid

Water drop:-

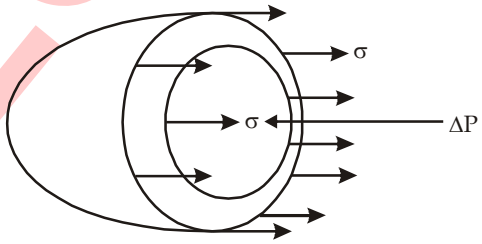


Force balancing in horizontal direction

$$\sigma(2\pi R) = \Delta P(\pi R^2)$$

$$\Delta P = \frac{2\sigma}{R}$$

(2) **Pressure inside soap bubble**

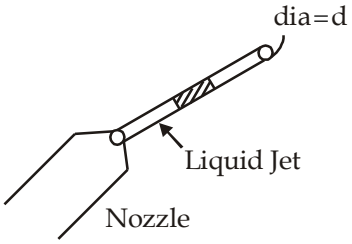


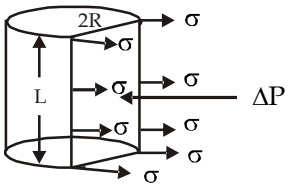
In soap bubble there are two interface between soap film and air one inside the bubble and other outside.

Hence $\sigma(2\pi R)+\sigma(2\pi R) = \Delta P(\pi R^2)$

$$\Delta P = \frac{4\sigma}{R}$$

(3) **Pressure inside liquid jet or jet of water**





$$\sigma(L)+\sigma(L) = \Delta P(2RL)$$

$$\sigma(2L) = \Delta P(2RL)$$

$$\Delta P = \frac{\sigma}{R}$$

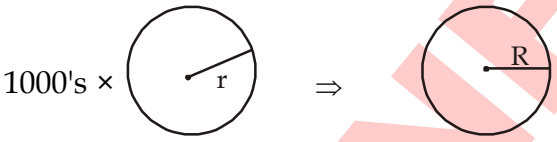
Surface Energy:-

The work done involved in the formation of surface is stored as form of energy and this energy is known as surface energy.

$E = \sigma A$ $\quad A \rightarrow$ surface area

Thus surface tension can also be defined as work done per unit increase in the surface area of the liquid surface. This work done is stored as surface energy.

Ex:-



Small drop Big drop

$$1000 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$$10r = R$$

$$E_1 = 1000 \times \sigma \times 4\pi r^2$$

$$E_1 = 4000\,\sigma\,\pi r^2 \quad \text{.....(i)}$$

$$E_2 = \sigma(4\pi R^2) = \sigma[4\pi(10r)^2]$$

$$E_2 = 400\,\pi r^2\,\sigma \quad \text{....(ii)}$$

$$E_1 - E_2 = 3600\,\sigma\,\pi r^2$$

$$E_1 > E_2$$

Note:-

A liquid droplet takes spherical shape because in spherical condition the surface area is minimum thereby the surface energy is minimum. Minimum surface energy to most stable state, thus liquid droplet takes spherical shape.

ZONE TECH

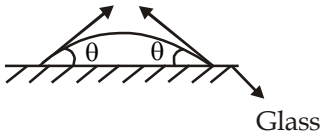
Wetting and non wetting surfaces:-

(1) Molecules of a liquid are held together by a force of attraction between them, which is known as cohesive force.

(2) The force of attraction between the molecules of different types is called adhesive force.

Case I :- Wetting surface

Adhesive force >>>> Cohesive force:-



If $\theta < 90°$ $\theta \rightarrow$ Angle of contact

For clean water $\Rightarrow \theta = 0°$

Liquid wets solid surface

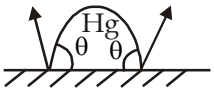
Case II :- Non wetting surface

Adhesive force <<< Cohesive force



$\theta = [130\text{-}140°]$ for Hg

If $\theta > 90°$ Liquid does not wet the surface



$\theta > 90°$

Wetting & non wetting surface is a mutual property of surface as well as liquid which will decide that the liquid is wet the surface or not.

Contact Angle for different interface

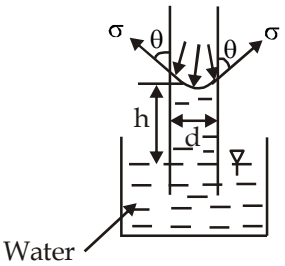
(1) Water-glass $= 0°$

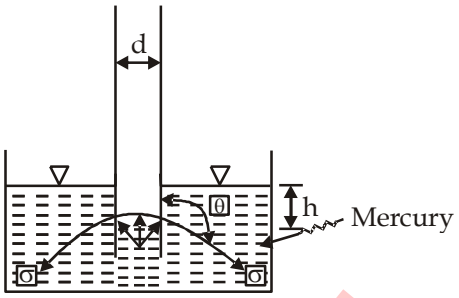
(2) Mercury-glass $= 130°$

(3) Kerosene-glass $= 26°$

Capillary Effect or capillarity:-

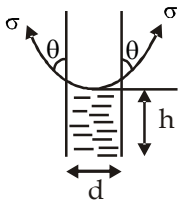
- Very very thin diameter tubes is capillary tube such that diameter < 1 cm is called capillary tube
- When a tube having a very very small diameter (capillary tubes inserted in a large reservoir of liquid then the liquid will start to rise or fall in the tubes this effect is known as capillary effect or capillarity.
- The rise or fall of a liquid in capillary tubes is decided by the direction of surface tension and the direction of surface tension will be decided by mutual behaviour of liquid and surface.



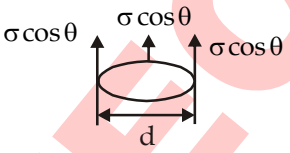


Capillary fall, $\theta > 90°$

(a) Capillary rise, $\theta < 90°$



(b) Capillary depression, $\theta > 90°$



Force balancing in vertical direction

$$\sigma\cos\theta(2\pi R) = \text{weight of liquid}$$

$$\sigma\cos\theta(2\pi R) = mg$$

$$= \rho vg$$

$$(\sigma\cos\theta)(2\pi R) = \rho\left[\pi R^2 \times h\right]g$$

$$2\sigma\cos\theta = \rho ghR$$

$$\Rightarrow \quad h = \frac{2\sigma\cos\theta}{\rho gR} \text{ or } \frac{4\sigma\cos\theta}{\rho gd}$$

θ = Angle of contact between liquid and glass tube

where

d = diameter of glass tube open at both end

h = height of the liquid in the tube

σ = surface tension of liquid

- Capillary Rise is inversely proportional to diameter of pipe.

Note:-

- Capillary effect of water is usually negligible in tube whose diameter is greater than 1 cm.
- When pressure measurements are made using manometer and barometer, it is important. Normally diameter > 10 mm is used as tube to avoid capillary effect.

ZONE TECH

Note:-

Angle of contact				
1. $\theta < 90°$	Cohesion < Adhesion	Liquid wets solid surface	Top surface of water is concave	Liquid rise in the capillary tube
2. $\theta > 90°$	Cohesion > Adhesion	Liquid does not wet the surface	Top surface of water is convex	Liquid falls in the capillary tube

Factors affect the rise of liquid in capillary tube:-

(1) Specific weight of liquid

(2) Diameter of the tube

(3) Surface tension of liquid

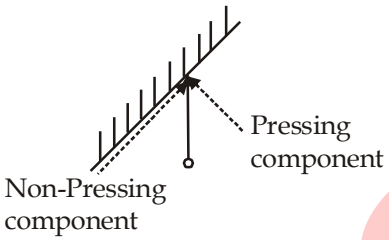
(4) Angle of Inclination (Angle of contact)

ZONE TECH

CHAPTER - 2 (FLUID PRESSURE AND ITS MEASUREMENT)

Pressure:-

Pressure is defined as normal force exerted by a fluid per unit area.

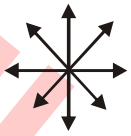


Mathematically:-

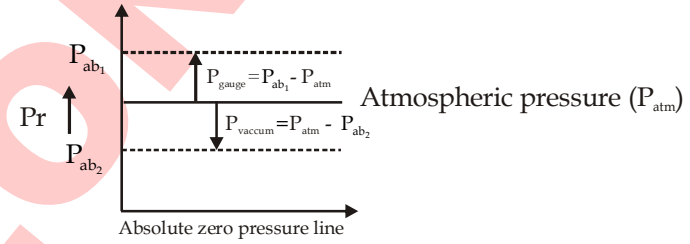
$$\sigma = \frac{F}{A}$$

F = External Normal Force or External Thrust or Pressure force

Pressure is a scalar quantity.



Different types of pressure:-



(1) **Atmospheric Pressure:-**

It is the pressure exerted by atmosphere. Its value is taken as 1.013 bar at mean sea level. It is measured by barometer. At mean sea level it is equal to 10.3 m head of water or 76 cm head of mercury (specific gravity 13.6)

(2) **Absolute pressure:-**

- It is the net pressure above zero pressure level.
- It is measured using Aneroid barometer.

(3) **Gauge Pressure:-**

- It is the value of pressure above atmospheric pressure.
- It is measured using manometer or Bourden gauge.

ZONE TECH

(4) **Vaccum pressure:-**

• It is the value of the pressure below atmospheric pressure.

$$P_{abs} = P_{gauge} + P_{atm}$$

$$P_{vaccum} = P_{atm} - P_{abs}$$

(-ve) P_{vaccum} is called negative pressure.

Unit of Pressure:-

(1) SI:- N/m^2

(2) Pascal:- $1\ Pa = 1\ N/m^2$

(3) Atm:- $1\ Atm = 101325\ Pa = 1.01325 \times 10^5\ Pa$

(4) Bar:- $1\ bar = 10^5 Pa$

(5) Kgf/cm²:- $1\frac{kgf}{cm^2} = \frac{9.81\,N}{10^{-4}\,m^2} = 9.81 \times 10^4\ N/m^2$

$$Atm > bar > \frac{kgf}{cm^2}$$

(6) psi:- Pound force per square inch

$$1\ psi = \frac{1lbf}{inch^2} = \frac{0.4536}{(2.54)^2}\frac{kgf}{cm^2}$$

$$1\ psi = \frac{0.4536}{(2.54)^2} \times 9.81 \times 10^4\ N/m^2$$

$1\ psi = 6894.76\ Pa$ According to the unit we fill the air in car and bikes tube

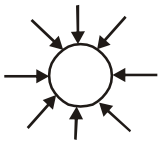
$1\ Atm = 101325\ Pa$ $30\ psi \approx 2\ atm$

$1\ Atm = 14.696\ psi$

Pascal's law :- It states that pressure or intensity of pressure in a static fluid is equal in all direction. External static pressure exerted on a fluid is distributed evenly throughout the fluid.

Any two points at same elevation in continuous mass of same static fluid will be at same pressure.

Pressure at a point:-



- Pressure at a point in static fluid is same in all directions (derived from pascal's law)
- When a fluid is at rest, it exerts normal force on the surface of contact.
- If a fluid exerted a tangential force on the surface, the surface would also exert tangential force on fluid.
- Pressure is a scalar quantity, it has magnitude but does not have a definite direction.

ZONE TECH

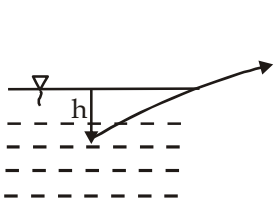
Pressure at a certain point in static fluid or Hydro-static pressure

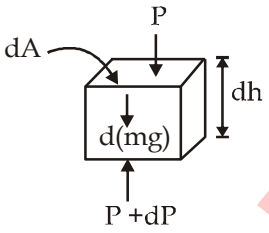
Consider a small fluid element as shown in figure

dA → Cross section area of fluid element

dh → Height of fluid element

h → Distance of fluid element from free surface





Force balancing in vertical direction

$$PdA - (P + dP)dA + dmg = 0$$

Because fluid is stationary (without any motion)

$$-dPdA = -d(mg)$$

$$dPdA = \rho\, dvg \qquad \{dv = dAdh\}$$

$$dPdA = \rho\, dAdhg$$

$$\frac{dP}{dh} = \rho g$$

where $\frac{dP}{dh}$ = Pressure gradient in downward direction

Integrate it

$$\int_0^P dP = \int_0^h \rho g dh$$

$$P = \rho g h \qquad P = fn(h)$$

$\frac{dP}{dh} = +\rho g$ for downward direction $\qquad$ $\frac{dP}{dh} = -\rho g$ for upward direction

Case 1:- For incompressible fluid:-

ρ = constant

$$\frac{dP}{dh} = \rho g$$

$dP \propto dz \rightarrow$ Linear relation

$$P = \rho g z$$

Case 2:- For compressible fluid:-

ρ is not constant

$$\frac{dP}{dh} = \rho g \qquad \because PV = MRT$$

$$\frac{dP}{dh} = \frac{P}{RT} g \qquad \because \rho = \frac{P}{RT}$$

$$\int_{P_{atm}}^{P_{abs}} \frac{dP}{P} = \int_0^h \frac{g}{RT} dh \Rightarrow ln\left[\frac{P_{abs}}{P_{atm}}\right] = \frac{g}{RT} h$$

$$P_{abs} = P_{atm}\, e^{\left[\frac{g}{RT} h\right]}$$

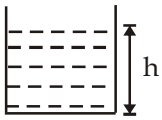
Note:-

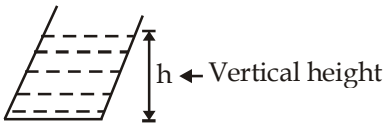
For static incompressible fluid the pressure increase linearly with depth and decrease linearly with height but for compressible fluid pressure varies exponentially.

ZONE TECH

Hydrostatic law :- The rate of increase of pressure in a vertically downward direction must be equal to specific weight of fluid at that point.

Pressure in terms of liquid column height:-





For H_2O:-

$P = \rho_w gh$

$1\ atm = 10^3 \times 9.81 \times h$

$101325 = 10^3 \times 9.81 \times h$

$h = 10.3\ m\ of\ H_2O$

For Hg:-

$P = \rho_{Hg} gh$

$1\ atm = 13.6 \times 10^3 \times 9.81 \times h$

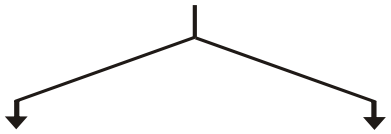
$101325 = 13.6 \times 10^3 \times 9.81 \times h$

$h = 760\ mm\ of\ Hg$

Torricelli

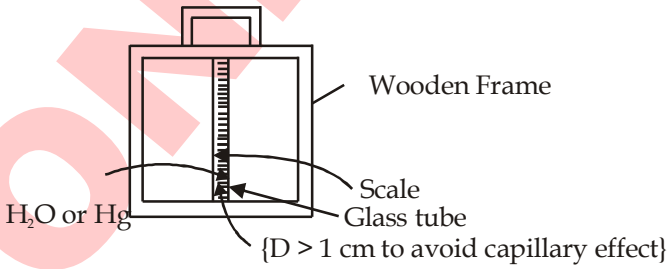
$1\ Torr = 1\ mm\ of\ Hg = 133.3\ Pa$

Pressure Measurement

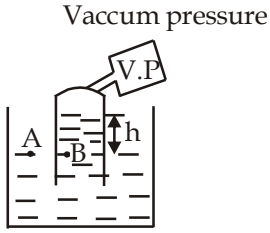


Conventional		Modern
→ Barometer	→	Mechanical gauge
→ Piezometer	→	Strain gauge Transducers
→ Manometer	→	Piezo-electric transducers

Barometer:- It is used to measure local atmospheric pressure.



Construction



$P_A = P_B$

$P_A = \rho_{Hg} gh + P_{Hg\ vapour}$

$P_{Atm} = \rho_{Hg} gh \quad \because P_{Hg\ vapour} \approx 0$

ZONE TECH

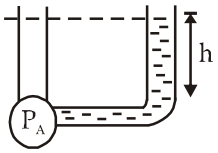
In case of water

$$P_{atm} = \rho_{H_2O} gh + P_{H_2O vapour} \rightarrow 2.7 \text{ m of } H_2O$$

We use Hg because it has high density and low vapour pressure.

Piezometer:-

$$P_A = \rho gh$$



- This is used for liquid only.
- For the measurement of low gauge pressure.
- Piezometer does not measure gas pressure because it does not have a free surface.
- Negative pressure cannot be measured by piezometer.

Manometer :- Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid.

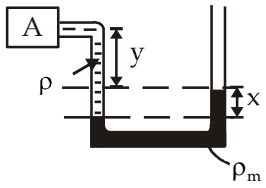
- Manometer are used for the measurement of low, medium and high gauge pressure as well as vaccum pressure for liquid and gas both.
- In manometer we uses an additional fluid known as manometric fluid.
- Manometer are worked on the liquid column balance principle.
- Manometric fluid has high surface tension



Simple U-tube manometer | Differential U-tube manometer

Simple U-tube manometer :- It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere.

For Gauge pressure:-



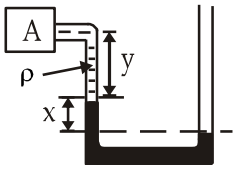
ρ_m = Density of manometric fluid

$$P_A + \rho g (x + y) = \rho_m g x$$

$$P_A = \rho_m g \times - \rho g (x + y)$$

P_A = +ve (pressure is high side of A)

For Vaccum pressure:-



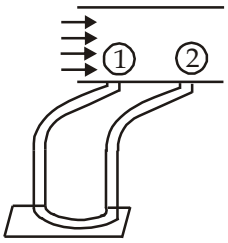
$$P_A + \rho g y + \rho_m g x = 0$$

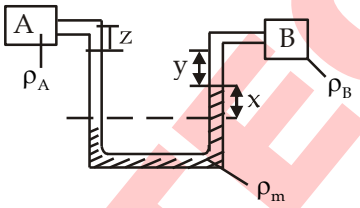
$$P_A = -(\rho g y + \rho_m g x)$$

P_A = -ve

Differential U-tube manometer:-

These manometers are used for the measurement of pressure difference between two points.



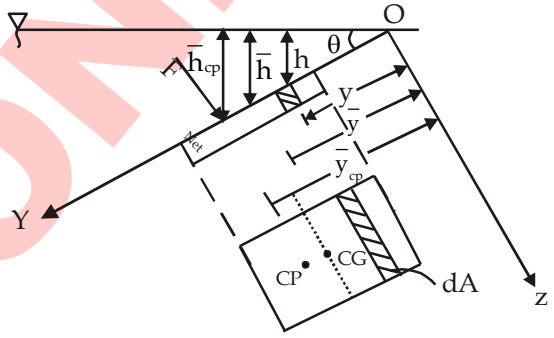


$$P_A + \rho_A g(x + y + z) = P_B + \rho_B g y + \rho_m g x$$

$$P_A + \rho_A g(x + y + z) - \rho_B g y - \rho_m g x = P_B$$

$$P_A - P_B = \rho_B g y + \rho_m g x - \rho_A g(x + y + z)$$

Pressure force on inclined surface:-



Let us consider a plane surface inclined at an angle θ with the horizontal.

A = Total area of the surface

$\bar{h}$ = Depth of C.G. of inclined area from free surface

h* or $\bar{h}_{cp}$ = Distance of centre of pressure from free surface of liquid

θ = Angle made by the plane of the surface with free liquid surface

$\bar{y}$ = Distance of C.G. of the inclined surface from O-Z

y* or $\bar{y}_{cp}$ = Distance of the centre of pressure from O-Z

ZONE TECH

$$P = \rho g h$$

$$dF = PdA$$

$$dF = \rho g h dA$$

Integrate it

$$F = \int \rho g h dA = \rho g \int h dA \qquad \left| \because h = y \sin\theta \right.$$

$$= \rho g \int y \sin\theta dA$$

$$F = \rho g \sin\theta \int y dA \qquad \left| \because \int y dA = Ay_1 + Ay_2 .... = A\bar{y} \right.$$

$$F = \rho g \sin\theta \bar{y} A \qquad \left| \because \bar{h} = \bar{y} \sin\theta \right.$$

$$F = \rho g \bar{h} A = F_{Net}$$

Location of Total pressure force:-

Taking moment about O

$$F_{net} . \bar{y}_{cp} = \int dF . y$$

$$= \int p dA . y$$

$$= \int \rho g h \, dA . y$$

$$\rho g \bar{h} A \frac{\bar{h}_{CP}}{\sin\theta} = \rho g \int y^2 \sin\theta dA \qquad \left(\because \bar{y}_{CP} = \frac{\bar{h}_{CP}}{\sin\theta} \right)$$

$$\bar{h} A \frac{\bar{h}_{cp}}{\sin\theta} = \sin\theta \int y^2 dA$$

$$\bar{h}_{cp} = \frac{\sin^2\theta}{\bar{h}A} I_{xx}$$

By using parallel axis theorem

$$= \frac{\sin^2\theta}{\bar{h}A} \left[I_{Gxx} + A \frac{\bar{h}^2}{\sin^2\theta} \right] \qquad \left(\because \bar{y} = \frac{\bar{h}}{\sin\theta} \right)$$

$$\bar{h}_{cp} = \frac{I_{GXX} \times \sin^2\theta}{\bar{h}A} + \bar{h}$$

For horizontal surface $\theta = 0$

$$\bar{h}_{cp} = \bar{h}, \qquad F_{gauge} = \rho g \bar{h} A$$

For vertical surface $\theta = 90°$

$$\bar{h}_{cp} = \bar{h} + \frac{\bar{I}_{Gxx}}{\bar{h}A}$$

Physical significance of $\bar{h}$ and $\bar{h}_{CP}$:-

(i) $$\bar{h}_{cp} = \bar{h} + \frac{\bar{I}_{Gxx} \times \sin^2\theta}{\bar{h}A}$$

$\frac{\bar{I}_{Gxx} \times \sin^2\theta}{\bar{h}A}$ is always positive, there for $\bar{h}_{CP}$ always greater than $\bar{h}$, this physcially mean that center of pressure (C_p) is always located below center of gravity (G).

(ii) $$\bar{h}_{cp} = \bar{h} + \frac{\bar{I}_{Gxx} \times \sin^2\theta}{\bar{h}A}$$

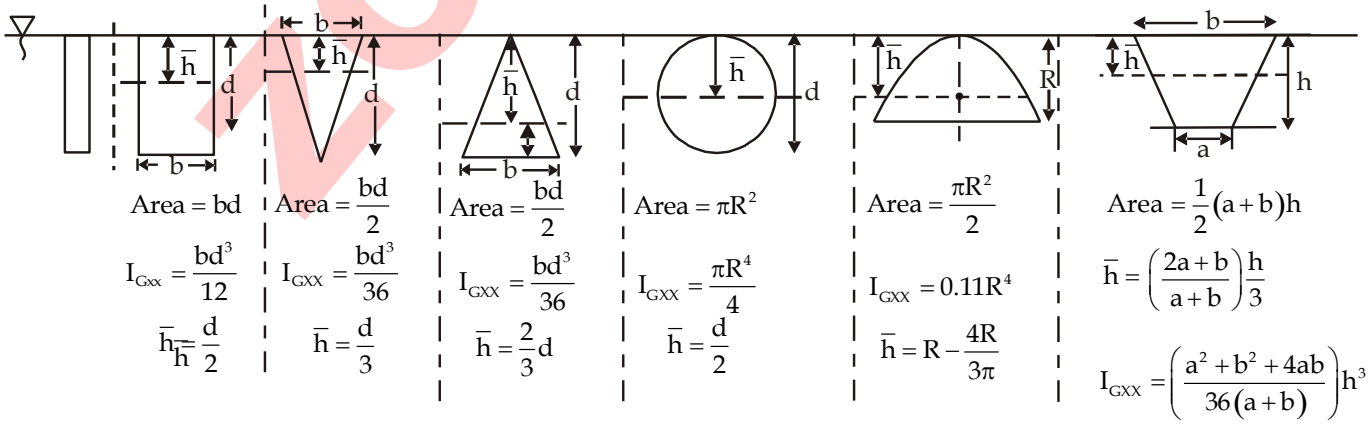
$$\bar{h}_{cp} - \bar{h} = \frac{\bar{I}_{Gxx} \times \sin^2\theta}{\bar{h}A}$$

$$\left(\bar{h}_{cp} - \bar{h}\right) \propto \frac{1}{\bar{h}} \Rightarrow \qquad \text{if } \bar{h}\uparrow \Rightarrow \left(\bar{h}_{cp} - \bar{h}\right)\downarrow$$

With the increase with depth of immersion ($\bar{h}$), $\frac{I_{Gxx} \sin^2\theta}{\bar{h}A}$ is decrease, this physically mean that with increase in depth of immersion the center of pressure (C_p) starts moving closer to center of gravity (G).

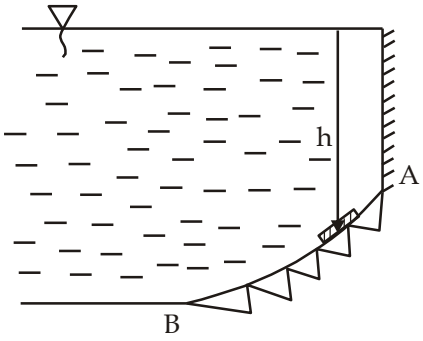
Centre of pressure :- Centre of pressure is defined as the point of application of the total pressure on the surface.

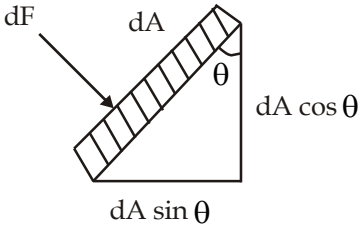
Different Shapes:-

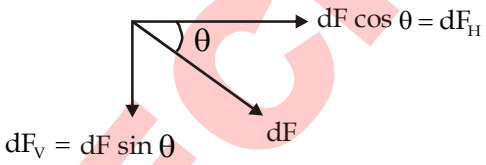


Area = bd	Area = $\frac{bd}{2}$	Area = $\frac{bd}{2}$	Area = πR^2	Area = $\frac{\pi R^2}{2}$	Area = $\frac{1}{2}(a+b)h$
$I_{Gxx} = \frac{bd^3}{12}$	$I_{GXX} = \frac{bd^3}{36}$	$I_{GXX} = \frac{bd^3}{36}$	$I_{GXX} = \frac{\pi R^4}{4}$	$I_{GXX} = 0.11R^4$	$\bar{h} = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$
$\bar{h} = \frac{d}{2}$	$\bar{h} = \frac{d}{3}$	$\bar{h} = \frac{2}{3}d$	$\bar{h} = \frac{d}{2}$	$\bar{h} = R - \frac{4R}{3\pi}$	$I_{GXX} = \left(\frac{a^2+b^2+4ab}{36(a+b)}\right)h^3$

Hydro static force on curved surface:-







Force acting on small area 'dA'

$$= dF = (\text{Area} \times \text{Pressure})$$

$$\Rightarrow dF = (\rho g h) \times dA$$

This force will have two component

Horizontal component $= (dF)_H = dF\cos\theta$

$$\Rightarrow (dF)_H = (\rho g h)\, dA\cos\theta$$

Vertical component $= (dF)_v = dF\sin\theta$

$$\Rightarrow (dF)_v = (\rho g h)\, dA\sin\theta$$

Total horizontal force on curved surface

$$= F_H = \int (dF)_H = \int (\rho g h)\, dA\cos\theta$$

$$\Rightarrow F_H = \rho g \int h\,(dA\cos\theta)$$

$$F_H = \rho g\ \bar{h}\,(A_v)$$

where:- A_v = Projected area of curved surface on vertical plane.

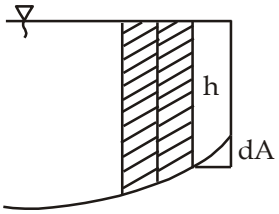
$\bar{h}$ = Depth of C.G. of projected area from free surface of water.

Total vertical force on curved surface

$$= F_v = \int (dF)_v = \int \rho g h\ dA\sin\theta$$

$$\Rightarrow F_v = \rho g \int h\,(dA\sin\theta)$$

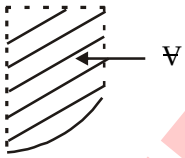
ZONE TECH



here project area is not
used because height is changed)

$F_v = \rho g \times$ (volume of water column above curved surface)

$F_v = \rho g \forall$

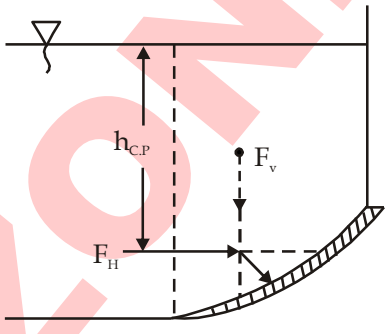


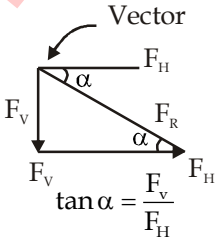
where:- $\forall$ = Volume of liquid above curved surface upto free surface of water.

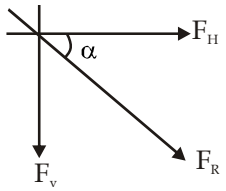
Resultant hydrostatic force on curved surface

$$= F_R = \sqrt{(F_H)^2 + (F_V)^2}$$

Location of resultant force:-







ZONE TECH

→ **Location of horizontal force from top:-**

$$\bar{h}_{CP} = \bar{h} + \frac{I_G}{A_v \bar{h}}$$

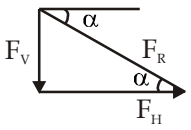
$\bar{h}$ = depth of C.G. of projected area

A_v = Projected area

→ **Location of vertical force:-**

Vertical force will act at C.G. of water column.

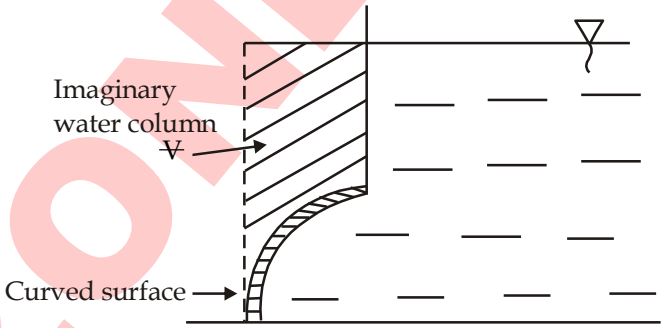
Angle of Resultant force with horizontal



$$\tan\alpha = \frac{F_v}{F_H}$$

$$\alpha = \tan^{-1}\left[\frac{F_V}{F_H}\right]$$

Special case:-



Vertical force on curved surface

F_V = weight of imaginary water column above curved surface upto free water surface

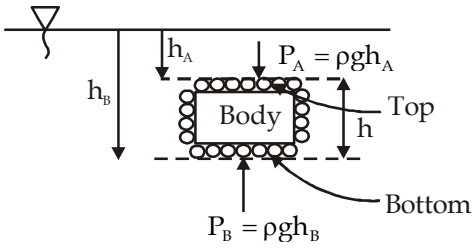
$\Rightarrow$ $F_V = \delta g \forall$ (direction of F_V will be upward)

$\forall$ = Volume of imaginary water column as shown in figure.

ZONE TECH

CHAPTER - 3 (BUOYANCY AND FLOATATION)

Hydro-static force on a body or Buoyant force:-



Force on the top surface of body $= P_A A$ $(A = \text{Area})$

Force on the bottom surface of body $= P_B A$

Net force acting on the body in vertical upward direction (or buoyancy force) $= (P_B - P_A)A$

$= (\rho g h_B - \rho g h_A)A$

$= \rho g A(h_B - h_A)$

$= \rho g h A$

$= \rho g \forall_{Displaced}$

$= \rho \forall_{Displaced} g$

$(\rho \forall_{Displaced} = \text{Mass of displaced fluid})$

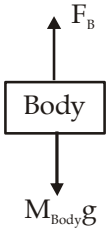
$= mg \Rightarrow$ weight of the displaced fluid

(Buoyancy Force = Net upward force = weight of liquid displaced = Upthrust)

Archimede's Principle:-

When a body submerged or imersed (fully or partially) inside the fluid then the fluid will exert a force in the vertical upward direction is known as upthrust or buoyancy force and the magnitude of this force is equal to the weight of the liquid displaced.

Force Analysis:-



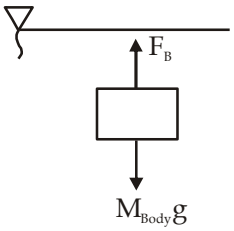
ZONE TECH

Upthrust $\Rightarrow F_B$

Down thrust $\Rightarrow M_{Body}g$

Different cases:-

Case I:- Submerged body



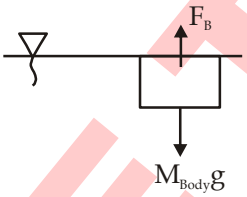
$$M_{Body}g = F_B$$

$$\rho_{Body} \forall_{Body} g = \rho \forall_{Displaced} g$$

$$\rho_{Body} = \rho_{Fluid} \qquad \left| \forall_{Displaced} = \forall_{Body} \right.$$

Relative density $= 1$

Case II:- Floating Bodies:-



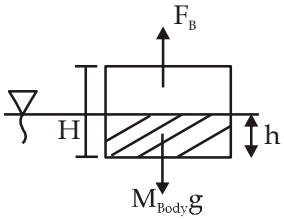
(a) $M_{Body}g = F_B$

$$\rho_{Body} \forall_{Body} g = \rho \forall_{Displaced} g$$

$$\rho_{Body} = \rho_{Fluid} \qquad \left(\because V_{body} = V_{displaced}\right)$$

Relative density $= 1$

(b) $M_{Body}g = F_B$



$$\rho_{Body} \forall_{Body} g = \rho \forall_{Displaced} g$$

$$R.D = \frac{\forall_{Displaced}}{\forall_{Body}}$$

Relative density < 1 $\qquad \left(\because V_{displaced} < V_{body}\right)$

Condition of floation, relative density ≤ 1

ZONE TECH

Depth of Immergence:-

$$h = ?$$

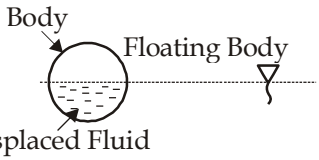
$$\rho_{Body} V_{Body} = \rho V_{Displaced}$$

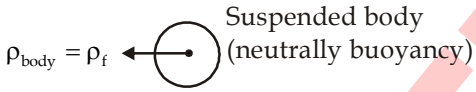
$$\rho_{Body} AH = \rho Ah$$

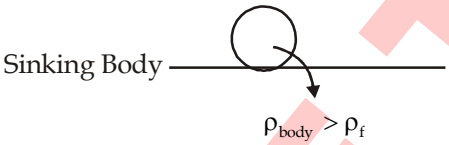
$$\rho_{Body} H = \rho h$$

$$\text{Relative density} = \frac{h}{H}$$

Note:-





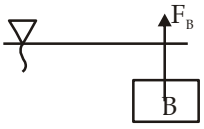


$\rho_{Body} = \rho$, $\rho_f \rightarrow$ Fluid Density

For fully submerged in fluid with constant density, the buoyancy force is independent of the distance of the body from the free surface it is also independent of the density of the solid body.

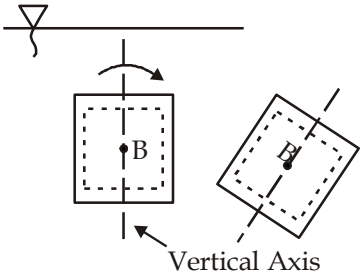
Center of Buoyancy:-

- It is a point at which buoyancy force acts.
- It is the point of center of gravity of displaced fluid.



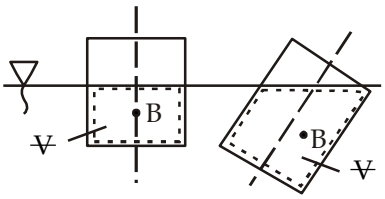
Effect of disturbance on center of buoyancy (B):-

Case I:- Submerged Body:-



ZONE TECH

Case II:- Floating body



Note:-

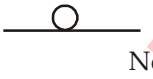
In submerged body the center of buoyancy remains on the vertical axis while a small disturbance whereas in floating body the center of buoyancy shift from the vertical axis of the body while a small disturbance and this is due to the change in shape of displaced fluid in floating body.

Stability:-

Stable Equilibrium:- When body is displaced from original position and it is tends to come back to its original position.



Neutral Equilibrium:- When body is displaced from its original position it remains in that new position.



Unstable Equilibrium:- When a body is displaced from its original position it continues to move away.



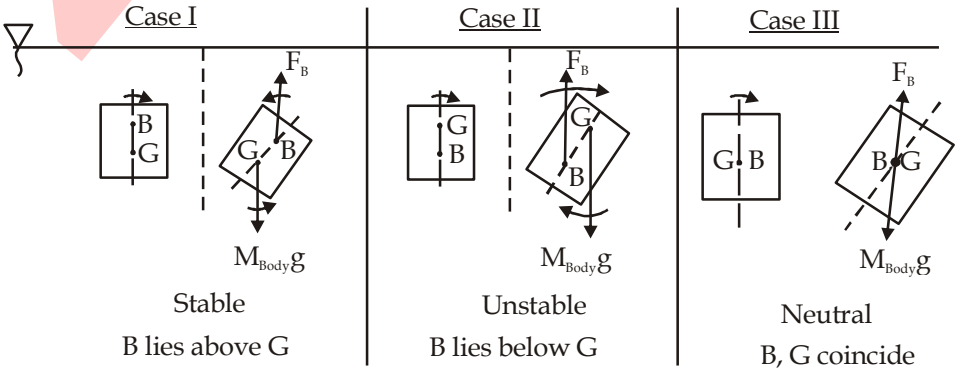
$$\text{Time of oscillation} \propto \frac{1}{\text{Stability}}$$

$$T = 2\pi\sqrt{\frac{R^2}{g(GM)}}$$

R = Radius of Gyration

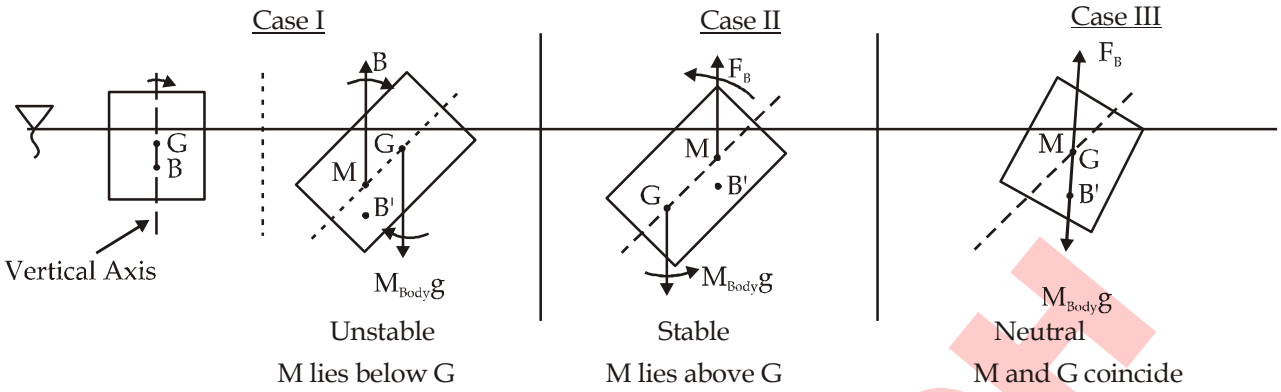
GM = Metacentric Height

Stability of submerged body:-



ZONE TECH

Stability of Floating body:-



Metacenter (M):- It is the point on vertical axis of the body at which the line of action of buyant force cuts the vertical axis of the body.

- Metacenter can lie inside or outside the body.
- It is the point about which the body oscillates.

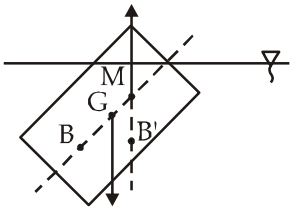
Metacentric Height (GM):-

It is the height of metacenter above the center of gravity.

$GM = +ve \rightarrow$ Stable

$GM = 0 \rightarrow$ Neutral

$GM = -ve \rightarrow$ Unstable



GM = Metacentric height

In general

$$GM = BM - BG$$

$$GM = \frac{I}{V_{Displaced}} - BG$$

where I = MOI of the surface of body cutted by the free surface of fluid.

Note:-

• M above G • BM > BG • GM = BM - BG = + ve	Stable Equilibrium
• M below G • BM < BG • GM = BM - BG = - ve	Unstable Equilibrium
• M at G • GM = 0	Neutral Equilibrium

ZONE TECH

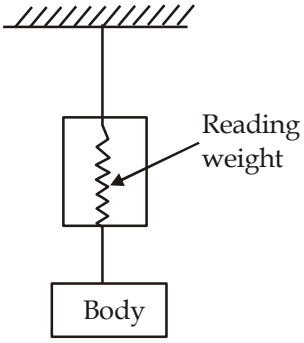
Metacentric height

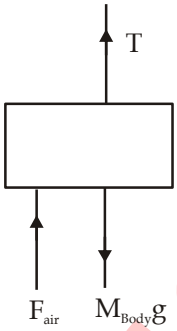
For (1) Passenger ship = (0.4 - 1.0) m

(2) War ship = (1.2 - 1.8) m

Real and Apparent weight:-

Real weight:- A weight of a body taken in air is considered as Real weight.



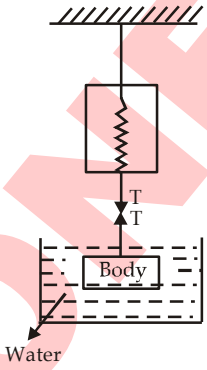


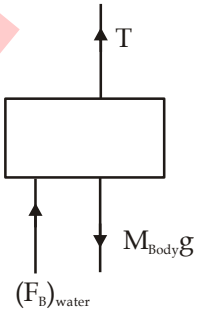
$T + F_B = M_{body} \times g$

Since F_B air is zero due to low density

$T = M_{Body}g$

Apparent weight:- A weight of a body taken in any fluid except air is considered as its apparent weight.





$T + (F_B)_{water} = M_{Body}g$

Apparent weight = Real weight - $(F_B)_{water}$

Buoyancy force = Real weight - Apparent weight

Condition of weightlesness :-

$T = 0$

So real weight $= F_{B\,water}$

$\rho_b \forall_b g = \rho \forall_{disp} . g$

$$\text{Relative density} = \frac{\forall_{displaced}}{\forall_{body}}$$

ZONE TECH

CHAPTER - 4 (FLUID KINEMATICS)

Fluid kinematics is defined as that branch of fluid mechanics which deals with motion of particles without considering the forces causing the motion and study by using parameters like velocity, angular velocity, vorticity, circulation.

The fluid motion is described by two methods:

(a) **Lagrangian Method :-** In this approach the motion of each particle is analysed. This is a microscopic study.

(b) **Eulerian Method :-** This is macroscopic study. In fluid mechanics we mostly follow Eulerian approach.

Different types of flows in fluid system:-

1. **Steady and Unsteady flow :-** If properties of fluid are not change with respect to time at a particular space that the flow is considered as steady flow. Otherwise, flow is unsteady.

Mathematically

Steady flow:- $\left.\frac{\delta F}{\delta t}\right|_{space} = 0 \quad \Rightarrow \quad \frac{\delta P}{\delta t} = 0$

$\frac{\delta V}{\delta t} = 0$

$\frac{\delta \rho}{\delta t} = 0$

Unsteady flow:- $\left.\frac{\delta F}{\delta t}\right|_{space} \neq 0$

2. **Uniform and Non-uniform flow:-** If properties of fluid are not change with respect to space at a particular time then the flow is considered as uniform flow. Otherwise, flow is non-uniform.

Mathematically

Uniform $\left.\frac{\delta F}{\delta s}\right|_{time} = 0$

Non-uniform $\left.\frac{\delta F}{\delta s}\right|_{time} \neq 0$

3. **Rotational and Irrotational flow:**

- When fluid particles rotate about their center of mass during movement. Flow is said to be rotational, otherwise irrotational.
- If we consider a flat plate, region closer to the boundary of plate is significantly affected by viscosity and hence is called viscous region (boundary layer region).
- Away from boundary layer, effect of viscosity is neglegible. Hence region is called inviscid region.

ZONE TECH

- Rotation of fluid particles in a flow field is caused by viscosity. Hence in viscous region flow will be rotational. If effect of viscosity is negligible, flow can be considered as irrotational as in inviscid region, rapidly accelerating region etc.
- A fluid may also be in rotational motion in the absence of viscosity in a flow field due to some rotational motion given to it earlier i.e. before entering the flow field.

 → Irrotational (Translation only)

 → Rotational (Translation + Rotation)

4. **Compressible and Incompressible Flow:-** If density of fluid changes then it is called compressible flow. Otherwise, incompressible flow.

For compressible flow

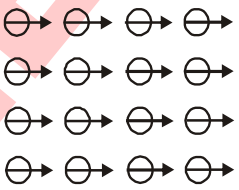
$$d\rho \neq 0$$

For incompressible flow

$$d\rho = 0$$

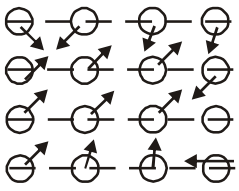
5. **Laminar and Turbulent Flow:-** Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream line and all the stream-lines are straight and parallel

In laminar flows the particles do not jump from one layer to another layer at in a particular plane, the direction vectors of the particles are in same direction.



Turbulent flow:- Turbulent flow is that type of flow in which the fluid particles move in a zig-zag way.

In turbulent flows particles jumped from one layer to another so the velocity vectors of the particles lies in a particular plane different in direction.



Random movement of the particles

5. **One, Two or three-dimensional flow with respect to space:-**

One-Dimensional Flow :-

If flow parameters varies in one dimension with respect to space only, the flow is one dimensional i.e $V = f(x, t)$

- In one dimensional flow, the fluid parameters (velocity, pressure, temperature, density, viscosity etc) remain constant throughout any cross section normal to flow direction but vary along longitudinal direction.
- Flow field is represented by streamlines which are essentially straight and parallel.

Two-Dimensional Flow:-

If flow parameter varies in two dimension with respect to space, the flow is two dimensional flow i.e. $V = f(x, y, t)$

In two dimensional flow, the flow velocity and other fluid parameters vary along two direction (longitudinal and vertical).

Three-Dimensional Flow:-

If flow parameters varies in three dimension with respect to space only. For 3-dimensional flow $V = f(x, y, z, t)$

Flow properties vary in all three direction.

Ex:- Flow at bend in pipe, flow within fluid machines, flow at an inlet to a nozzle.

Type of flow	Steady	Unsteady
1 - D	$V = f(x)$	$V = f(x, t)$
2 - D	$V = f(x, y)$	$V = f(x, y, t)$
3 - D	$V = f(x, y, z)$	$V = f(x, y, z, t)$

Note:-

1. Flow through a long constant diameter pipe at a constant flow rate → Steady uniform flow.
2. Flow through a long diameter pipe at either decreasing or increasing flow rate → Unsteady uniform flow.
3. Flow through a tapering pipe at constant flow rate → Steady non-uniform flow.
4. Flow through a tapering pipe at either decreasing or increasing flow rate → Unsteady non-uniform flow.

Rate of flow or discharge :- It is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel.

Consider a liquid flowing through a pipe in which

A = Cross-sectional area of pipe

V = Average velocity of fluid across the section

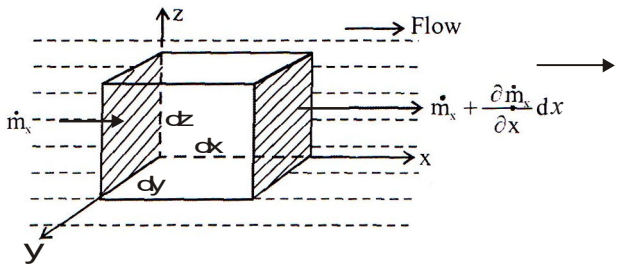
Then discharge $Q = A \times V$

Continuity equation :-

- Continuity equation is based on principle of conservation of mass.
- This is the only equation which indicates whether the flow is possible or not.
- For the flow to be possible continuity equation must be satisfied.

Consider a fluid element as show in figure.

$$(\dot{m}_{in} - \dot{m}_{out}) = \dot{m}_{storage}$$



In general 3-D flow

$$\vec{V} = u\hat{i} + v\hat{j} + +w\hat{k}$$

u, v, w $\rightarrow$ velocity component in x,y, z direction

$$\vec{V} = F(x, y, z, t)$$

From conservation of mass–

$$(\dot{m}_{in} - \dot{m}_{out})_x = \dot{m}_x - \left(\dot{m}_x + \frac{\partial \dot{m}}{\partial x} dx\right)$$

$$= -\frac{\partial \dot{m}_x}{\partial x} dx = -\frac{\partial}{\partial x}[(\rho(dydz)u)dx] \qquad (\because dv = dx\,dy\,dz)$$

$$(\dot{m}_{in} - \dot{m}_{out})_x = -\frac{\partial}{\partial x}(\rho u dv) \qquad .....(1)$$

Similarly,

$$(\dot{m}_{in} - \dot{m}_{out})_y = -\frac{\partial}{\partial x}(\rho v\, dv) \qquad .....(2)$$

$$(\dot{m}_{in} - \dot{m}_{out})_z = -\frac{\partial}{\partial z}(\rho w\, dv) \qquad .....(3)$$

by adding all above 3 equations we get

$$(\dot{m}_{in} - \dot{m}_{out}) = -dv\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial x}(\rho v) + \frac{\partial}{\partial x}(\rho w)\right]$$

$\because$ $(\dot{m}_{in} - \dot{m}_{out}) = \dot{m}_{storage}$

and $\frac{\partial m_{st}}{\partial t} = \frac{\partial}{\partial t}(\rho dv) = dv.\frac{\partial \rho}{\partial t}$

$\Rightarrow$ $$-dv\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] = dv.\frac{\partial \rho}{\partial t}$$

$\Rightarrow$ $$\boxed{\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{d\rho}{dt} = 0}$$

ZONE TECH

This equation is known as 3-D General continuity equation.

(1) Steady Flow, $\frac{\partial \rho}{\partial t} \to 0$

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0$$

(2) Steady, Incompressible flow $(\rho = \text{constant})$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

(3) Steady, Incompressible, 2-D flow

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

(4) 1-D, steady flow

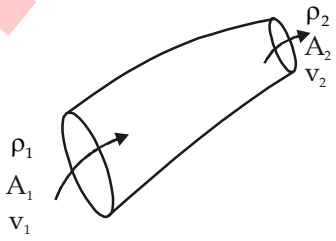
$$(\dot{m}_{in}-\dot{m}_{out})_x=0 \qquad \vec{v}=u\hat{i}$$

$$\frac{\partial}{\partial x}(\rho A u)=0 \qquad (\because v=u)$$

$\Rightarrow$ $$\frac{\partial}{\partial x}(\rho A v)=0$$

$$\rho_1 A_1 v_1 = \text{constant}$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$



Case I:- If flow is incompressible

$$\rho_1 = \rho_2$$

$$A_1 v_1 = A_2 v_2$$

Case II:- If flow is compressible

$$\rho_1 \neq \rho_2$$

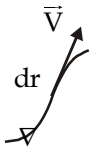
$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

ZONE TECH

Representation of flow :-

Stream Line:-

It is imaginary curve drawn in a flowing fluid such that tangent drawn at any point on this line directly represents the direction of the velocity vector of the fluid particle at that point.



Equation of stream line

Take a small position vector dr along stream line.

$$\vec{dr} = (dx)\hat{i} + (dy)\hat{j} + (dz)\hat{k}$$

$$\vec{V} = \left[\left(u\hat{i}\right) + \left(v\hat{j}\right) + \left(w\hat{k}\right)\right]$$

Since velocity vector and direction vector parallel to each other so cross product of both will be zero.

$$\vec{dr} \times \vec{V} = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ dx & dy & dz \\ u & v & w \end{vmatrix} = 0$$

$$\hat{i}\underbrace{(wdy - vdz)} - \hat{j}\underbrace{(wdx - udz)} + \hat{w}\underbrace{(vdx - udy)}$$
$$\Downarrow \quad\quad \Downarrow \quad\quad \Downarrow$$
$$0 \quad\quad 0 \quad\quad 0$$

$$\frac{dy}{v} = \frac{dz}{w}, \frac{dx}{u} = \frac{dz}{w}, \frac{dx}{u} = \frac{dy}{v}$$

Equation of stream line $\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$

Thus the equation of a stream line in x-y plane

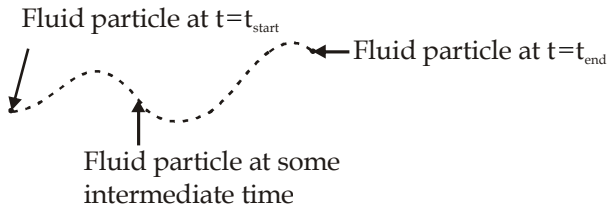
$$\frac{dx}{u} = \frac{dy}{v} \Rightarrow \frac{dy}{dx} = \frac{v}{u}$$

Note:-

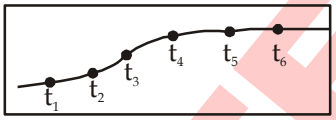
1. A streamline can neither intersect itself nor two streamline can cross each other.
2. The spacing of streamline varies inversely with velocity. This is because the flow (discharge) between streamline cannot go out of the space between streamline. Hence, as the spacing between streamline increase, velocity decrease.

ZONE TECH

Path line:-

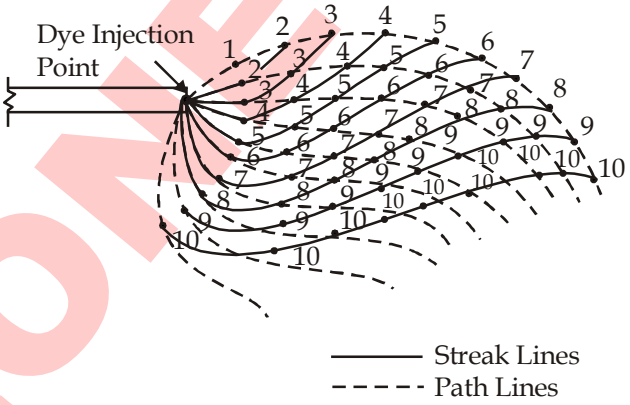


- It is the line traced by a single fluid particle as it move through space over a period of time.
- A pathline can intersect itself at different times.
- Path line is actual line and stream line is imaginary line.
- Path line is a lagrangian concept.
- Path line is not an instantaneous limit involves a finite time period. The figure below shows the position of particles at different time instants. When all these positions are jointed we get path line.



Streak line or filament line:-

- It is the line formed by joining all the particles that have crossed a given point in the flow field over a period of time.
- It is used for tracing the movement of gases.



Acceleration of fluid particles:-

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

where u, v and w are components of velocity in x, y and z direction

$$\vec{V} = fn\,(x, y, z, t)$$

So $$\vec{V} = u\,(x, y, z, t)\,\hat{i} + v(x, y, z, t)\,\hat{j} + w(x, y, z, t)\,\hat{k}$$

$$a_x = \frac{dv}{dt} = \frac{\delta u}{\delta x}\frac{dx}{dt} + \frac{\delta u}{\delta y}\frac{dy}{dt} + \frac{\delta u}{\delta z}\frac{dz}{dt} + \frac{\delta u}{\delta t}$$

$$a_x = u\frac{\delta u}{\delta x} + v\frac{\delta u}{\delta y} + w\frac{\delta u}{\delta z} + \frac{\delta u}{\delta t}$$

Similarly,

$$a_y = u\frac{\delta v}{\delta x} + v\frac{\delta v}{\delta y} + w\frac{\delta v}{\delta z} + \frac{\delta v}{\delta t}$$

$$a_z = u\frac{\delta w}{\delta x} + v\frac{dw}{\delta y} + w\frac{\delta w}{\delta z} + \frac{\delta w}{\delta t}$$

$$a = \sqrt{a_x^{\,2} + a_y^{\,2} + a_z^{\,2}} \text{ unit}$$

Total acceleration = Convective acceleration or Advective acceleration (a_c) + local or temporal acceleration (a_T).

Local or temporal acceleration :- It is the rate of increase of velocity with respect to time at a given point in a flow field.

Convective acceleration :- It is the rate of change of velocity due to the change of position or fluid particles in a fluid flow.

For steady flow $\Rightarrow$ Total acceleration = Convective acceleration

For uniform flow $\Rightarrow$ Total acceleration = Temporal acceleration

For steady and uniform flow $\Rightarrow$ Total acceleration = 0

Ex:-

1. Flow through a long constant diameter pipe at a constant flow rate $\rightarrow a_C = 0, a_T = 0$.
2. Flow through a long diameter pipe at either decreasing or increasing flow rate $\rightarrow a_C = 0, a_T \neq 0$
3. Flow through a tapering pipe at constant flow rate $\rightarrow a_C \neq 0, a_T = 0$
4. Flow through a tapering pipe at either decreasing or increasing flow rate $\rightarrow a_C \neq 0, a_T \neq 0$

Types of motion :-

A fluid particle while moving may undergo anyone or combination of following four types of displacements

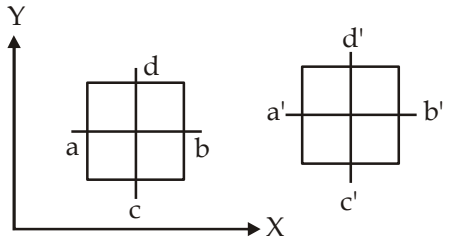
(i) Linear translation or Pure translation

(ii) Linear deformation

(iii) Angular deformation

(iv) Rotation

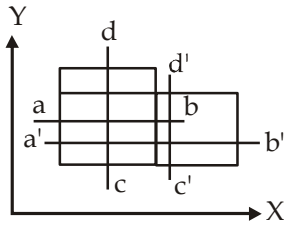
(1) Linear Translation:- It is defined as the movement of a fluid element in such a way that it moves bodily from one position to another position.



Condition for L.T.

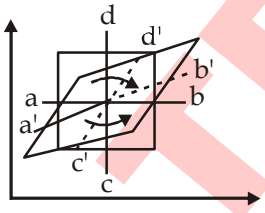
- Bodily motion
- Two axis should be perpendicular to each other before and after.
- ab//a'b' and dc//d'c'

(2) **Linear strain or deformation:-** It is defined as the deformation of a fluid element in linear direction when the element moves.



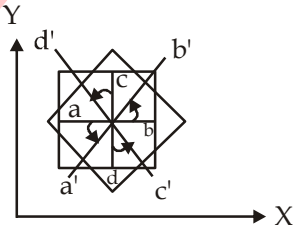
- Bodily motion not follow.
- Two axis should be perpendicular to each other before and after.
- ab//a'b' and dc//d'c'

(3) **Angular Deformation or shear strain:-** It is defined as the average change in the angle contained by two adjacent sides.



- ab and cd move in opposite direction
- (Shear strain or Deformation rate) $= \frac{1}{2}\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right]$

(4) **Pure Rotation:-** It is defined as the movement of a fluid element in such a way that both of its axes (horizontal as well as vertical) rotate in the same direction.



- ab and cd move in same direction
- Rotate about its own mass center.

Rotation in X-Y plane

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$$\omega_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]$$

ZONE TECH

$$\omega_y = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right]$$

$$\omega = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

or

$$\omega = \frac{1}{2}\left(\vec{\nabla} \times \vec{V}\right)$$

$$= \frac{1}{2} \text{ Curl } \vec{V}$$

$$\omega = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$$

$$\omega = \sqrt{\omega_x^2 + \omega_y^2 + \omega_z^2}$$

Irrotational flow:-

$$\left.\begin{aligned} \omega_x &= 0 \\ \omega_y &= 0 \\ \omega_z &= 0 \end{aligned}\right\}$$ for 3-D flow all of these should be zero

$\omega_z = 0$ for 2-D flow $\Rightarrow$ $\omega_z = 0 \rightarrow$ Irrotational flow

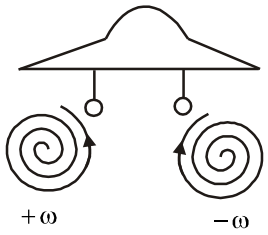
$\omega_z \neq 0 \rightarrow$ Rotational flow

Vorticity (ζ):-

Vorticity is the vector that describe the local spining motion of a fluid particle near some point as would be seen by an observer located at that point and travelling with the flow. Mathematically it is equal to twice of the rotation.

Vorticity for a 2-D flow field is always perpendicular to the plane at rotation.

Vorticity is the measurement of rotation of a fluid particle.



$$\zeta = \omega - (-\omega)$$

$$\zeta = 2\omega$$

Vorticity = 2 × Rotation

$$\zeta = 2 \times \frac{1}{2} \text{Curl } \vec{V}$$

$$\zeta = \text{Curl } \vec{V}$$

$$\zeta = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

Circulation (Γ)

Flow along a close curve is define as a circulation.

Mathematically:- It is equal to one integral of the tangential velocity component taken completely along the close curve.

Or

It is the line integral of velocity vector along the closed loop path.

$$\Gamma = \oint \vec{V} \, \vec{dr}$$

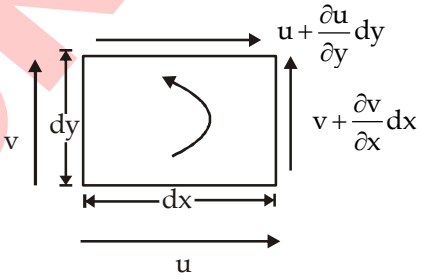
$$\vec{V} = u\hat{i} + v\hat{j}$$

$$\vec{dr} = dx\hat{i} + dy\hat{j}$$

$$\Gamma = \oint \left(u\hat{i} + v\hat{j}\right)\left(dx\hat{i} + dy\hat{j}\right)$$

$$\Gamma = \oint udx + vdy$$

For 2-D



$$\Gamma = udx + \left(v + \frac{\partial v}{\partial x}dx\right)dy - \left(u + \frac{\partial v}{\partial y}dy\right)(dx) - v(dy)$$

$$\Gamma = \frac{\partial v}{\partial x}dxdy - \frac{\partial u}{\partial y}dydx$$

$$\Gamma = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)(dx\,dy)$$

Γ = vorticity × area

That circulation is strength of vorticity in a particular area.

Velocity potential function (ϕ):-

It is a scalar function at space and time such that its negative derivative with respect to any direction gives the component of velocity along that direction. (-ve) sign show the flow take place in the direction of decreasing potential

$$\phi = fn(x, y, z, t)$$

$$-\frac{\partial\phi}{\partial x} = u$$

$$-\frac{\partial\phi}{\partial y} = v$$

$$-\frac{\partial\phi}{\partial z} = w$$

Properties of ϕ:-

1. For Irrotational flow:-

In X-Y plane $\omega_z = 0$

So $$\frac{1}{2}\left|\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right| = 0$$

$$= \frac{1}{2}\left|-\frac{\partial^2\phi}{\partial x\partial y} + \frac{\partial^2\phi}{\partial x\partial y}\right| = 0$$

Since ϕ is a continuous function (i.e. difference with respect to x or y result will be same)

$$\frac{\partial^2\phi}{\partial x\partial y} = \frac{\partial^2\phi}{\partial x\partial y} \qquad \text{So } w_z = 0$$

- ϕ exist only for Irrotational flows. Hence Irrotational flows are also known as potential flow.
- If $\nabla^2\phi = 0$ then flow is rotational

2. **Steady Incompressible flow:-**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \Rightarrow \text{Laplace equation}$$

$$\nabla^2\phi = 0$$

If ϕ satisfy laplace equation then the flow will be steady, incompressible and irrotational flow.

3. **Equipotential line:-**

A line along which ϕ = constant is called equipotential line

$$d\phi = 0$$

$\phi = f(x, y)$ Line will be in 2-D

For steady flow $t = 0$

$$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy$$

$$\frac{\partial \phi}{\partial x} = - u$$

$$\frac{\partial \phi}{\partial y} = - v$$

$$0 = - udx - vdy$$

$udx + vdy = 0 \Rightarrow$ equation of equipotential line

$$\text{Slope} = \frac{dy}{dx} = \frac{-u}{v} \qquad \text{.....(i)}$$

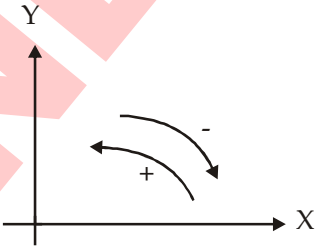
Stream Function (ψ):-

It is defined as a scalar functon of space and time such that its partial derivative with respect to any direction gives the component of velocity at right angle to this direction (counter clock wise direction taken as positive).

$$\psi = f(x, y, t)$$

$$\frac{\partial \psi}{\partial x} = v$$

$$\frac{\partial \psi}{\partial y} = -u$$



Important properties:-

1. **Steady and Incompressibe flow:-** $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$$u = -\frac{\partial \psi}{\partial y}$$

$$v = +\frac{\partial \psi}{\partial x}$$

$$-\frac{\partial^2 \psi}{\partial x \partial y} + \frac{\partial^2 \psi}{\partial y \partial x} = 0$$

Since ψ is a continuous function so ψ exist only for incompressible flow.

2. **Calculate ω_z for Irrotational flow:-**

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

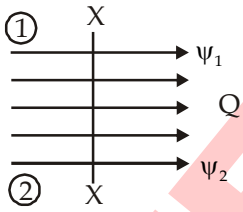
$$= \frac{1}{2}\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right] \quad \text{Laplace equation}$$

If ψ satisfy laplace equation $w_z = 0$ so the flow become irrotational.

Note:- If ψ satisfy laplace equation then the flow is irrotational. if $\nabla^2\Psi = 0$ then flow is a irrotational

If $\nabla^2\Psi \neq 0$ then flow is rotational

3. **Discharge along a line:-**



$Q = \psi_1 - \psi_2\ m^2/sec$

Q = Discharge Per unit width

Discharge across the line joining two point at which the value of stream function is ψ_1 and ψ_2 respectively is given as $Q = |\psi_1 - \psi_2|$ absolute value (always +ve)

4. **Stream line:-** A line along which ψ = constant is called stream line.

ψ = constant

So $\partial\psi = 0$

$\therefore$ $\psi = f(x, y)$

$$v = \frac{d\psi}{dx}$$

$$-u = \frac{\partial\psi}{\partial y}$$

$$d\psi = \frac{\partial\psi}{\partial x}dx + \frac{\partial\psi}{\partial y}dy$$

$$0 = vdx - udy = 0$$

Slope of stream line $\frac{dy}{dx} = \frac{v}{u}$ ....(ii)

Relation between equipotential and stream line slope

Slope of stream line $\frac{dy}{dx} = \frac{v}{u}$

Slope of equipotential line $\frac{dy}{dx} = \frac{-u}{v}$

(Slope of equipotential line) × (Slope of stream line)

$$-\frac{u}{v} \times \frac{v}{u} = -1 \quad \text{....(iii)}$$

Note:- Equipotential line and stream line are perpendicular to each other or intersect at right angle.

Velocity Potential Function (ϕ)	Stream Function(ψ)
1. In a flow ϕ may exist or not	Always exist in a flow
2. ϕ may be exist in 2-D or 3-D	Defined for 2-D flow only
3. If ϕ exist the boundation with ϕ $V^2\phi = 0$	No any boundation (always satisfy continuity equation)
4. Physical significance irrotational flow	Physical significance is Q (discharge) in per unit width.
5. Slope of Equi-Potential line $\frac{dy}{dx} = -\frac{u}{v}$	Slope of stream function line $\frac{dy}{dx} = \frac{v}{u}$

Complex potential

The conditions $\frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y}$ and $\frac{\partial\phi}{\partial y} = -\frac{\partial\psi}{\partial x}$ are satisfied if $\phi + i\psi$ is taken, as function of z i.e.

$$f(z) = w, \text{ where } z = x + iy$$

$$\therefore \quad w = \phi + i\psi = f(z) = f(x + iy)$$

$$\phi + i\psi = f(x + iy) \qquad \ldots (i)$$

Differentiate equation (i) with respect to x

$$\frac{\partial\phi}{\partial x} + i\frac{\partial\psi}{\partial x} = f'(x + iy) \qquad \ldots (a)$$

Differentiate equation (i) with respect to y

$$\frac{\partial\phi}{\partial y} + i\frac{\partial\psi}{\partial y} = f'(x + iy) \quad (i)$$

Put the value $f'(x + iy)$ from equation (a)

$$\frac{\partial\phi}{\partial y} + i\frac{\partial\psi}{\partial y} = i\left(\frac{\partial\phi}{\partial x} + i\frac{\partial\psi}{\partial x}\right)$$

$$\frac{\partial\phi}{\partial y} + i\frac{\partial\psi}{\partial y} = i\frac{\partial\phi}{\partial x} - \frac{\partial\psi}{\partial x}$$

Equating the real and imaginary part

$$\frac{\partial\phi}{\partial y} = -\frac{\partial\psi}{\partial x} \text{ and } \frac{\partial\psi}{\partial y} = \frac{\partial\phi}{\partial x}$$

Hence ϕ and ψ satisfied the cauchy-riemann equation then w is analytic function of z and w is known as complex potential.

ZONE TECH

CHAPTER - 5 (FLUID DYNAMICS)

Dynamics of fluid flow is the study of fluid motion with the forces causing the motion.

Dynamic behaviour of fluid flow is analysed by newton's second law of motion $\vec{F} = M\vec{a}$

In the fluid flow various forces acting are

(a) Gravity Force $(\vec{F_g})$

(b) Pressure Force $(\vec{F_p})$

(c) Viscous Force $(\vec{F_v})$

(d) Force due to compressibility $(\vec{F_c})$

(e) Force due to turbulence $(\vec{F_t})$

(f) Minor forces like surface tension $(\vec{F_\sigma})$

- When all forces are taken into account the equation of motion is called newton's equation of motion.
- When compressibility and other minor forces are neglected, it is called Reynold's equation of motion

$$\vec{F_g} + \vec{F_p} + \vec{F_v} + \vec{F_t} = M\vec{a}$$

- When compressibility, turbulence and minor forces are neglected and only gravity, pressure and viscosity is taken into account, the equation of motion is called navier-stock equation of motion

$$\vec{F_g} + \vec{F_p} + \vec{F_v} = M\vec{a}$$

- When only gravity and pressure force are considered the equation of motion is called Euler's equation of motion

$$\vec{F_g} + \vec{F_p} = M\vec{a}$$

- Dynamics of fluid motion when only pressure and gravity is accounted for, is governed by momentum principle (Euler's equation) and energy principle (Bernoulli equation).
- Bernoulli equation - Integration of Euler's equation of motion along a stream line under steady incompressible condition is called bernoulli equation.

Euler equation is an equation in which the forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of a fluid element along a stream-line as

Consider a stream-line in which flow is taking place in s-direction as shown in figure. Consider a cylinderical element of cross-section dA and length ds.

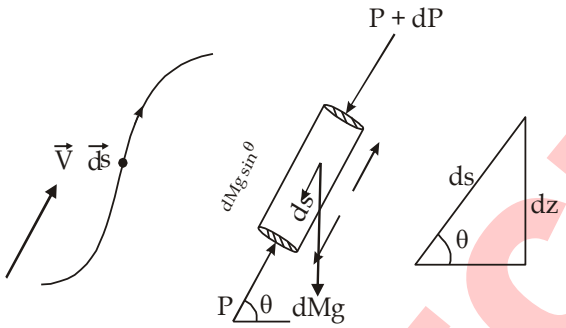
ZONE TECH

Assumptions.

1. Flow is streamlined $[\nabla \times \vec{V} = 0]$

2. Flow is invisid $[F_{Viscous} = 0]$

Let consider a stream lined flow.



writing eq^n in s-direction

$$(P.dA) - (P + dP)dA - dMg\sin\theta = dM.a_s$$

$$-dPdA - \rho dA.(dz)g = \rho\, dA\, ds.a_s$$

$$dP + \rho ds a_s + \rho g\, dz = 0$$

$\because$ $$a_s = v\frac{\delta V}{\delta S} + \frac{\delta V}{\delta t}$$

$$\boxed{dP + \rho.ds\left(V\frac{\delta V}{\delta S} + \frac{\delta V}{\delta t}\right) + \delta g\, dz = 0} \qquad \text{.....(1)}$$

Equation (1) is Euler's equation for stream lined flow

For steady flow $\frac{\delta V}{\delta t} = 0$ and $\frac{\delta V}{\delta S} = \frac{dV}{ds}$

$$dP + \rho ds\, v\left(\frac{dV}{dS}\right) + \rho g\, dz = 0$$

$$dP + \rho V\, dV + \rho g\, dz = 0$$

$$\boxed{\frac{dP}{\rho} + VdV + gdz = 0} \qquad \text{.....(2)}$$

Equation (2) is Euler's equation for steady and streamlined flow.

Bernoullis equation from Euler's Equation

Bernoulli's equation is obtained by integration the Euler's equation of motion as

$$\int\frac{dP}{\rho} + \int gdz + \int vdv = \text{constant}$$

ZONE TECH

If flow is incompressible, ρ is constant and

$$\therefore \quad \frac{P}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

$$\text{or} \quad \frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

$$\text{or} \quad \frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant} \qquad \text{... (1)}$$

Equation (1) is Bernoullis' equation in which

$\frac{p}{\rho g}$ = Pressure energy per unit weight of fluid or pressure head.

$\frac{v^2}{2g}$ = kinetic energy per unit weight or kinetic head.

z = potential energy per unit wieght or potential head.

The following are the assumptions made in the derivation of Bernoulli's equation.

- The fluid is ideal, i.e., viscoity is zero
- The flow is steady
- The flow is incompressible
- The flow is irrotational
- The flow is in strem line

Different forms of Bernoulli equation:-

1. **Ist (Energy per unit volume):-**

$$P + \frac{1}{2}\rho v^2 + \rho gz = \text{constant}$$

$P \rightarrow$ Pressure energy per unit volume

$\frac{1}{2}\rho v^2 \rightarrow$ Kinetic energy per unit volume

$\rho gz \rightarrow$ Potential energy per unit volume

$$\text{Unit} = \frac{\text{Joule}}{m^3} = \frac{N \times m}{m^3} = \frac{N}{m^2}$$

2. **2nd (Energy per unit weight or Head):-**

$$\frac{P}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

$\frac{P}{\rho g} \rightarrow$ Pressure energy per unit weight or pressure head

$\frac{v^2}{2g} \rightarrow$ Kinetic energy per unit weight or kinetic head.

$z \rightarrow$ Potential energy per unit weight or Datum head

3. **3rd (Energy per unit Mass):-**

$$\frac{P}{\rho}+\frac{v^2}{2}+gz=\text{constant}$$

$\frac{P}{\rho} \rightarrow$ Pressure energy per unit volume

$\frac{v^2}{2} \rightarrow$ Kinetic energy per unit volume

$gz \rightarrow$ Potential energy per unit volume

Bernoulli Theorem:- It states that in a steady, ideal flow of an incompressible fluid the total energy at any point of the fluid is constant. The total energy consists of pressure energy, kinetic energy and potential enegy or datum enegy. These energies per unit weight of the fluid are:

$$\text{Pressure energy} = \frac{p}{\rho g}$$

$$\text{Kinetic energy} = \frac{v^2}{2g}$$

$$\text{Datum energy} = z$$

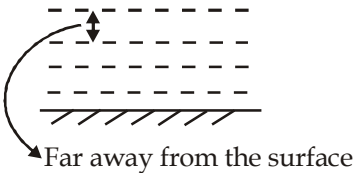
Thus mathematically, Bernoulli's theorem is written as

$$\frac{p}{w}+\frac{v^2}{2g}+z = \text{Constant}$$

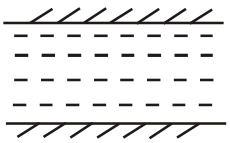
Note:- Bernoulli's equation is based on conservation of energy.

Application:-

1.



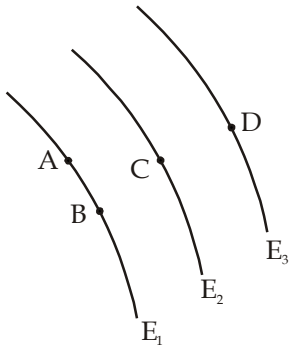
2.



Pipe Flow

ZONE TECH

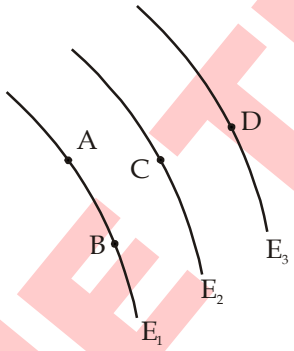
3. Irrotational



$$E_A = E_B = E_C = E_D$$

$$E_1 = E_2 = E_3$$

4. Rotational flow:-



$$E_A = E_B \neq E_C \neq E_D$$

$$E_1 \neq E_2 \neq E_3$$

- For irrotational flow the bernoulli's equation can be applied between any two points in the flow.
- For rotational flow bernoulli equation can be applied when the point lies on the same stream line.

$$\text{Total Head} = \frac{P}{\rho g} + \frac{v^2}{2g} + z$$

$$\text{Piezometric Head} = \frac{P}{\delta g} + z$$

- We don't apply bernoulli equation near the plate surface because there is viscous force due to which energy losses occur.
- In internal flow bernoulli equation is valid only at center line in pipe flow. We don't apply bernoulli equation near the surface of pipe due to highly viscous zone.
- In external flow bernoulli equation valid outside of boundary layer.

ZONE TECH

Flow Measurement Devices:-

(1) Venturimeter

(2) Orifice Meter

(3) Pitot Tube

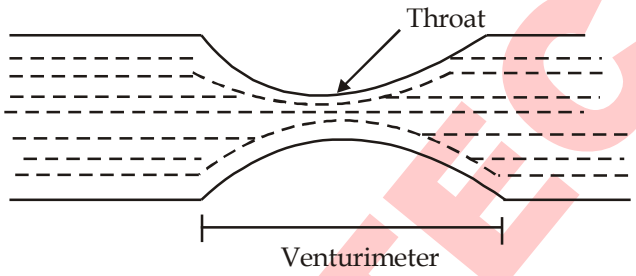
Arrangements:-

(1) With Piezometer

(2) With differential U-tube manometer

(1) **Venturimeter:-**

It is used for measurement of discharge (m^3/sec).



Coefficient of discharge (C_d)

$$C_d \in [0.97 - 0.99]$$

$$C_d = \frac{Q_{Actual}}{Q_{theoritical}}$$

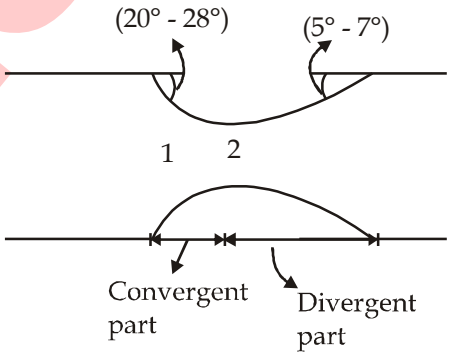
Construction Details:-

It consists of three parts :

(i) A short converging part

(ii) Throat

(iii) Diverging part



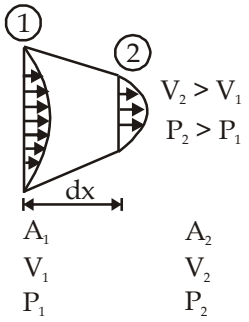
$$D_2 \in \left[\frac{D_1}{3} \text{ to } \frac{D_1}{2}\right]$$

Convergent angle = 20° - 28°

Divergent angle = 5° - 7°

Length of Divergent part = (3 - 4) × length of convergent part

Converging Part



By continuity equation

$$A_1V_1 = A_2V_2$$

$$A_1 > A_2$$

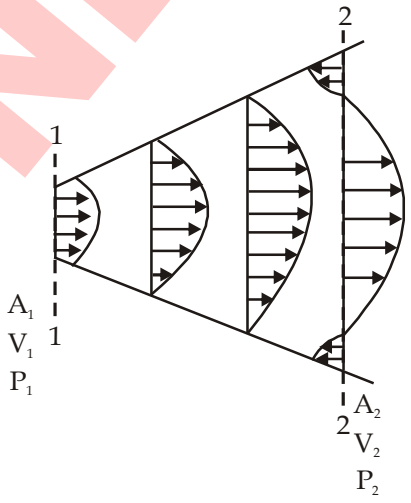
$$V_2 > V_1$$

So $P_2 < P_1$

$\therefore$ $dP = P_2 - P_1 \Rightarrow$ -ve

$$\frac{dP}{dx} < 0 \rightarrow \text{Flow favourable condition}$$

Diverging Part



$$V_2 < V_1$$

$$P_2 > P_1 \Rightarrow dP = +\text{ ve}$$

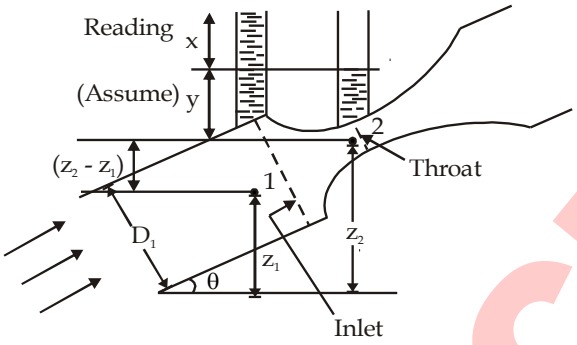
$$\frac{dP}{dx} > 0 \rightarrow \text{Adverse condition of flow}$$

ZONE TECH

So the diverging angle of the diverging section of venturimeter is kept so small and divergent length is larger than convergent length to reduce the probability of flow separation.

Installation:-

With piezometer:-



Theoretical Discharge

Consider a venturimeter fitted in a pipe through which a fluid is flowing (say water), as shown in Fig.

Let D_1 = diameter at inlet or at section (1),

D_2 = diameter at throat or at section (2),

p_1 = pressure at section (1)

p_2 = pressure at section (2)

v_1 = velocity of fluid at section (1),

v_2 = velocity of fluid at section (2),

A_1 = area at section (1) $= \frac{\pi}{4}D_1^2$

A_2 = area at section (2) $= \frac{\pi}{4}D_2^2$

and D_2, p_2, v_2, A_2 are corresponding values at section (2)

Apply general equation between 1 and 2

$$\frac{P_1}{\rho g}+\frac{v_1^2}{2g}+z_1 = \frac{P_2}{\rho g}+\frac{v^2}{2g}+z_2$$

$$\frac{v_2^2}{2g}-\frac{v_1^2}{2g} = \left(\frac{P_1}{\rho g}+z_1\right)-\left(\frac{P_2}{\rho g}+z_2\right)$$

$\because$ $Q_{th} = A_1V_1 = A_2V_2$

$$V_1 = \frac{Q_{th}}{A_1},\ V_2 = \frac{Q_{th}}{A_2}$$

$$\frac{Q^2_{th}}{2g}\left[\frac{1}{A_2^2}-\frac{1}{A_1^2}\right] = h$$

$$\frac{Q^2_{th}}{2g}\left[\frac{A_1^2-A_2^2}{A_1^2A_2^2}\right] = h$$

ZONE TECH

$$Q^2_{th} = \frac{A_1^2 A_2^2}{A_1^2 - A_2^2} 2gh$$

$$Q_{th} = \frac{A_1 A_2}{A_1^2 - A_2^2}\sqrt{2gh}$$

Actual Discharge:-

$$Q_{actual} = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g(h - h_L)}$$

$$Q_{actual} = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh} \times \sqrt{\frac{h - h_L}{h}}$$

$$C_d = \sqrt{\frac{h - h_L}{h}}$$

$$Q_{actual} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

$$C_d = \sqrt{\frac{h - h_L}{h}}$$

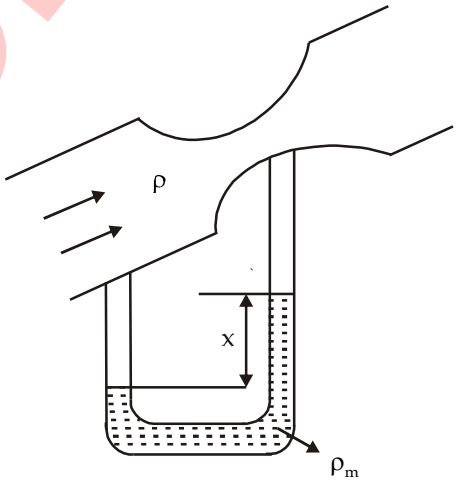
Note:-

1. With Piezometer:-

$$h = x$$

2. With simple difference U tube manometer

$$\rho_m > \rho$$

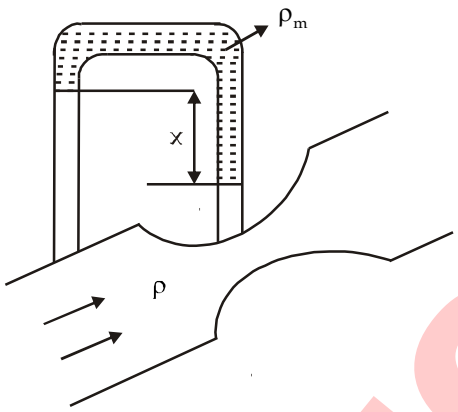


$$h = x\left[\frac{\rho_m}{\rho} - 1\right]$$

ZONE TECH

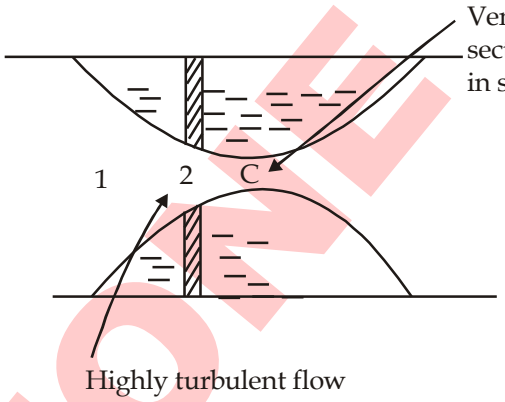
2. With Inverted difference U tube manometer

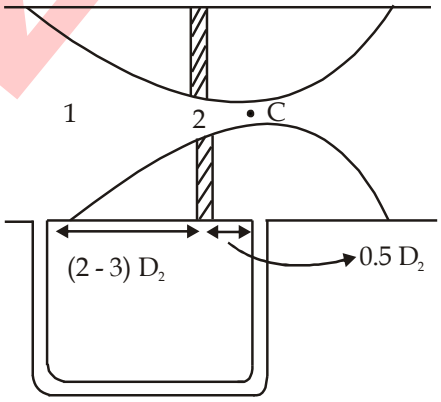
$\rho_m < \rho$



$$h = x\left(1 - \frac{\rho_m}{\rho}\right)$$

Orifice Meter:- It is used for measurement of discharge (m^3/sec).

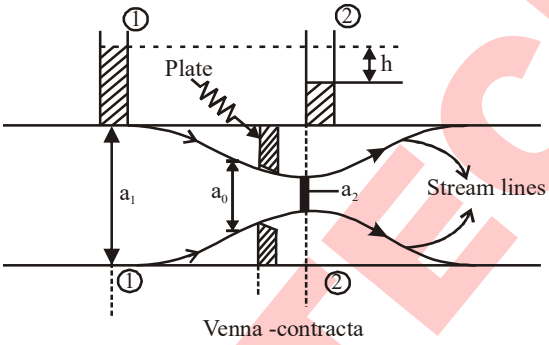




$D_2 \in [(0.4 - 0.7)D_1]$

ZONE TECH

- It is used to measure discharge through a pipe.
- In this case a circular plate with concentric sharp edged hole is installed in a pipe such that the plate is perpendicular to the axis of pipe.
- Concept of discharge measurement is same as that in venturimeter but it is a cheaper arrangement as compared to venturimeter.
- In this case, only a small length of pipe is affected hence if there is space restriction, orificementer can be used in place of venturimeter.
- However, the disadvantage is that the head loss is more in this case due to flow separation. The region where flow area is minimum is called venna-contracta



a_1 = cross-sectional area of section (1)-(1)

a_0 = cross-sectional area of opening

a_2 = cross-sectional area of flow at venna-contracta

Coefficient of contraction $(C_C) = \frac{a_2}{a_0}$

As it is difficult to find out a_2 we take $\boxed{a_2 = C_C.a_0}$

Applying Bernoulli eq. between (1) and (2)

$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + Z_2 \qquad (\gamma = \rho g)$$

$$\left(\frac{P_1}{\gamma} + Z_1\right) - \left(\frac{P_2}{\gamma} + Z_2\right) = \frac{V_2^2 - V_1^2}{2g}$$

$$h = \frac{V_2^2 - V_1^2}{2g} = \frac{Q^2\left(\frac{1}{a_2^2} - \frac{1}{a_1^2}\right)}{2g} \qquad (Q = a_1v_1 = a_2v_2)$$

$$\boxed{Q_{th} = \frac{a_1a_2}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}}$$

$$Q_{th} = \frac{C_c a_1 a_0}{\sqrt{a_1^2 - a_0^2 C_C^2}}\sqrt{2gh}$$

Q_{actual} is taken as

$$Q_{actual} = \frac{C_d a_1 a_0 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

C_d = Coefficient of discharge for orifice meter [0.62 to 0.67]

3. **Pitot Tube :-** It is a device used for measuring the velocity of flow at any point in a pipe or a channel. It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of the kinetic energy into pressure energy. In its simplest form, the pitot-tube consists of a glass tube, bent at right angles as shown in Fig.

The lower end, which is bent through 90° is directed in the upstream direction as shown in Fig. The liquid rises up in the tube due to the conversion of kinetic energy into pressure energy. The velocity is determined by measuring the rise of liquid in the tube.

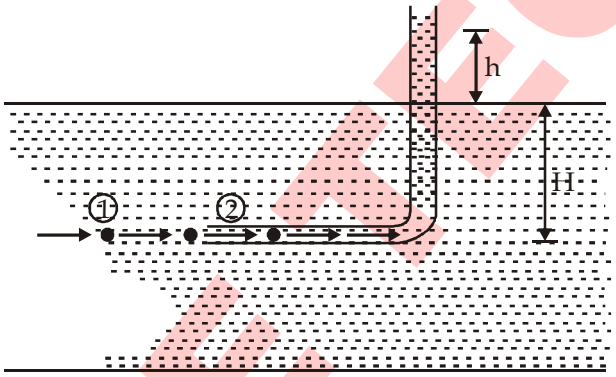


Fig. Pitot-tube

Consider two points (1) and (2) at the same level in such a way that point (2) is just as the inlet of the pitot-tube and point (1) is far away from the tube.

Let p_1 = intensity of pressure at point (1)

v_1 = velocity of flow at (1)

p_2 = pressure at point (2)

v_2 = velocity at point (2), which is zero

H = depth of tube in the liquid

h = rise of liquid in the tube above the free surface.

Applying Bernoulli's equation at point (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

But $z_1 = z_2$ as points (1) and (2) are on the same line and $v_2 = 0$.

$\frac{p_1}{\rho g}$ = pressure head at (1) = H

$\frac{p_2}{\rho g}$ = pressure heat at (2) = (h + H)

Substituting these values, we get

$$\therefore \quad H + \frac{v_1^2}{2g} = (h + H)$$

$$\therefore \quad h = \frac{v_1^2}{2g} \text{ or } v_1 = \sqrt{2gh}$$

This is theoretical velocity. Actual velocity is given by

$$(v_1)_{act} = C_v\sqrt{2gh}$$

where C_v = Co-efficient of pitot-tube

$\therefore$ Velocity at any point $v = C_v\sqrt{2gh}$

Note:-

1. **If Pipe is inclined**

$$\frac{v^2}{2g} = \left(\frac{P_2}{\delta g} + z_2\right) - \left(\frac{P_1}{\delta g} + z_1\right)$$

Kinetic Head = Stagnation Piezometric Head - Static Piezometric Head

2. If Pipe is Horizontal:-

$$z_1 = z_2$$

$$\frac{v^2}{2g} = \frac{P_2}{\delta g} - \frac{P_1}{\delta g}$$

Kinetic Head = Stagnation Pressure Head - Static Pressure Head

Note:-

Discharge (Q) = Area (A) × Velocity (V)

$$Q_{Actual} = A_{actual} \times V_{actual} \quad .....(1)$$

$$Q_{th} = A_{th} \times V_{th} \quad .....(2)$$

$$\frac{\text{Equation}(1)}{\text{Equation}(2)} \Rightarrow \frac{Q_{actual}}{Q_{th}} = \frac{A_{actual}}{A_{th}} \times \frac{V_{actual}}{V_{th}}$$

Relation between C_d, C_V and C_C

Coefficient of Velocity (C_v):- It is defined as the ratio between the actual velocity of a jet of liquid at vena contracta and the theoretical velocity of jet. It is denoted by C_v.

$$C_v = \frac{\text{Actual velocity of jet at vena-contracta}}{\text{Theoretical velocity}} = \frac{V}{\sqrt{2gH}}$$

Where V = Actual velocity

$\sqrt{2gH}$ = Theoretical velocity

The value of C_v varies from 0.95 to 0.99 for different, on fleets depending on the shape,, size of the orifice and on the head under which flow takes place. Generally the value of C_v = 0.98 is taken for sharp-edged orifices.

ZONE TECH

H.O. : M-28, Madhuban Colony, Tonk Phatak, Jaipur, Rajasthan Pin: 302015
Contact No. 141-2597591 Ph. No. 9828747676

ZONE TECH Best Institute For Assistant & Junior Engineer
www.zonetech.in SSC-JE · State-AE/JE · RRB-SSE/JE

B.O.: F-26/A, Behind Honda Pinkcity Showroom
Gatta Stand, Tonk Phatak, Jaipur, Rajasthan Pin : 302015
Ph. No. 9462447676 · Web : www.zonetech.in

Coefficients of Contraction (C_c):- It is defined as the ratio of the area of the vena-contracta to the area of the orifice, it is denoted by C_c.

Let a = Area of orifice and

a_c = Area of jet at vena-contracta

Then $$C_c = \frac{\text{Area of jet at vena - contracta}}{\text{Area of orifice}} = \frac{a_c}{a}$$

The value of C_c varies from 0.61 to 0.69 depending on shape and size of the orifice and head of liquid. In general, the value of C_c may be taken as 0.64.

Coefficients of Discharge (C_d):- It is defined as the ratio of the actual discharge from an orifice to the theoretical discharge from the orifice. It is denoted by C_d. If Q is actual discharge and Q_{th} is the theoretical discharge then mathematically, C_d is given as

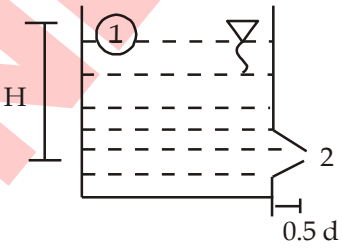
$$C_d = \frac{Q}{Q_{th}} = \frac{\text{Actual velocity} \times \text{Actual area}}{\text{Theoretical velocity} \times \text{Theoretical area}}$$

$$= \frac{\text{Actual velocity}}{\text{Theoretical velocity}} \times \frac{\text{Actual area}}{\text{Theoretical area}}$$

$$C_d = C_v \times C_c$$

The value of C_d varies from 0.61 to 0.65. For general purpose the value of C_d is takne as 0.62.

Flow through Orifice:- It is used for the measurement of velocity.



Area of orifice = a

Velocity through orifice = V

Apply energy equation between 1 and 2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V^2}{2g} + z_2$$

Because $V_1 = \frac{A_2}{A_1} \times V_2$ but $\frac{A_2}{A_1} \approx 0$ so $V_1 \approx 0$

$$\frac{V^2}{2g} = (z_1 - z_2)$$

ZONE TECH

$$\frac{V_{th}^2}{2g} = H$$

$$V_{th} = \sqrt{2gH}$$

$$V_{actual} = C_V\sqrt{2gH}$$

C_v = Coefficient of velocity

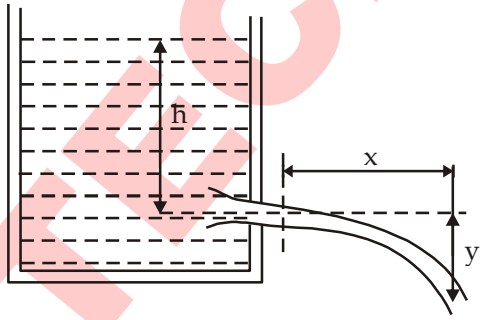
Actual Discharge $= A_{actual} \times V_{actual}$

$$= C_c a \times C_v\sqrt{2gH} = C_d a\sqrt{2gH}$$

Experimental Determination of the cofficient of velocity for an orifice:-

$v_x = v$ (Velocity in x direction)

$v_y = 0$ (Velocity in y direction)



Velocity at orifice

$$x = ut + \frac{1}{2}gt^2$$

$$x = vt \quad \text{....(1)} \quad \text{(Because acceleration in x-direction} = 0)$$

$$y = ut + \frac{1}{2}gt^2$$

$$y = \frac{1}{2}gt^2 \quad \text{....(2)} \quad (\because v = 0)$$

From equation (1) and (2)

$$y = \frac{1}{2}g\frac{x^2}{v^2} \Rightarrow v = \sqrt{\frac{x^2 \times g}{2y}} \quad \text{.....(A)}$$

Equation (A) represents the actual velocity of flow of jet at vena contracta. The theoretical velocity of jet emerging from an orifice under a constant head h is

$$V_{th} = \sqrt{2gh}$$

Therefore coefficient of velocity (C_V)

$$C_V = \frac{V_{actual}}{V_{th}} = \frac{\sqrt{\frac{x^2 \times g}{2y}}}{\sqrt{2gh}} = \sqrt{\frac{x^2}{4hy}}$$

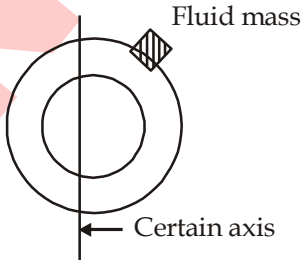
ZONE TECH

Device	Measurement
1. Venturimeter	Rate of flow (Discharge)
2. Flow Nozzle	Rate of flow (Discharge)
3. Orifice Meter	Rate of flow (Discharge)
4. Bend Meter	Rate of flow (Discharge)
5. Retometer	Rate of flow (Discharge)
6. V-notch Weir	Discharge
7. Critical Depth Meter	Discharge (open channel)
8. Pitot Tube	Velocity
9. Current Meter	Velocity
10. Hot wire Anemometer	Air and gas velocity
11. Pitometer logs	Submarine and ship speed
12. Hydrometer	Specific gravity (Relative Density)
13. Tachometer	Rotational speed

Vortex Flow:-

When a mass of fluid rotate about a certain axis such a motion of flow of fluid mass is known as vortex flow.

Note:- It is not necessary by rotating about certain axis fluid mass should be rotate about its centre of mass.



Vertex flow are of two types:-

1. Free Vortex 2. Force Vortex

1. **Free Vortex**

$\vec{T}_{Ext} = 0$

T = Rate of change of angular momentum

$\frac{d}{dt}\vec{j} = 0$

$\vec{j} = \text{constant}$

$Mvr = \text{constant}$

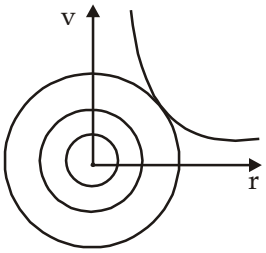
$vr = \text{constant}$

So hyperbolic curves formed

$$v \propto \frac{1}{r}$$

(i) Flow is irrotational

(ii) Bernoulli's equation is valid



2. **Forced Vortex**

$$\vec{T}_{Ext} \neq 0$$

$$\omega = 0$$

$$\omega = \frac{v}{r}$$

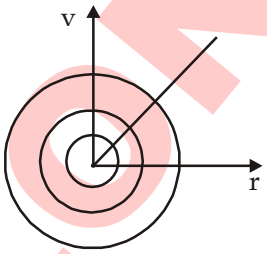
$$\frac{v}{r} = \text{constant}$$

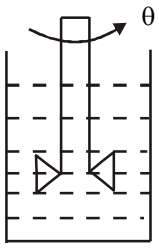
So linear curve formed

$$v \propto r$$

(i) Flow is rotational

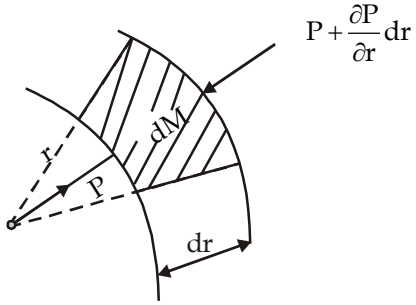
(ii) Bernoulli's equation is not valid





Ex:- Juicer

Fundamental equation for vortex flow:-



$\vec{F}\,\text{pressure} + \vec{F}\,\text{gravity} + \vec{F}\,\text{viscous} = \vec{F}\,\text{exit}$

$$PdA - \left(P + \frac{\partial P}{\partial r}dr\right)dA = dM\vec{a}_r$$

$$-\frac{\partial P}{\partial r}drdA = -dMa_r \qquad \because\ a_r = \frac{V^2}{r}$$

$$\frac{\partial P}{\partial r}drdA = \rho d\forall\left(\frac{V^2}{r}\right)$$

$$\frac{\partial P}{\partial r}drdA = \rho\,(drdA)\left(\frac{V^2}{r}\right)$$

$$\frac{\partial P}{\partial r} = \frac{\rho V^2}{r}$$

$P = fn(r)$

and also

$P = \text{function}\,(z)$

$P = fn\,(r, z) \qquad \because \quad \frac{\partial P}{\partial r} = \frac{\rho V^2}{r}$

$dP = \frac{\partial P}{\partial r}dr + \frac{\partial P}{\partial z}dz \qquad \frac{\partial P}{\partial z} = -\rho g$

$$\boxed{dP = \frac{\rho V^2}{r}dr - \rho g dz} \qquad \text{....(A)}$$

Equation (A) is Fundamental equation of vortex flow.

Free Vortex Flow:-

$$V = \frac{C}{r}$$

$$dP = \frac{\rho V^2}{r}dr - \rho g\,dz$$

$$dP = \rho\frac{C^2}{r^3}dr - \rho g\,dz$$

$$\int_1^2 dP = \rho C^2\int_1^2\frac{1}{r^3}dr - \rho g\int_1^2 dz$$

$$P_2 - P_1 = \frac{\rho C^2}{2}\left[\frac{-1}{r^2}\right]_1^2 - \rho g(Z_2 - Z_1)$$

ZONE TECH

$$P_2 - P_1 = \frac{\rho}{2}C^2\left[\frac{1}{r_1^2} - \frac{1}{r_2^2}\right] - \rho g(Z_2 - Z_1)$$

$$\frac{C}{r_1} = V_1, \quad \frac{C}{r_2} = V_2$$

$$P_2 - P_1 = \frac{\rho}{2}\left(V_1^2 - V_2^2\right) - \rho g(Z_2 - Z_1)$$

$$P_2 + \frac{1}{2}\rho V_2^2 + \rho g Z_2 = P_1 + \frac{1}{2}\rho V_1^2 + \rho g Z_1$$

$\Rightarrow$ Bernoulli valid

Free Vortex Flow = Irrotational

Forced Vortex Flow:-

$$V = r\omega$$

$$dP = \frac{\rho V^2}{r}dr - \rho g\, dz$$

$$dP = \rho\omega^2 r dr - \rho g dz$$

$$\int_{P_1}^{P_2} dP = \rho\omega^2\int_1^2 r dr - \rho g\int_1^2 dz$$

$$P_2 - P_1 = \frac{\rho}{2}\omega^2\left[r_2^2 - r_1^2\right] - \rho g(Z_2 - Z_1)$$

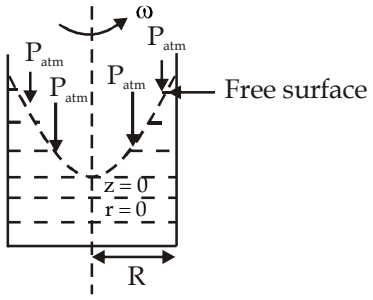
$$P_2 - P_1 = \frac{\rho}{2}\left[V_2^2 - V_1^2\right] - \rho g(Z_2 - Z_1)$$

$$P_2 - \frac{1}{2}\rho V_2^2 + \rho g Z_2 = P_1 - \frac{1}{2}\rho V_1^2 + \rho g Z_1$$

$\Rightarrow$ Bernoulli not valid

Forced Vortex Flow = Rotational

Equation of free surface in force vortex flow:-



ZONE TECH

By fundamental equation

$$dP = \frac{\rho V^2}{r} dr - \rho g dz$$

Apply at free surface

$$dP = P_{atm} - P_{atm} = 0$$

$$0 = \frac{\rho V^2}{r} dr - \rho g dz$$

$$\frac{\rho V^2}{r} dr = \rho g dz$$

$$dz = \frac{V^2}{rg} dr \qquad \omega = \frac{V}{r}$$

$$\because V = \omega r$$

$$\therefore dz = \frac{\omega^2 r^2}{rg} dr$$

$$dz = \frac{\omega^2 r}{g} dr$$

Integrate it

$$\int_0^z dz = \frac{\omega^2}{g} \int_0^r r dr$$

$$z = \frac{\omega^2 r^2}{2g} \qquad \text{.....(A)}$$

Equation (A) of free surface in force vortex flow

Shape = Parabolic

ZONE TECH

CHAPTER - 6 (DIMENSIONAL ANALYSIS)

It is a mathematical technique used in research work for design and for conducting model tests. Its objective to develop a fundamental relationship, modeling and similitude studies.

Fourier Principle of Dimensional Homogenity (DH):-

It states that if any equation or function has to represent a physical phenomenon, then the equation has to be dimensionally homogenous. Further the equation will be dimensionally homogeneous if the dimensions of the terms on both sides of equation are equal.

Method of Dimensional Analysis:- There are two methods of dimensional analysis

1.Rayleigh's method

2.Buckingham's π-theorem.

1. **Rayleigh's Method:-**

This method is used for determining the expression for a variable which depends upon maximum three or four variables only.

Let X is a variable, which depends on X_1, X_2 and X_3 variables. Then according to Rayleigh's method.

$$X = fn(X_1, X_2, X_3 ....X_n) \qquad .....(ii)$$

or

$$X = K\left[X_1^a, X_2^b .....X_n^n\right] \qquad .....(ii)$$

K = constant and a, b & c are arbitary powers

a, b, c,.....n can be determined by fourier principle of dimensional homogenity.

Ex :- The time period (t) of a pendulum depends upon the length (L) of the pendulum and acceleration due to gravity (g). Derive an expression for the time period.

$$T = K\, L^a g^b$$

$$T = K\, L^a\, [LT^{-2}]^b$$

$$T = K\, [L^{a+b}\, T^{-2b}]$$

Compare

$$L^\circ T^1 = K\, [L^{a+b}\, T^{-2b}]$$

$$a + b = 0,\ -2b = 1 \Rightarrow b = -1/2$$

$$\Rightarrow \quad a = 1/2$$

Relation $T = K[L]^{1/2}[g]^{-1/2}$

$$T = K\sqrt{\frac{L}{g}}$$

By Experiments $T = 2\pi\sqrt{\frac{L}{g}}$ $K = 2\pi$

2. **Buckingham-π-Method:-**

It is based upon Bukingham-π-theorem it states that:-

(i) If there are m physical variables (dependent and independent) in any physical phenomenon.

(ii) And these can be expressed completely in terms of n fundamental dimension (M, L, T etc)

(iii) Then the relationship among the physical variables can be described in terms of (m-n) dimensionless and independent π-terms.

Mathematical Modal

Let $X_1, X_2, X_3, \ldots X_n$ are the variables involved in a physical problem. Let X_1 be the dependent variable and $X_2, X_3, \ldots X_n, \ldots X_n$ are the independent variables on which X_1 depends. Then X_1 is a function of $X_2, X_3, \ldots X_n$ and mathematically it is expressed as

$$X_1 = f(X_2, X_3, \ldots X_n)$$

$$f_1(X_1, X_2, \ldots X_3 \ldots, X_n) = 0$$

According to Buckingham's π-theorem, equation can be written in terms of number of dimensionless groups or π-terms in which number of π-terms is equal to (m - n). Hence eqaution becomes as

$$f(\pi_1, \pi_2, \pi_3 \ldots\ldots \pi_{m-n}) = 0$$

m = Total number of variable

n = Number of fundamental dimensions involved

Number of π-Terms = m-n

Guidelines for selecting Repeating Variables:-

(1) Select the first repeating variable from those describing the geometry of flow. Ex:- size and shape of fluid passage or of the moving body such as diameter, length, etc.

(2) Select the second repeating variable from those representing the fluid properties such as density, voscosity, surface tension, elasticity and vapour pressure.

(3) Select the third repeating variable from those specifying the fluid motion such as velocity, acceleration, discharge, pressure, force, power.

(4) Repeating variables should consider all basic fundamental dimensional parameters.

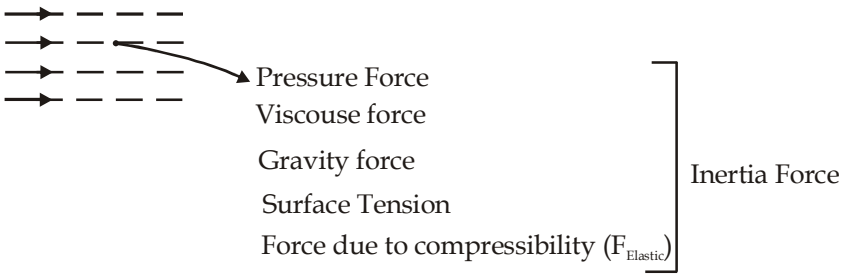
(5) They should be selected from independent variables

Choice of Repeating Variables:-

(i) L, v, ρ (ii) L, v, μ (iii) d, v, ρ (iv) d, v, μ

Different types of forces in a fluid flow system:-

For the fluid flow problems, the forces acting on a fluid mass may be any one, or a combination of the several of the following forces :



ZONE TECH

1. Inertia Force:-

$$F_i = Ma$$

$$F_i = \rho L^3 \frac{V}{T}$$

$$F_i = \rho L^2 \frac{L}{T} \times V$$

$$F_i = \rho L^2 V^2$$

2. Viscouse Force:-

$$F_v = \tau A$$

$$F_v = \mu \frac{du}{dy} A$$

$$F_v = \mu \frac{V}{L} L^2$$

$$F_v = \mu VL$$

3. Pressure Force:-

$$F_p = PA$$

$$F_p = PL^2$$

4. Gravity Force:-

$$F_g = Mg$$

$$F_g = \rho L^3 g$$

5. Surface Tension:-

$$F_\sigma = \sigma L$$

6. Elastic Force:-

$$F_e = KA$$

$$F_e = KL^2$$

Dimensionless Numbers:-

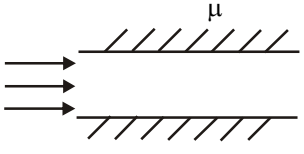
- Reynold Number
- Euler Number
- Froude Number
- Weber Number
- Mach Number

(1) Reynolds Number (Re)

$$Re = \frac{\text{Inertia Force } (F_i)}{\text{Viscous Force } (F_v)} = \frac{\rho V^2 L^2}{\mu VL}$$

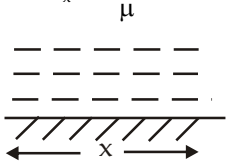
$$Re = \frac{\rho VL}{\mu}$$ L → Characteristic Dimension

$$Re = \frac{\rho VD}{\mu}$$



Pipe Flow

$$Re_x = \frac{\rho Vx}{\mu}$$



Surface Flow

Application:-1. Pipe Flow

2. Flow near the surface

ZONE TECH

(2) Euler Number (E_u)

$$E_u = \sqrt{\frac{\text{Inertia Force } (F_i)}{\text{Pressure Force } (F_p)}} = \sqrt{\frac{\rho V^2 L^2}{PL^2}}$$

$$E_u = \frac{V}{\sqrt{P/\rho}}$$

Application:-1. Pipe Flow

2. Aeroplane in air

(3) Froude Number (Fr)

$$Fr = \sqrt{\frac{\text{Inertia Force } (F)_i}{\text{Gravity Force}(F_g)}} = \sqrt{\frac{\rho V^2 L^2}{\rho L^3 g}} = \frac{V}{\sqrt{Lg}}$$

$$Fr = \frac{V}{\sqrt{Lg}}$$

Application:-1. River Flow

2. Open channel flow

3. Spill ways

(4) Weber Number (We)

$$We = \sqrt{\frac{\text{Inertia Force}(F_i)}{\text{Surface Tension Force}(F_\sigma)}} = \sqrt{\frac{\rho V^2 L^2}{\sigma L}}$$

$$We = \frac{V}{\sqrt{\frac{\sigma}{\rho L}}}$$

Application:- Capillary tube

(5) Mach number (Mc)

$$Mc = \sqrt{\frac{\text{Inertia Force}(F_i)}{\text{Force due to compressibility } (F_{Elastic})}} = \sqrt{\frac{\rho V^2 L^2}{KL^2}}$$

$$Mc = \frac{V}{\sqrt{K/\rho}} = \frac{V}{C}$$

where $V \rightarrow$ Velocity of flow

$C = \sqrt{K/\rho} \rightarrow$ Velocity of sound in fluid

ZONE TECH

Application:- Compressible flow

Note:-

If Mc < 0.3 → Gases flow can be treated as incompressible.

If Mc > 0.3 → Gases flow cannot be treated as incompressible.

If there is no heat transfer

Mc < 1 → Subsonic

Mc > 1 → Supersonic

Mc = 1 → Sonic

Mc >> 1 → Hypersonic

Model analysis :-

The model is the small scale replica of the actual structure or machine. The actual structure or machine is called Prototype. it is not necessary that the models should be smaller than the prototypes (through in most of cases it is), they may be larger than the prototype. The study of models of actual machines is called Model analysis.

Modeling and Similitude Studies:- Similitude is defined as the similarity between the model and its prototype in every respect, which means that the model and prototype have similar properties or model and prototype are completely similar.

Generally to see the performance of any prototype we perform experiment on the models rather than to make prototype.

Generally (not always) size of model is smaller than the prototype.

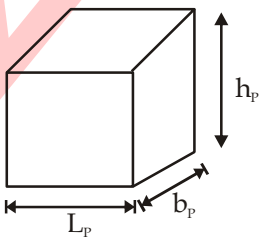
"Model should be similar to prototype and the models can be considered as similar to the prototype when the models has following three types of similarity."

→ Geometrical Similarity

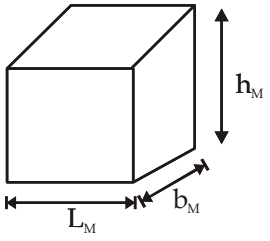
→ Kinematic Similarity

→ Dynamic Similarity

(1) Geometrical Similarity:- In this similarity each and every dimension of prototype and model is scaled in the same ratio. This is similarity of dimension.



Prototype



Model

$$\frac{L_P}{L_M} = \frac{b_P}{b_M} = \frac{h_P}{h_M} = L_r \rightarrow \text{Length scale ratio or scale ratio}$$

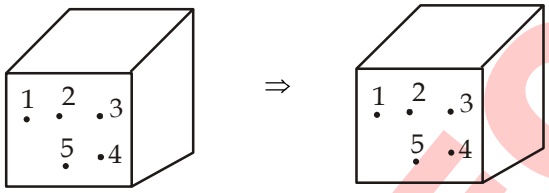
Area Scale Ratio:-

$$\frac{A_P}{A_M} = (L_r)^2$$

Volume Scale Ratio:-

$$\frac{V_P}{V_M} = (L_r)^3$$

(2) **Kinematic Similarity:-** To maintain kinematic similarity the ratio of velocity and accelertion at corresponding points of model and prototype should be same (both in direction and magnitude). This is similarity of motion



$$\frac{(V_1)_P}{(V_1)_M} = \frac{(V_2)_P}{(V_2)_M} = \frac{(V_3)_P}{(V_3)_M} \text{ ..... } (V_r) = \frac{L_r}{T_r}$$

$$\frac{(a_1)_P}{(a_1)_M} = \frac{(a_2)_P}{(a_2)_M} = \frac{(a_3)_P}{(a_3)_M} \text{ ..... } (a_r) = \frac{V_r}{T_r} = \frac{L_r}{T_r^2}$$

$$\frac{(Q_1)_P}{(Q_1)_M} = \frac{(Q_2)_P}{(Q_2)_M} = \frac{(Q_3)_P}{(Q_3)_M} \text{ ..... } (Q_r) = \frac{L_r^3}{T_r}$$

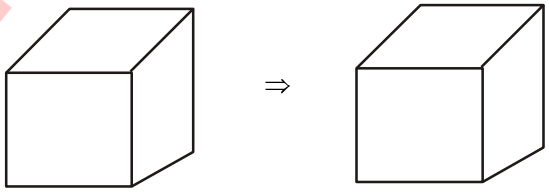
where $V_r \rightarrow$ velocity scale ratio

$T_r \rightarrow$ Time scale ratio

$a_r \rightarrow$ Accleration scale ratio

$Q_r \rightarrow$ Discharge scale ratio

Dynamic Similarity:- To maintain dynamic similarity the ratio of different forces at the corresponding point of model and prototype should be same (both magnitude and direction). This is similarity of forces



Prototype Model

$$\frac{(F_u)_P}{(F_u)_M} = \frac{(F_P)_P}{(F_P)_M} = \frac{(F_g)_P}{(F_g)_M} = \frac{(F_\sigma)_P}{(F_r)_M} \text{ .....} (F_r) \rightarrow \text{Force scale ratio} = \frac{\rho_r L_r^4}{T_r^2}$$

$$\frac{(P)_P}{P_M} = F_r . V_r = \frac{\rho_r L_r^5}{T_r^3} \rightarrow \text{Power scale ratio}$$

(1) For Kinematic Similarity	Geometrical Similarity must exist
(2) For Dynamic Similarity	(i) Geometrical similarity must exist (ii) Kinematic similarity must exist but not the sufficient condition for dynamic similarity

For complete similarity geometrical, kinematic and dynamic similarity must exist.

Model Law:-

For dynamic similarity between model and prototype, ratio of dimensionless number (Re, Fr, We, Eu Mc) should be same for model and prototype.

- But it is difficult to satisfy all the forces ratio. Hence models are designed on the basis of ratio of forces which are dominating in the phenomenon.
- Thus we have various model laws depending on the forces which are dominant in the flow.

(1) Reynold's Model law:-

The situation where apart from inertia force only viscous force are dominant, Reynold's model law is used i.e. Reynold's number of model should be equal to Reynold's number of prototype.

i.e. $(Re)_M = (Re)_P$

$$\frac{\rho_M V_M L_M}{\mu_M} = \frac{\rho_P V_P L_P}{\mu_P}$$

$$\frac{\rho_r V_r L_r}{\mu_r} = 1$$

Reynold's Model law is used in:-

- Pipe flow (where viscosity has significant effect like laminar flow).
- Flow around completely submerged objects like submarines, airplanes, automobiles.
- Parachutes when there is drag (i.e. when not falling in vacuum).

Thus in this law:-

(a) $V_r = \frac{\mu_r}{\rho_r L_r} = \text{Velocity ratio}$

(b) $T_r = \frac{L_r}{V_r} = \frac{L_r}{\frac{\mu_r}{\rho_r L_r}} = \frac{\rho_r L_r^2}{\mu_r} = \text{Time ratio}$

(c) $a_r = \frac{V_r}{T_r} = \frac{\frac{\mu_r}{\rho_r L_r}}{\frac{\rho_r L_r^2}{\mu_r}} = \frac{\mu_r^2}{\rho_r^2 L_r^3} = \text{Acceleration ratio}$

ZONE TECH

(d) $F_r = M_r a_r = \frac{(\rho_r L_r^3)\mu_r^2}{\rho_r^2 L_r^3} = \frac{\mu_r^2}{\rho_r} =$ Force ratio

(e) $P_r = F_r V_r = \frac{\mu_r^3}{\rho_r^2 L_r} =$ Power ratio

(f) $Q_r = \frac{L_r^3}{T_r} = V_r \times L_r^2 = \frac{\mu_r}{\rho_r L_r} \times L_r^2 = \frac{\mu_r L_r}{\rho_r} =$ Discharge ratio

Note:- If μ and ρ are same then $\frac{\rho_r L_r V_r}{\mu_r} = 1 \Rightarrow L_r V_r = 1$

(2) **Froude's Law:-** The law is applicable when in addition to inertial force gravity forces are dominant.

In this case $(Fr)_M = (Fr)_P$

$$\frac{V_r}{\sqrt{g_r L_r}} = 1$$

If model and prototype are at same location i.e. if g is constant then

$\frac{V_r}{\sqrt{L_r}} = 1$, L = Length parameter

L = Hydraulic mean depth $= \frac{A}{T}$ (in case of open channel)

Froude's Model law is applicable in:-

(a) Free surface flow such as flow over spillway, weirs, channels (open channel) hydraulic jump.

(b) Flow of liquid jet from orifices.

(c) Flow over weir and notches.

(d) Motion of ship in rough and turbulent sea (motion resistance is due to waves for meal in sea due to gravity)

Note:- In case of ship (gravity as well as viscous forces both are predominant) hence Reynolds and Froude Law both should be used.

In this case.

(1) $V_r = \sqrt{L_r} =$ Velocity ratio

(2) $T_r = \frac{L_r}{V_r} = \frac{L_r}{\sqrt{L_r}} = \sqrt{L_r} =$ Time ratio

(3) $a_r = \frac{V_r}{T_r} = \frac{\sqrt{L_r}}{\sqrt{L_r}} = 1 =$ Acceleration ratio

(4) $F_r = M_r a_r = \rho_r L_r^3 =$ Force ratio

(5) $P_r = F_r V_r = \rho_r \times L_r^3 \times \sqrt{L_r} = \rho L_r^{7/2} =$ Power ratio

(f) $Q_r = \frac{L_r^3}{T_r} = \frac{L_r^3}{\sqrt{L_r}} = L_r^{2.5} =$ Discharge ratio

ZONE TECH

(3) Euler's Model Law:-

When apart from inertial forces only pressure for is dominant, this law is applicable. In this case

$$(Eu)_M = (Eu)_P$$

$$\frac{V_r}{\sqrt{P_r / \rho_r}} = 1$$

This law is applicable:-

- In pipe flow at high pressure (in which viscous effect is negligible as in turbulent flow).
- In case of cavitation.
- Pressure due to sudden closure of valve.

(4) Weber Model Law:-

When apart from inertial force surface tension force is dominant.

$$(We)_M = (We)_P$$

$$\frac{V_r}{\sqrt{\frac{\sigma_r}{\rho_r L_r}}} = 1$$

It is applicable in:-

- Capillary rise in narrow passages.
- Rising bubble seepage flow through soil.
- Flow of blood in veins and arteries.

(5) Mach Law:-

When apart from inertial force compressibility forces are dominant, we use Mach law.

Compressibility force are predominant when Mach number ≥ 0.3 i.e. speed of movement $\geq (0.3 \times$ Velocity of sound)

In this case

$$(M)_M = (M)_P$$

$$\frac{V_r}{\sqrt{K_r / \rho_r}} = 1$$

Applicable in:-

- Aerodynamic testing such as launching of missiles rockets.
- Under water testing of torpedoes.
- Water hammer pressure.
- Flow of gases through pipe at high velocity.

Theory of Distorted Models or River Model Law:-

In River Models Distorted models are used.

A model is said to be distorted if it is not geometrically similar. For distorted model different scale ratio for horizontal and vertical direction are used.

In the case of river models if undistorted model is used. The depth will become very small and hence there will be difficulty in measurement.

Depth become so small that flow model may not remain turbulent as is there in case of orginal river.

River, as the model will not be similar to river. Result obtained from distorted model cannot be directly transferred to river prototype.

But this type of arrangement gives some approximately understanding of the flow situations. Hence it is

$(L_r)_H$ = Scale ratio in horizontal direction.

$$= \frac{L_P}{L_M} = \frac{B_P}{B_M}$$

$(L_r)_V$ = Scale ratio in vertical direction $= \frac{h_P}{h_M}$

In Distorted model for calculating velocity ratio relation, scale ratio of vertical direction is used.

$$\text{Velocity ratio } (V_r) = \sqrt{(L_r)_V} = \sqrt{\frac{h_P}{h_M}}$$

$$A_r = \frac{B_P}{B_M} \times \frac{h_P}{h_M} = (L_r)_V \times (L_r)_H = \text{Area ratio}$$

Discharge ratio $Q_r = A_r V_r = (L_r)_H (L_r)_V . \sqrt{(L_r)_V}$

$$Q_r = (L_r)_H . (L_r)_V^{3/2}$$

Model law for Rotodynamic Machines

$$\frac{Q_r}{N_r D_r^3} = 1$$

$$\frac{\sqrt{H}}{N_r D_r} = 1$$

$$\frac{P_r}{\rho_r N_r^3 D_r^5} = 1$$

where Q_r = Discharge ratio

N_r = Rotational speed ratio

D_r = Dia ratio

H_r = Head ratio

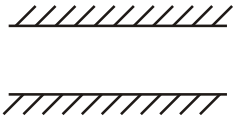
P_r = Power ratio

Note:- $$\frac{P}{\rho N^3 D^5} = F\left[\frac{Q}{ND^3}, \frac{gH}{N^2 D^2}\right]$$

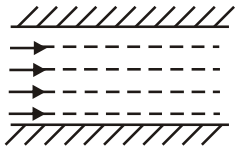
Each of the above parameters will have same value in model and prototype.

CHAPTER - 7 (PIPE FLOW)

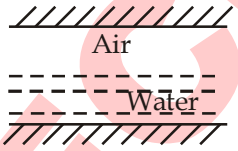
Pipe:-



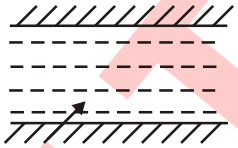
It is the conduit generally having a circular cross section. The shape of cross section may be circular, square, triangular etc.



Pipe Flow



Open Channel



Viscous force is highly dominant

Re < 2000 → Laminar flow

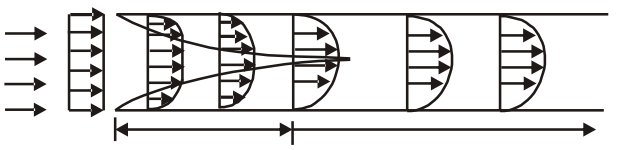
Re Number > 4000 → Turbulent flow

Hydraulic Mean dia:- $\frac{4A}{P}$

For circular pipe $= \frac{4\frac{\pi}{4}D^2}{\pi D} = D$

Hydraulic Mean Depth (m) $= \frac{\text{Wetted Area (A)}}{\text{Wetted Perimeter (P)}}$

For circular cross section, $m = \frac{\frac{\pi}{4}D^2}{\pi D} = \frac{D}{4}$



ZONETECH

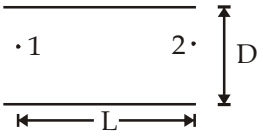
Head Losses:- When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as

1. Major Losses
2. Minor Losses

1. **Major Losses:-** This is due to internal friction between the layer.

$$\text{Total Head} = \frac{P}{\rho g} + \frac{v^2}{2g} + z$$

Darcy-Weisbach formula :-



$$h_f = \frac{fLV^2}{2gD} \qquad \text{.....(i)}$$

where V = average velocity

f = friction factor

f = 4f'

f' = Coefficient of friction

Coefficient of friction

For laminar flow

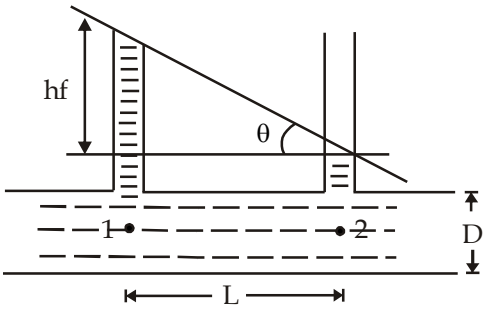
$$f' = \frac{16}{Re}$$

For Turbulent Flow

$$f' = \frac{0.079}{(Re)^{1/4}}$$

$\because$ $f = 4f' \Rightarrow f = \frac{64}{Re}$

Chezy's Formula



ZONE TECH

$V = C\sqrt{mi}$

C = Chezy constant

m = Hydraulic Mean depth = $\frac{D}{4}$

i = slope $\Rightarrow$ $\tan\theta = i = \frac{h_f}{L}$ = Hydrualic gradient

$V = C\sqrt{\frac{D}{4} \times \frac{h_f}{L}}$

$V^2 = C^2\left(\frac{D}{4} \times \frac{h_f}{L}\right)$

$$h_f = \frac{4LV^2}{C^2D} \quad \text{....(ii)}$$

From equation (i) and (ii), $\frac{f}{2g} = \frac{4}{C^2} \Rightarrow C = \sqrt{\frac{8g}{f}}$

where C = Chezy's constant

g = gravitational acceleration

f = friction factor

Dimension of chezy's constant = $(L)^{1/2}T^{-1}$

2. **Minor Losses:-**

(i) Losses due to sudden expansion.

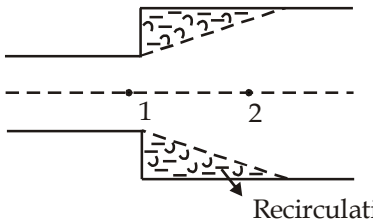
(ii) Losses due to exit.

(iii) Losses due to sudden contraction.

(iv) Losses due to entry.

(v) Losses due to bend in pipe.

(i) **Losses due to sudden expansion:-**

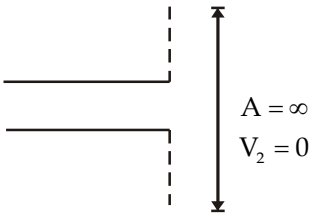


Recirculation zone (P')

Assumption:- $P_1 = P'$

$$h_L = \frac{(V_1 - V_2)^2}{2g}$$

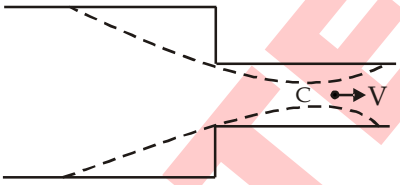
(ii) **Due to exit:-**



$$h_L = \frac{(V_1 - V_2)^2}{2g}$$

$$h_L = \frac{V^2}{2g}$$

(iii) **Due to sudden contraction:-**



$$h_L = \frac{(V_c - V)^2}{2g}$$

$$h_L = \frac{\left(\frac{V}{C_c} - V\right)^2}{2g}$$

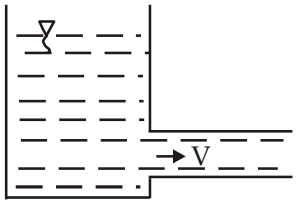
$$h_L = \frac{V^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$$

If C_c is not given

$$\left(\frac{1}{Cc} - 1\right)^2 = 0.5$$

$$h_L = \frac{0.5V^2}{2g}$$

(iv) **Due to entry:-**



$$h_L = \frac{0.5V^2}{2g}$$

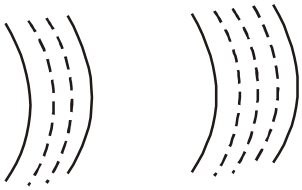
ZONE TECH

(v) **Due to pipe bend:-**

$$h_L = \frac{KV^2}{2g} \qquad K = \text{coefficient of pipe bend}$$

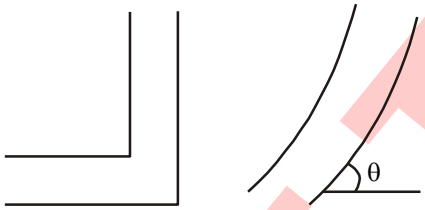
Effect of K:-

(I) Dia:-



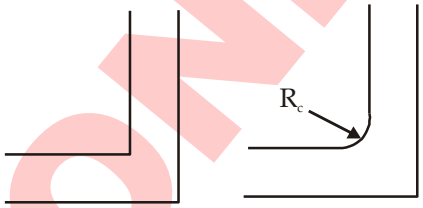
$D\uparrow = K\downarrow$

(II) Angle



$\theta\uparrow = K\uparrow$

(III) Radius of curvature (R_c):-



$R_c\uparrow = K\downarrow$

Head Loss in terms of discharge:-

$$h_f = \frac{fLV^2}{2gD} \qquad \text{....(A)}$$

We know that

$$Q = AV$$

$$Q = \frac{\pi}{4} D^2 \times V$$

$$V = \frac{4Q}{\pi D^2} \qquad \text{....(B)}$$

ZONE TECH

By equation (A) and (B)

$$h_f = \frac{fL\left(\frac{4Q}{\pi D^2}\right)^2}{2gD}$$

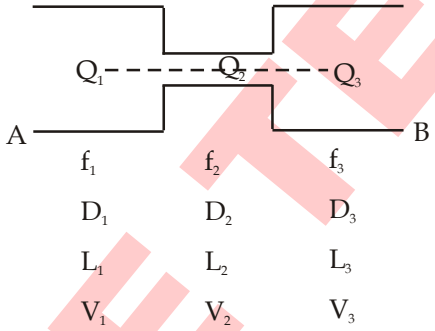
$$h_f = \frac{16fLQ^2}{2g\pi^2 D^5} \qquad \left(\because \frac{16}{2g\pi^2} = \frac{1}{12}\right)$$

$$h_f = \frac{1}{12}\frac{fLQ^2}{D^5}$$

Connection in Pipe

1. Series
2. Parallel

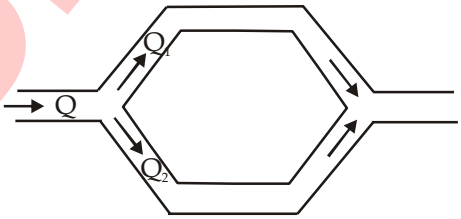
1. **Series connection**



$Q_1 = Q_2 = Q_3 = Q \rightarrow$ Remain constant

$h_{f_{AB}} = h_{f_1} + h_{f_2} + h_{f_3}$

2. **Parallel connection**

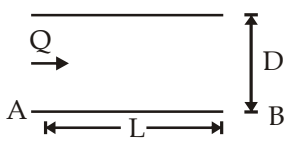


$Q = Q_1 + Q_2$

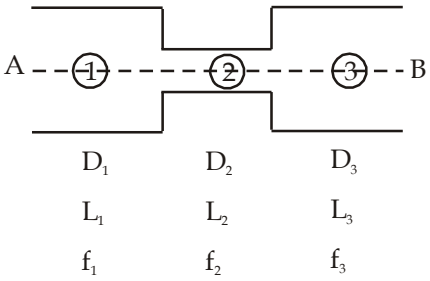
$h_{f_1} = h_{f_2} \rightarrow$ Remain constant

Equivalent Pipe:- It is a pipe of certain length and diameter having the same discharge under same head loss.

For Series Connection:-



ZONE TECH



$$h_{f_{eq}} = h_{f_1} + h_{f_2} + h_{f_3}$$

$$\frac{fL_{eq}Q^2}{12D_{eq}^5} = \frac{f_1L_1Q^2}{12D_1^5} + \frac{f_2L_2Q^2}{12D_2^5} + \frac{f_3L_3Q^2}{12D_3^5}$$

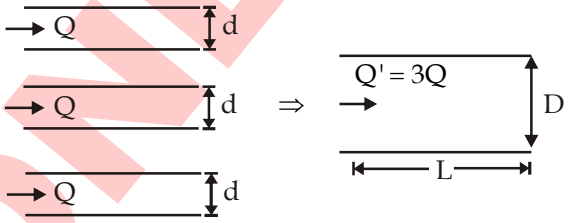
$$\frac{fL_{eq}}{D_{eq}^5} = \frac{f_1L_1}{D_1^5} + \frac{f_2L_2}{D_2^5} + \frac{f_3L_3}{D_3^5}$$

If $f_1 = f_2 = f_3 = f$

$$\frac{L_{eq}}{D_{eq}^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}$$

This is known as dupit's equation

For Parallel Connection:- Discharge is increasing but head loss for each pipe is same.



$$Q' = Q + Q + Q = 3Q$$

Relation between D and d

We know that in parallel connection

$$h_f = h_{f_{eq}} \Rightarrow \frac{fLQ^2}{12d^5} = \frac{fL(3Q)^2}{12D^5}$$

$$D^5 = 9d^5$$

$$D = (3^2)^{1/5}d$$

$$D = 1.55d$$

If number of pipe are 'n'

$$D_{eq} = (n)^{2/5}d$$

ZONE TECH

Lines:-

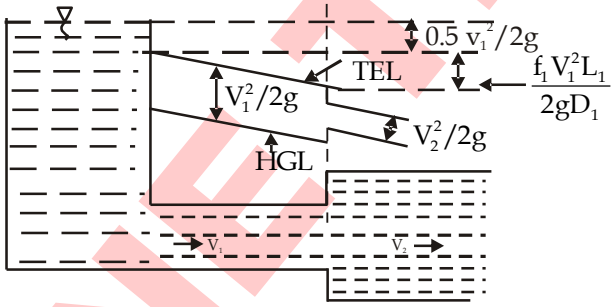
1. Total energy line or energy gradient line.
2. Hydraulic gradient line.

1. **Total energy line:-**

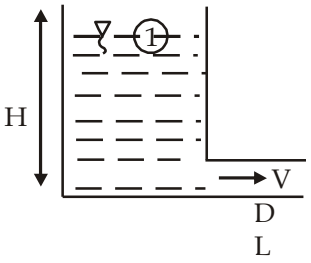
- It is a line joining the point representing the value of total head at various cross section of pipe in pipe flow.
- These lines always go down in the direction of flow until or unless some energy supplied externally. Ex:- pump.

2. Hydraulic gradient line:-

- It is the line joining the point representing the value of piezometric head at the various cross section of pipe in pipe flow.
- These line may go up or down in direction of flow.
- These line always below the total energy line.
- In a uniform diameter pipe this line always parallel to total energy line and vertical gap between the two line at any section of pipe represent the value of kinetic head.



Power Transmission through pipe:-



At point 1

$$\text{Energy (Joule)} = mgH$$

$$\text{Power (watt)} = \dot{m}gH \qquad m = \text{mass flow rate} = \rho A V$$

$$(\text{Power})_{\text{Input}} = \dot{m}gH$$

$$P_{\text{output}} = \dot{m}g\,(H - h_f)$$

$$P_{output} = \rho AVg\,(H - h_f)$$

$$P_{output} = \rho AVg\left(H - \frac{fLV^2}{2gD}\right)$$

$$P_{output} = \rho Ag\left(HV - \frac{fLV^3}{2gD}\right)$$

For maximum power output

$$\frac{dP_{output}}{dV} = 0$$

$$\frac{d}{dV}\left[\rho Ag\left(HV - \frac{fLV^3}{2gD}\right)\right] = 0$$

$$\rho Ag\left[H - \frac{3fLV^2}{2gD}\right] = 0$$

$$\rho Ag\,(H - 3h_f) = 0$$

$$\rho Ag \neq 0$$

$$H - 3h_f = 0$$

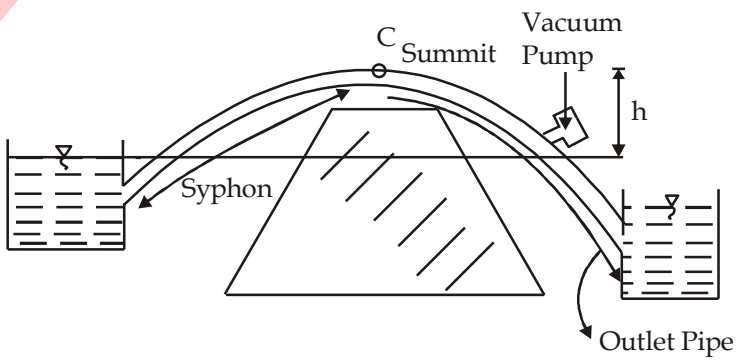
$h_f = \frac{H}{3} = 33.33\% \rightarrow$ minimum head losses to fixed maximum power output

$$(P_{output})_{max} = H - h_f$$

$$H - \frac{H}{3} = \frac{2H}{3} = 66.67\%$$

$(P_{output})_{max} = \frac{2H}{3} = 66.67\% \rightarrow$ maximum power output

Syphon:-



Syphon is a long bent pipe which is used to transfer liquid from a reservoir at a higher elevation to another reservoir at a lower level when the two reservoirs are separated by a hill or high level ground as shown in figure.

The point C which is at the highest of the syphon is called the summit. As the point C is above the free surface of the water in the tank A, the pressure at C will be less than atmospheric pressure. Theoretically, the pressure at C may be reduced to - 10.3 m of water but in actual practice this pressure is only - 7.6 m of water or 10.3 - 7.6 = 2.7 m of water absolute. If the pressure at C becomes less than 2.7 m of water absolute, the dissolved air and other gases would come out from water and collect at the summit.

$\frac{P_s}{\rho g} > \frac{P_v}{\rho g}$ $\frac{P_s}{\rho g}$ = Pressure head at summit

$\left(\frac{P_s}{\rho g}\right)_{min} = \frac{P_v}{\rho g}$ $\frac{P_v}{\rho g}$ = Vapour pressure head

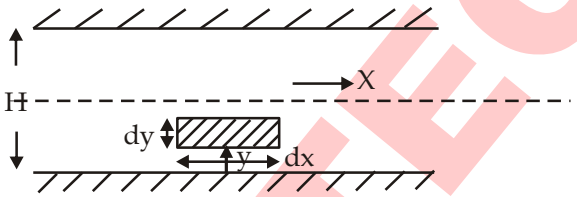
ZONE TECH

CHAPTER - 8 (LAMINAR FLOWS)

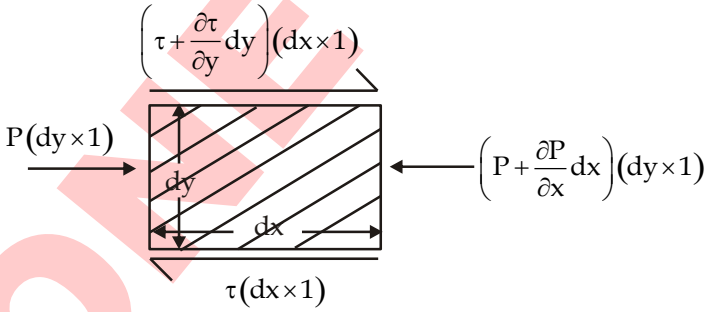
Laminar Flows

- Flow in between parallel flat plates
- Laminar flow through pipe
- Correction factors

1. **Flow in between parallel flat plates:-** Consider two parallel fixed plates kept at a distance 't' apart as shown in figure. A viscous fluid is flowing between these two plates from left to right. Consider a fluid element of length dx and thickness dy at a distance y from the lower fixed plate.



Width = 1



Momentum Equation in X-direction

$$F_{PX} + F_{GX} + F_{VX} = F_{iX}$$

$$Pdy - \left(P+\frac{\partial P}{\partial x}dx\right)dy - \tau dx + \left(\tau+\frac{\partial\tau}{\partial y}dy\right)dx = dm.a_x$$

No change in a_x due to uniform cross section and laminar flow $\Rightarrow a_x = 0$

$$-\frac{\partial P}{\partial x}dxdy + \frac{\partial\tau}{\partial y}dydx = 0$$

$$\frac{\partial P}{\partial x} = \frac{\partial\tau}{\partial y}$$

For Newtonian Fluids

$$\tau = \mu \frac{du}{dy}$$

$$\frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) = \frac{\partial P}{\partial x}$$

$$\mu \frac{\partial^2 u}{\partial y^2} = \frac{\partial P}{\partial x}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial P}{\partial x}$$

Integrate it

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{\partial P}{\partial x} y + C_1$$

Again Integrate it

$$u = \frac{1}{\mu}\frac{\partial P}{\partial x}\frac{y^2}{2} + C_1 y + C_2 \qquad .....(1)$$

Apply Boundary conditions:-

at $y = 0, u = 0$

$$C_2 = 0$$

at $y = H, u = 0$

$$C_1 H = \frac{-1}{\mu}\frac{\partial P}{\partial x}\frac{H^2}{2}$$

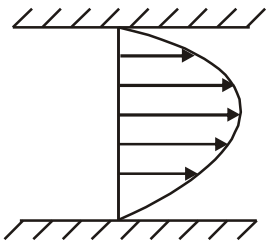
$$C_1 = \frac{-1}{\mu}\frac{\partial P}{\partial x}\frac{H}{2}$$

$$u = \frac{1}{\mu}\frac{\partial P}{\partial x}.\frac{y^2}{2} - \frac{1}{\mu}\frac{H}{2} y \frac{\partial P}{\partial x}$$

$$u = \frac{1}{2\mu}\frac{\partial P}{\partial x}\left[y^2 - Hy\right]$$

$$u = -\frac{1}{2\mu}\frac{\partial P}{\partial x}\left[Hy - y^2\right]$$

Hence we can say velocity profile is parabolic.



ZONE TECH

Maximum velocity

at $y = \frac{H}{2}$, $u = u_{max}$

$$u_{max} = \frac{-1}{2\mu}\frac{\partial P}{\partial x}\left[H \times \frac{H}{2} - \frac{H^2}{4}\right]$$

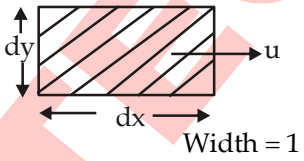
$$u_{max} = \frac{-1}{2\mu}\frac{\partial P}{\partial x}\frac{H^2}{4}$$

$$u_{max} = \frac{-H^2}{8\mu}\frac{\partial P}{\partial x}$$

Mean velocity:- (Mass flow rate same at this velocity)

$$\dot{m} = \int_0^H d\dot{m}$$

$$\dot{m} = \int_0^H \rho(dy \times 1)u$$



$$\rho(H \times 1)U_{mean} = \int_0^H \rho\left[\frac{-1}{2\mu}\frac{\partial P}{\partial x}\left(Hy - y^2\right)\right]dy$$

$$H\,U_{mean} = \frac{-1}{2\mu}\frac{\partial P}{\partial x}\int_0^H \left(Hy - y^2\right)dy$$

$$H.\,U_{mean} = \frac{-1}{2\mu}\frac{\partial P}{\partial x} \times \left[\frac{H^3}{2} - \frac{H^3}{3}\right]$$

$$H.\,U_{mean} = \frac{-1}{2\mu}\frac{\partial P}{\partial x}\frac{H^3}{6}$$

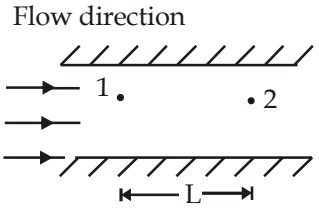
$$U_{mean} = \frac{-H^2}{12\mu}\frac{\partial P}{\partial x} = \frac{U_{max}}{1.5}$$

$\Rightarrow$ $U_{max} = 1.5\,U_{mean}$

Pressure Drop:- (Head Loss)

In pressure drop we use mean velocity

$$U_{mean} = \frac{-H^2}{12\mu}\frac{\partial P}{\partial x}$$



$$\frac{12\mu\ U_{mean}}{H^2}dx = -\ dP$$

Integrate it

$$\frac{12\mu\ U_{mean}}{H^2}\int_0^L dx = -\int_1^2 dP$$

$$\frac{12\mu\ U_{mean}L}{H^2} = -\ (P_2 - P_1)$$

$$\frac{12\mu\ U_{mean}L}{H^2} = P_1 - P_2$$

$$\frac{P_1 - P_2}{\rho g} = \frac{12\mu\ U_{mean}L}{\rho g H^2}$$

In case of inclined:- $\left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_2}{\rho g} + Z_2\right) = \frac{12\mu\ U_{mean}L}{\rho g H^2} = h_f$

Shear stress distribution:-

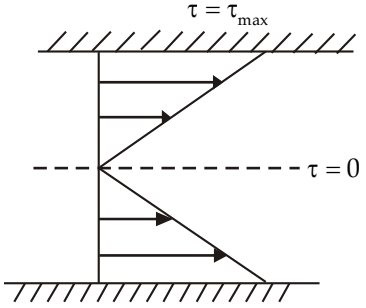
$$\tau = \mu\frac{du}{dy}$$

$$\tau = \mu\frac{d}{dy}\left[\frac{-1}{2\mu}\frac{\partial P}{\partial x}\left(Hy - y^2\right)\right]$$

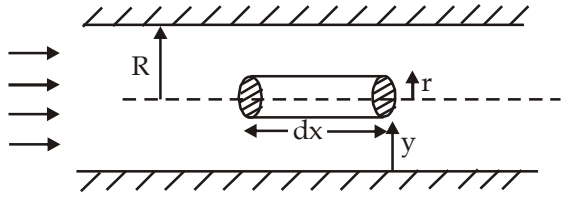
$$\tau = \mu \times \frac{-1}{2\mu}\frac{\partial P}{\partial x}\left(H - 2y\right)$$

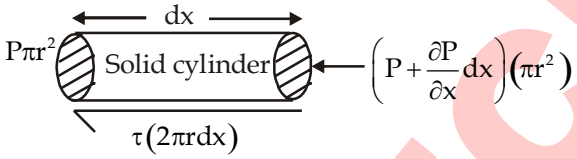
$$\tau = \frac{-1}{2}\frac{\partial P}{\partial x}\left(H - 2y\right)$$

Hence we can say shear stress distribution profile is linear profile.



Laminar flow through pipe:- Consider a horizontal pipe of radius R. The viscous fluid is flowing from left to right in the pipe as shown in figure. Consider a fluid element of radius r, sliding in a cylindrical fluid element of radius (r + dr). Let the length of fluid element be dx.





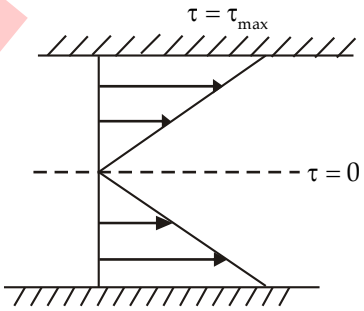
Momentum Equation in X-direction

$$\rho\pi r^2-\left(P+\frac{\partial p}{\partial x}dx\right)\pi r^2-\tau(2\pi r dx)=dma_x$$

No change in a_x due to uniform cross section and laminar flow ($a_x=0$)

$$\left[\frac{-\partial P}{\partial x}dx\right]\pi r^2=\tau\,2\pi r dx$$

$$\tau=\frac{-r}{2}\frac{\partial P}{\partial x} \qquad .....(1)$$



Velocity Profile:-

For newtonian fluid $\tau=\mu\frac{du}{dy}$ .....(2)

By equation (1) and (2)

$$\mu\frac{du}{dy}=\frac{-r}{2}\frac{\partial P}{\partial x}$$

ZONE TECH

$$\frac{\partial u}{\partial y} = \frac{-r}{2\mu}\frac{\partial P}{\partial x}$$

$$y + r = R$$

$$dy + dr = 0$$

$$dy = - dr$$

$$\frac{-\partial u}{\partial r} = \frac{-r}{2\mu}\frac{\partial P}{\partial x}$$

Integrate it

$$u = \frac{r^2}{4\mu}\frac{\partial P}{\partial x} + C$$

Apply Boundary condition

At $r = R,\ u = 0$

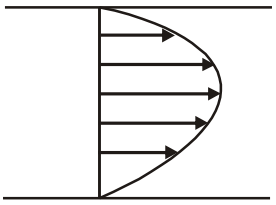
$$C = -\frac{R^2}{4\mu}\frac{\partial P}{\partial x}$$

$$u = \frac{r^2}{4\mu}\frac{\partial P}{\partial x} - \frac{R^2}{4\mu}\frac{\partial P}{\partial x}$$

$$u = -\frac{1}{4\mu}\frac{\partial P}{\partial x}\left(R^2 - r^2\right)$$

$$u = -\frac{R^2}{4\mu}\frac{\partial P}{\partial x}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

Hence we can say velocity profile is parabolic profile.



Maximum velocity:-

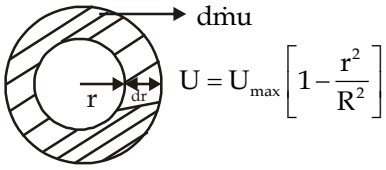
$$r = 0$$

$$U_{max} = -\frac{R^2}{4\mu}\frac{\partial P}{\partial x}$$

$$U = U_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

ZONE TECH

Mean velocity:-



$$\dot{m} = \int d\dot{m}$$

$$d\dot{m} = \rho(dA)u$$

$$= \rho(2\pi r dr)u$$

$$\dot{m} = \int_0^R d\dot{m}$$

$$\rho(\pi R^2)u_{mean} = \int_0^R \rho(2\pi r dr)u$$

$$U_{mean} = \frac{2}{R^2}\int_0^R u r dr$$

$$U_{mean} = \frac{2}{R^2}\int_0^R U_{max}\left[1-\left(\frac{r}{R}\right)^2\right] r\,dr$$

$$U_{mean} = \frac{2U_{max}}{R^2}\int_0^R \left[r-\left(\frac{r^3}{R^2}\right)\right] dr$$

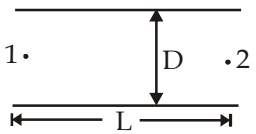
$$U_{mean} = \frac{2U_{max}}{R^2}\left[\frac{R^2}{2}-\frac{R^2}{4}\right]$$

$$U_{mean} = \frac{2U_{max}}{R^2}\left(\frac{R^2}{4}\right)$$

$$U_{mean} = \frac{U_{max}}{2}$$

$$U_{mean} = -\frac{R^2}{8\mu}\frac{\partial P}{\partial x}$$

Pressure Head Loss:-



ZONE TECH

$$U_{mean} = -\frac{R^2}{8\mu}\frac{\partial P}{\partial x}$$

$$U_{mean} = -\frac{D^2}{32\mu}\frac{\partial P}{\partial x}$$

$$\frac{32\ \mu\ U_{mean}\ dx}{D^2} = -\ dP$$

Integrate it

$$\frac{32\ \mu\ U_{mean}}{D^2}\int_0^L dx = -\int_1^2 dP$$

$$\frac{32\ \mu\ U_{mean} L}{D^2} = P_1 - P_2$$

$$\frac{P_1 - P_2}{\rho g} = \frac{32\ \mu\ U_{mean} L}{\rho g D^2} \rightarrow \text{Hagen Poiseullie's equation}$$

Darcy-weisbach & Hagen Poiseullie:-

$$h_f = \frac{32\ \mu\ U_{mean} L}{\rho D^2 g}$$

$$\frac{f L U_{mean}^2}{2gD} = \frac{32\ \mu\ U_{mean} L}{\rho g D^2}$$

$$f = \frac{64\mu}{U_{mean} D\rho}$$

$$f = \frac{64}{\frac{\rho VD}{\mu}}$$

$$f = \frac{64}{Re}$$

We know that

$$f = 4f' \Rightarrow f' = \frac{f}{4}$$

$$f' = \frac{64}{Re} \times \frac{1}{4}$$

$$f' = \frac{16}{Re}$$

where f = friction factor

f' = Coefficient of friction

So we can apply any equation Darcy Weisbach or Hagen Poiseullie equation for calculation of head loss.

ZONE TECH

Wall Shear Velocity (V*):-



$$\tau = -\frac{r}{2}\frac{\partial P}{\partial x}$$

Shear at wall

At $r = R, \quad \tau = \tau_0$

$$\tau_0 = -\frac{R}{2}\frac{\partial P}{\partial x}$$

$$-\frac{\partial P}{\partial x} = \frac{2\tau_0}{R}$$

$$-\int_1^2 dP = \frac{2\tau_0}{R}\int_0^L dx$$

$$P_1 - P_2 = \frac{2\tau_0}{R}L$$

$$\frac{P_1 - P_2}{\rho g} = \frac{2\tau_0 L}{\rho g R}$$

$$h_f = \frac{4\tau_0 L}{\rho g D}$$

$$\frac{fLV^2}{2gD} = \frac{4\tau_0 L}{\rho g D}$$

$$V^2 = \frac{8\tau_0}{\rho f}$$

$$\frac{\tau_0}{\rho} = \frac{fV^2}{8} \qquad \text{....(1)}$$

$$V^* = \sqrt{\frac{\tau_0}{\rho}} = V\sqrt{\frac{f}{8}}$$

Wall Shear Stress

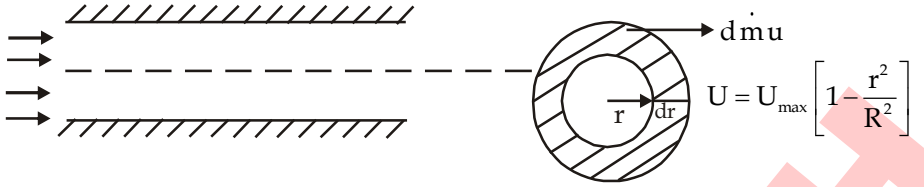
$$\frac{\tau_0}{\rho} = V^2\frac{f}{8}$$

$$\tau_0 = \frac{f\rho V^2}{8}$$

ZONE TECH

Correction Factor:-

Momentum Correction Factor (β):-It is defined as the ratio of momentum of flow per second based on actual velocity across a section to momentum of the flow per second based on average velocity across the same section.



$$d\dot{m} = \rho(2\pi r dr)u$$

$$\beta = \frac{\text{Actual momentum / sec. through the pipe cross-section}}{\text{Momentum / sec. on the basis of mean velocity through same cross-section of pipe}}$$

$$\beta = \frac{\int_0^R d\dot{m}u}{\dot{m}U_{mean}}$$

$$d\dot{m} = \rho(2\pi r dr)u$$

$$\beta = \frac{\int_0^R \rho 2\pi r dr.U_{max}^2\left[1-\frac{r^2}{R^2}\right]^2}{\rho(\pi R^2)U_{mean}^2}$$

$$\beta = \frac{2\,U_{max}^2}{R^2\,U_{mean}^2}\int_0^R\left(1-\frac{r^2}{R^2}\right)^2 rdr$$

$$\beta = \frac{2\,U_{max}^2}{R^2\,\frac{U_{max}^2}{4}}\int_0^R\left(1+\frac{r^4}{R^4}-\frac{2r^2}{R^2}\right)rdr$$

$$\beta = \frac{8}{R^2}\left[\frac{R^2}{2}+\frac{R^2}{6}-\frac{R^2}{2}\right]$$

$$\beta = \frac{8}{6} = 1.33$$

Energy Correction factor or kinetic energy Correction factor(α):-Kinetic energy correction factor is defined as the ratio of the kinetic energy of the flow per second based on actual velocity across a section to the kinetic energy of the flow per second based on average velocity across the same section.

$$\alpha = \frac{\text{Actual kinetic energy / sec. through the pipe cross-section}}{\text{kinetic energy / sec. on the basis of mean velocity through the same cross-section of pipe}}$$

$$\alpha = \frac{\int_0^R \frac{1}{2} d\dot{m} u^2}{\frac{1}{2}\dot{m} U_{mean}^2}$$

$$\alpha = \frac{\int_0^R \rho(2\pi r dr)\, U_{max}^3 \left[\left(1-\frac{r^2}{R^2}\right)\right]^3}{\rho(\pi R^2) U_{mean}^3}$$

$$\alpha = \frac{2U_{max}^3}{R^2\, U_{mean}^3} \int_0^R \left(1-\frac{r^2}{R^2}\right)^3 r dr$$

$$\alpha = \frac{2U_{max}^3}{R^2\, \frac{U_{mean}^3}{8}} \int_0^R \left(1-\frac{r^2}{R^2}\right)^3 r dr$$

We integrate it then

$$\int_0^R \left(1-\frac{r^2}{R^2}\right)^3 dr = \frac{1}{8}R^2$$

$$\alpha = \frac{2}{R^2} \times 8 \times \frac{1}{8} R^2$$

$$\alpha = 2.0$$

	α	β
Laminar	2.0	1.33
Turbulent	1.33	1.2

ZONE TECH